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Comparison of Analysis and Optimization Methods for  
Core-Megacolumn-Outrigger Skyscrapers

James B. Peterson

A thesis submitted to the faculty of  
Brigham Young University  
in partial fulfillment of the requirements for the degree of  
Master of Science

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December 2011

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## ABSTRACT

### Comparison of Analysis and Optimization Methods for Core-Megacolumn-Outrigger Skyscrapers

James B. Peterson  
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Master of Science

The goal of this research is to compare performance of three analysis methods and three optimization methods for core-megacolumn-outrigger, or CMO skyscrapers. The three analysis methods include a 1D stick analysis, 2D frame analysis, and 3D finite element analysis. The three optimization methods include a trial and error optimization, optimality criteria optimization, and genetic algorithm. Each of these methods was compared by applying an example CMO skyscraper. The 1D stick analysis proved to be the most accurate when compared with the 3D finite element results. The genetic algorithm was recommended as the best optimization method in this research. The 1D stick method in this thesis introduces a new analysis involving an outrigger modification factor. The comparison of these optimization methods for skyscrapers has not been reported in the literature.

Keywords: James B. Peterson, skyscraper, 1D stick analysis, structural optimization



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I would like to thank my wonderful wife for supporting me through my graduate degree and also my incredible parents who helped pay for my education early on here at BYU and have developed me into the man I am today. I also thank the BYU football team, for lessons on leadership and hard work. Lastly I would like to thank Amy McCall and the other students who worked with our group on this project.



# TABLE OF CONTENTS

<b>LIST OF TABLES .....</b>	<b>vii</b>
<b>LIST OF FIGURES .....</b>	<b>ix</b>
<b>1 Introduction.....</b>	<b>1</b>
1.1 History of Skyscrapers.....	1
1.1.1 First Generation .....	1
1.1.2 Second Generation .....	3
1.1.3 Third Generation.....	4
1.2 Contribution to Knowledge .....	9
<b>2 Literature Review .....</b>	<b>11</b>
2.1 1D Analysis Methods for Tall Buildings.....	11
2.2 Optimization Methods Applied to Tall Buildings .....	13
<b>3 Example CMO Skyscraper .....</b>	<b>17</b>
3.1 Geometry .....	17
3.2 Member Dimensions and Properties.....	19
3.2.1 Core.....	19
3.2.2 Megacolumns .....	20
3.2.3 Outriggers .....	20
3.2.4 Flooring System .....	21
3.3 Vertical Loads.....	21
3.4 Lateral Loads .....	26
3.4.1 Wind Load .....	26
3.4.2 Seismic Load.....	28
<b>4 Analysis Methods .....</b>	<b>31</b>



4.1	1D Stick Method.....	31
4.1.1	Story Statics .....	32
4.1.2	Story Deflections .....	32
4.1.3	Moment of Inertia and Outrigger Modification .....	33
4.1.4	Spreadsheet Implementation.....	36
4.2	2D Frame Method.....	37
4.2.1	Frame Model.....	37
4.2.2	The Core Spine .....	38
4.2.3	2D Frame Program.....	39
4.3	3D Finite Element Method .....	40
<b>5</b>	<b>Optimization Methods .....</b>	<b>45</b>
5.1	Trial and Error Method .....	45
5.2	Optimality Criteria Method .....	46
5.2.1	Kuhn-Tucker Conditions .....	46
5.2.2	Algorithm.....	47
5.3	Genetic Algorithm Method.....	48
<b>6</b>	<b>Results .....</b>	<b>53</b>
6.1	Analysis .....	53
6.2	Optimization .....	59
<b>7</b>	<b>Conclusions.....</b>	<b>63</b>
7.1	Analysis .....	63
7.2	Optimization .....	63
	<b>REFERENCES.....</b>	<b>65</b>

## LIST OF TABLES

Table 3-1: Concrete Properties .....	19
Table 3-2: Thickness Core Walls.....	19
Table 3-3: Cross Sectional Area Megacolumns.....	20
Table 3-4: Cross Sectional Areas of Outrigger Chords .....	21
Table 3-5: Steel Properties.....	21
Table 3-6: Floor Dead Load Calculation .....	22
Table 3-7: Tributary Area Calculation.....	23
Table 3-8: Combined Dead and Live Point Loads.....	24
Table 3-9: Cladding Weight Calculation .....	24
Table 3-10: Self Weight of Core and Megacolumn.....	25
Table 3-11: Vertical Point loads .....	25
Table 3-12: Exposure Coefficients .....	27
Table 3-13: Lateral Wind Forces .....	27
Table 3-14: Dead Load Weights.....	28
Table 3-15: Lateral Seismic Forces .....	30
Table 5-1: Design Variables for Trial and Error.....	45
Table 5-2: Member Area Ranges for Continuous Values.....	49
Table 5-3: Genetic Algorithm Parameters 1D Stick Model.....	52
Table 5-4: Genetic Algorithm Parameters 2D Frame Model.....	52



## LIST OF FIGURES

Figure 1-1: Empire State Building.....	2
Figure 1-2: World Trade Center .....	3
Figure 1-3: Tube in Tube Plan View WTC .....	4
Figure 1-4: Exterior Frame Tube in Tube.....	5
Figure 1-5: CMO Skyscraper Diagram.....	6
Figure 1-6: Petronas Towers.....	7
Figure 1-7: Plan View Petronas Towers .....	7
Figure 1-8: Tapei 101 diagram .....	8
Figure 3-1: Elevation and Plan Views of CMO Skyscraper .....	18
Figure 3-2: Plan View of Degenerated Skyscraper.....	18
Figure 3-3: Floor Plan Layout of Beams .....	22
Figure 3-4: Tributary Area for Core and 1 Megacolumn .....	23
Figure 4-1: Transformation of Skyscraper to Vertical Cantilever .....	32
Figure 4-2: Statics for One Story .....	33
Figure 4-3: Deflections for One Story .....	33
Figure 4-4: Plan View of Skyscraper.....	34
Figure 4-5: Outrigger Modification Factor .....	35
Figure 4-6: 2D Frame Model .....	37
Figure 4-7: Plan of 2D Frame Model.....	38
Figure 4-8: Elevation of 2D Frame Model .....	39
Figure 4-9: Plan View of Skyscraper Model in ADINA .....	40
Figure 4-10: Zoomed in View of Frame (left) and Meshed Frame (right) .....	41
Figure 4-11: Colored Member Types .....	42

Figure 4-12: Zoomed in View of Shell Elements in the Core Walls .....	43
Figure 4-13: Lateral Loads on the 3D Frame from ADINA.....	43
Figure 5-1: Typical Chromosome.....	49
Figure 6-1: Lateral Force versus Height.....	54
Figure 6-2: Safety Factor Analysis .....	54
Figure 6-3: Wind Displacement.....	55
Figure 6-4: Seismic Displacement.....	56
Figure 6-5: Wind Deflection from 3D Finite Element.....	57
Figure 6-6: Seismic Deflections from 3D Finite Element .....	58
Figure 6-7: Deflection in Core.....	59
Figure 6-8: Optimized Designs for 1D Stick Model.....	60
Figure 6-9: Optimized Designs for 2D Frame Model.....	61

# **1 Introduction**

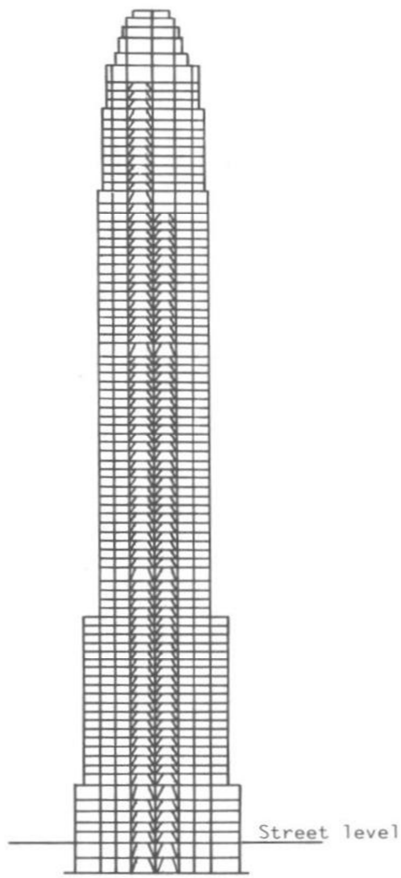
Many different analysis and optimization methods are used in skyscraper design. These methods help in minimizing deflection, maximizing height, and providing cost efficient designs for the next age of high-rise buildings. This research will look at three analysis methods along with three optimization methods for designing core-megacolumn-outrigger (CMO) skyscrapers.

## **1.1 History of Skyscrapers**

The construction of tall structures began in ancient times with the pyramids. Ancient peoples built large buildings out of stone and brick. Modern day skyscrapers have evolved with the use of steel and high strength concrete. This section will outline and discuss the progression of tall building design in order to understand how the CMO skyscraper came into existence.

### **1.1.1 First Generation**

The first generation of skyscraper is the steel braced frame. This type of skyscraper is built entirely out of steel. An example of this type of steel frame skyscraper is the Empire State Building seen in Figure 1-1.



**Figure 1-1: Empire State Building**

The Empire State Building was built in 1931 to a height of 381 meters. The sketch on the left in Figure 1-1 shows the ‘cage-like’ construction of these types of steel frame buildings. Many columns and braces are placed at each floor to stiffen the building against lateral loads. These columns and braces obstruct the space within the building. The small spaces for windows on the exterior are seen in the right picture of Figure 1-1. Floor space is limited because of the need for many members within the interior of the building. The interior columns create small pockets for offices and living space. Although this building was one of the tallest of its time, there were still strides to be made in the efficiency of tall buildings.

### 1.1.2 Second Generation

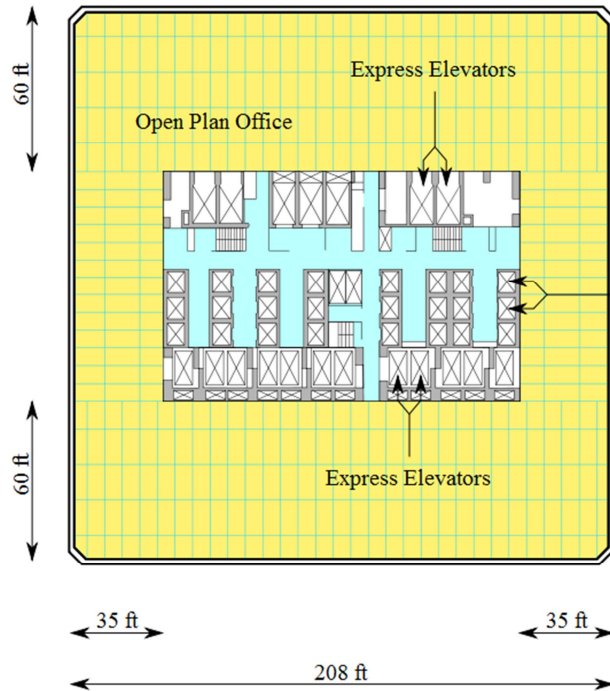
The steel framed tube is the second generation of skyscrapers. This skyscraper is also referred to as ‘tube in tube’ construction. The tube has maximum moment of inertia because it places material away from the axis of bending. An example of this type of construction is the World Trade Center.



**Figure 1-2: World Trade Center**

The World Trade Center in Figure 1-2 was built in 1970, at a height of 417 m. It was taller than the Empire State Building, and also more efficient in design. This is shown in the floor plan in Figure 1-3.





**Figure 1-3: Tube in Tube Plan View WTC**

Column free space is created between the core tube and the exterior tube. An open floor plan is created by this model of skyscraper by using long span steel floor trusses. Elevators and restrooms are allocated to the interior tube area while office and living space can be utilized between the interior and exterior tubes. However, the condition of tight exterior window space remains because of the numerous columns located on the perimeter of the exterior tube as shown in Figure 1-4.

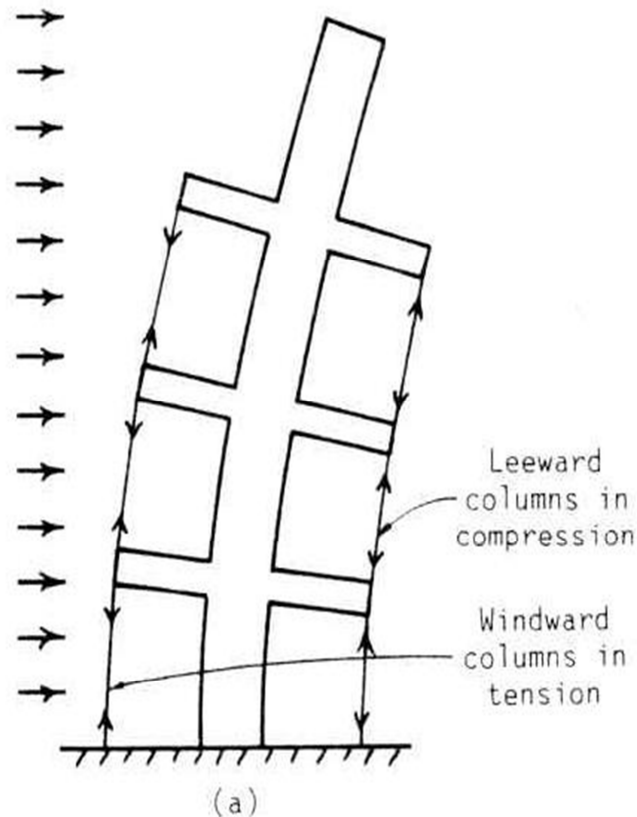
### **1.1.3 Third Generation**

The concrete core-megacolumn-outrigger, or CMO design is the third generation of skyscraper. This type of skyscraper construction has been used from 1990 to the present. This type of design was made possible through the development of high strength concrete. Mega-

columns were developed to replace the many small exterior columns in the framed tube model. This creates column free space on the exterior of the building. The large moment of inertia created by the exterior mega-columns limits the deflection of the building due to lateral loading. In order to integrate the concrete core with the megacolumns, an outrigger system is used.



**Figure 1-4: Exterior Frame Tube in Tube**



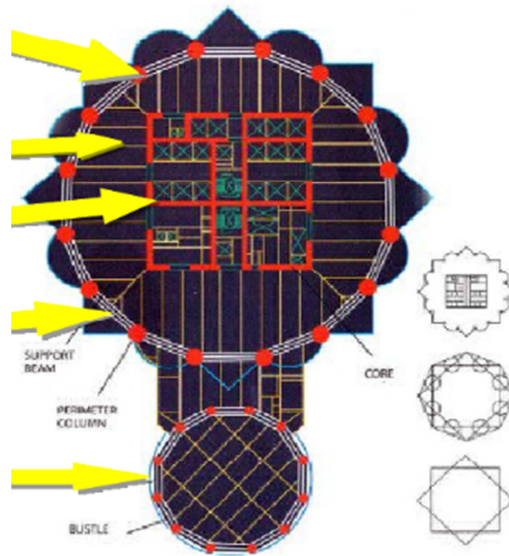
**Figure 1-5: CMO Skyscraper Diagram**

Figure 1-5 shows this integrated system of core, columns, and outriggers. The system bends under the load as one complete force resisting system integrating the full moment of inertia of the mega-columns. The interior and exterior spaces are both maximized in the skyscraper. An example of the CMO is the Petronas Towers in Figure 1-6.

The Petronas Towers were built in 1998, at a height of 452 meters. Each tower includes an interior concrete core with exterior mega-columns and connecting outriggers. The floor plan for one of the towers is seen in Figure 1-7.



**Figure 1-6: Petronas Towers**



**Figure 1-7: Plan View Petronas Towers**

The interior concrete core in this skyscraper is made up of rectangular thick core walls seen in the middle of the circular floor plan above. This shows the flexibility of design for the concrete core. The smaller circles around the perimeter represent the mega-columns. These two elements are integrated periodically about every 15-20 stories by an outrigger system up the height of the building.

Outriggers vary in design from building to building. Some include a single member extending from each mega-column to the core, while others involve a belt truss system similar to the one seen in Taipei 101 shown in Figure 1-8.

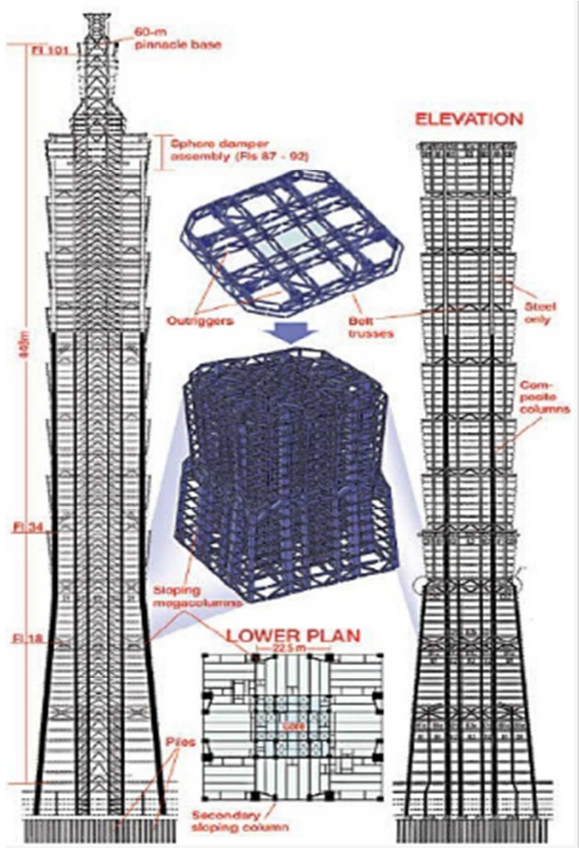


Figure 1-8: Tapei 101 diagram

The top middle drawing in Figure 1-8 shows a plan view of what the belt truss might look like within the skyscraper. All megacolumns are connected to the core through the outrigger system.

## **1.2 Contribution to Knowledge**

This goal of this research is to compare performance of three analysis methods and three optimization methods for CMO skyscrapers. The analysis methods are 1D stick, 2D frame, and 3D finite element. The 1D stick method introduces a new type of analysis involving an outrigger modification factor. The optimization methods are trial and error, optimality criteria (or gradient-based Lagrange multiplier), and genetic algorithm. The comparison of these optimization methods for skyscrapers has not been reported in the literature.

This research will be used in future research of a new urban model for the 21<sup>st</sup> century known as the Greenplex model. The Greenplex model incorporates skyscrapers, skybridges, and an envelope, with green technologies. Multiple CMO skyscrapers connected by skybridges will form a larger structure. This structure will be enclosed within an envelope to minimize surface area and maximize efficiency for heating and cooling. The building will be sustained with green technologies such as ground source heat pumps, hydronic heating and cooling, onsite waste water treatment, solar/ wind energy harvesting, rain/ grey water recycling, etc.



## **2 Literature Review**

A review of literature in the field of analysis and optimization of tall buildings will be discussed in this chapter. Since frame analysis and finite element analysis are well understood, only 1D analysis methods for tall buildings will be reviewed. Only optimality criteria and genetic algorithm applications to tall buildings will be reviewed.

### **2.1 1D Analysis Methods for Tall Buildings**

In performing a 1D stick analysis (Zalka 2002) created an equivalent column by relating stiffness of individual elements to deformations. This analysis uses the axial, shear, and bending deformations of each element in order to find the global deformation of the system. The difference in this reference is to calculate combined stiffness of systems of frameworks. This means frameworks containing a combination of columns, walls, and coupled shear walls. The 1D method in this reference was compared for accuracy with an example problem using 3D finite element software AXIS VM. The formulas and method for the 1D analysis had an average absolute error of 6%. The analysis in this paper is similar in finding combined equivalent stiffness while the procedure in the reference uses more complicated formulas for individual elements.

Potzta, G., Kollar, L. P., (2003) use what are called sandwich or replacement beams and apply them to examples of plane and flexural-torsional buckling. The sandwich beam studies the



case of multiple lateral resisting systems connected horizontally along the height of a building. This involves a local bending stiffness value not found in other methods. The error of the sandwich beam was less than 1% when compared with the exact finite element solution. Kaviani, P., Rahgozar, R., Saffari, H., (2008) used this same method for members with variation in cross section to calculate natural period of multi-storey buildings. Comparable results were found. The method in this reference finds a 1D beam for comparison but uses a different method for finding its stiffness.

The continuum method, or transforming a structure into a continuous beam (Zalka, K.A., 2000) and the Timoshenko-beam method (Timoshenko, S.P., Gere, J.M., 1961) both had less accuracy than the sandwich beam research posed above. Both of these methods in essence calculate the global bending stiffness of the structure and transform into a single beam for analysis. In each of these cases an example problem is given in which much smaller frames than a full height skyscraper are analyzed.

A one dimensional finite element method was developed by Bozdogan, K. B., (2011). The purpose was to integrate the step changes of properties along the height of the building. The 1D method in this reference proved to increase its accuracy with the increase of height of the building. Similar to this thesis, the research in this reference provides a decrease in member size along the height of the building.

Analyzing an approximate method to matrix stiffness methods of rigid frames Smith, B. S., Kuster, and M., Hoenderkamp, J. C. D., (1984) shows a comparison of 1D and 2D analysis. The method was found by tests on multiple structures and gave reasonably accurate results within 10% of computer stiffness solutions.

O. A. Pekau, L. Lin and Z. A. Zielinski (1996) performed a 1D analysis on a tube in tube type skyscraper. The interior tube is modeled as a thin-walled beam element at each story. This finite story method is then used which is explained in their previous research (O. A. Pekau, Z. A. Zielinski and L. Lin 1995). This method is based on “nodal displacement fields obtained from two-story substructures and intended to approximate shear, bending and torsion components of global deformations.” The analysis compares with the 1D analysis in this research for finding an approximate stiffness at each story.

Chajes, M. J., Zhang, L., and Kirby, J. T., (1996) uses a continuum model to analyze the dynamic response of a 47-storey building. Results from the 1D model proved to be within 8% of the measured values from an actual earthquake. This shows the application of the 1D model to various types of structural response. Again, the method is comparable to the method in this thesis in finding the equivalent stiffness for a continuous beam.

The references reviewed above did not mention the involvement of belt/ outrigger trusses; in order to address this type of analysis the following was reviewed. Rahgozar, R., Ahmadi, A. R., Sharifi, Y. (2009) analyze the use of belt trusses within a skyscraper. This research reveals the effectiveness of different types of outrigger systems in resisting loads within a building. For simplification purposes the outrigger system assumed in this thesis is a group of two story trusses connecting each mega-column to core.

## **2.2 Optimization Methods Applied to Tall Buildings**

The IFC 2 Tower in Hong Kong was optimized with an algorithm that uses optimality criteria method, and compared results with those from the trial and error method (C-M. Chan, 2001). The minimum cost design problem was subject to drift constraints due to lateral wind

loads. The optimality criteria method proved to provide a cheaper, more efficient building. The study also suggests that the optimal design would not have been easily achieved without the use of the algorithm. This building is an example of a CMO skyscraper.

An example of a tall steel frame building was optimized by the optimality criteria method and compared with the trial and error method (C-M. Chan, 1995). The algorithm is automated to use discrete values taken from commercially manufactured steel shapes in order to minimize the weight of the 50-story asymmetrical steel frame building. The effectiveness again proves to be much better than the trial and error method for large buildings.

Another example for the optimality criteria method is developed for a 60-story, 7 bay framework (C-M. Chan, 1992). Chan assigns discrete optimal section sizes to the structural members and again illustrates the efficiency of such methods for optimization design.

The optimality criteria method was used to design two large frameworks, one made up of 30 members and the other of 105 members (Tabak EI, Wright PM. 1981). Size, stress, and displacement constraints are placed on the designs. The optimal criteria are created based on the Kuhn-Tucker conditions and stress gradients are included in the optimization for the rigid plane frames.

Another optimality criteria method is the drift control method which uses displacement participation factors integrated into an optimization problem in order to determine the amount of material to be modified (Park, H. S., Park, C. L., 1997). This reference evaluates how each section of the building influences how much the buildings will deflect. By recognizing the most influential parts, deflection can be minimized and the building is optimized. The drift of the building is evaluated against an initial design. The purpose is to show the effectiveness of the design method against a time consuming trial and error method.

A series of optimizations were performed on a 3 bay, 26-storey tall building using a genetic algorithm (K. Murawski, T. Arciszewski, K. De Jong, 2000). The cross-over mutation rates were analyzed for convergence to the optimal solution. Dead, live, and wind loads were incorporated into the design process. The algorithm in each case proved to provide sufficient optimal solutions. A comparison was also given for total weight versus the number of generations developed. The feasibility of evolutionary computation in structural design is proven.

The Burj Kalifa had many of its components designed by an optimality criterion method (Baker et al. 2009). Baker comments that “Wall and column sizes were optimized using virtual work / LaGrange multiplier methods, resulting in a very efficient structure.” The use of such methods for the tallest building in the world shows the effectiveness of such optimization methods on large structural design problems.

The interior service core has become a common practice for designing tall buildings. ‘In general the more time spent on the core design, the more efficient and sustainable the building can be’ (Ali and Armstrong, 2008). This reference merely discusses that optimizing the core area can provide the best results in resisting loads within a tall building. No example problem or specific method was used.

Jr WU and QS LI, (2004) performed an optimization and analysis of the CMO skyscraper. The optimization however was only applied to spacing of outriggers within the structure and not to the optimization of megacolumn and core.



### **3 Example CMO Skyscraper**

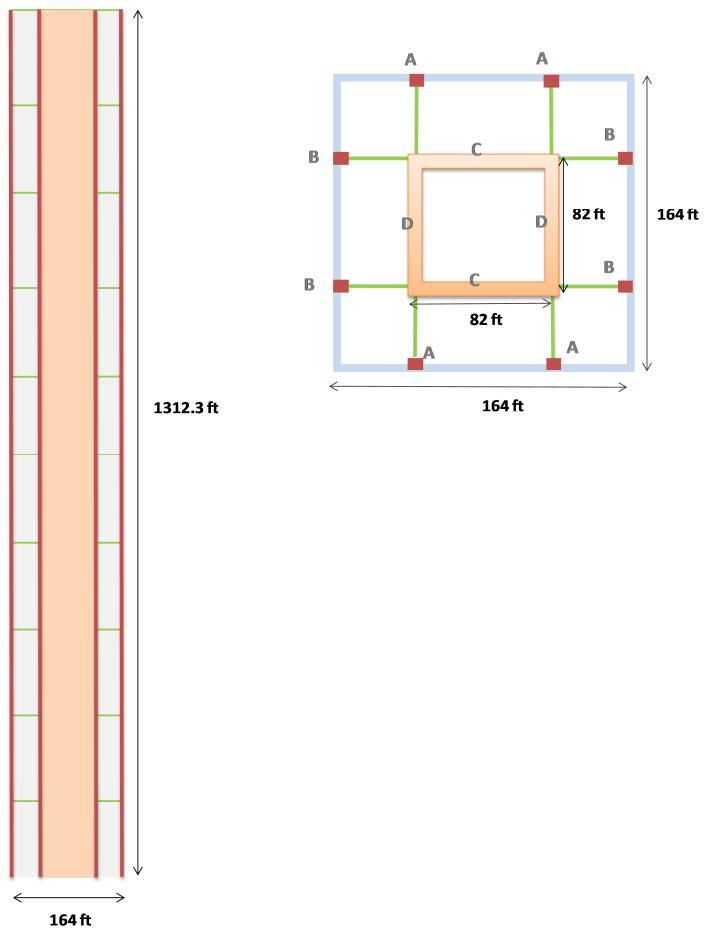
The example CMO skyscraper used in the comparison of analysis and optimization methods in this thesis will now be described in detail.

#### **3.1 Geometry**

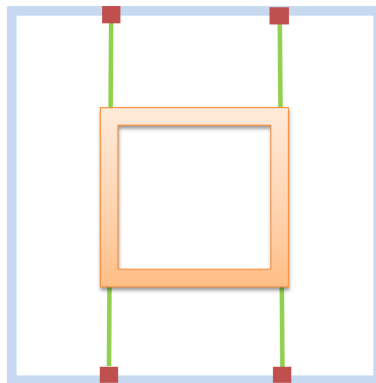
The plan and elevation views are shown in Figure 3-1. It has a width of 164 feet, and an interior core of 82 feet. The height of the building is 1312.3 feet with 90 stories. Each story is 14.6 feet tall.

The eight mega-columns can be seen in the plan view in Figure 3-1, two on each side of the square building. The outriggers are those members spanning from the eight megacolumns to the interior square core at intervals of 10 stories.

The 2D frame analysis has limitations for modeling the out of plane megacolumns. For simplification purposes, the model must be degenerated to the plan view seen in Figure 3-2. This model is used for all of the analysis and optimization methods for comparison purposes. In addition, the vertical loads from the flooring system applicable to the excluded columns will be neglected.



**Figure 3-1: Elevation and Plan Views of CMO Skyscraper**



**Figure 3-2: Plan View of Degenerated Skyscraper**

## 3.2 Member Dimensions and Properties

### 3.2.1 Core

The interior square core shown in the plan view of Figure 3-1 consists of high strength concrete. The concrete properties are shown in Table 3-1. The assumed cost of concrete used in this research for the cost of structure calculation is \$4.50/ft<sup>3</sup>. This calculation is described in Chapter 6.

**Table 3-1: Concrete Properties**

strength	11.6	ksi
modulus	6300	ksi
density	0.138	kcf

The core thicknesses used for analysis are shown in Table 3-2. Core thickness decreases moving up the height of the building.

**Table 3-2: Thickness Core Walls**

stories	ft
1 to 10	7.2
11 to 20	7.0
21 to 30	5.9
31 to 40	4.7
41 to 50	3.6
51 to 60	2.6
61 to 70	1.6
71 to 80	1.0
81 to 90	1.0



### 3.2.2 Megacolumnns

The exterior columns consist of the same high strength concrete used in the core. The concrete properties can be seen in Table 3-1. Megacolumnns decrease in cross sectional area moving up the building. These cross sectional areas are listed in Table 3-3.

**Table 3-3: Cross Sectional Area Megacolumnns**

stories	ft <sup>2</sup>
1 to 10	8.7
11 to 20	7.6
21 to 30	6.5
31 to 40	5.4
41 to 50	4.3
51 to 60	4.3
61 to 70	4.3
71 to 80	3.3
81 to 90	3.3

### 3.2.3 Outriggers

The outriggers connect to the interior core and mega-columns every 10 floors. The outriggers are steel trusses with a depth equal to one story. Calculation of the outrigger area and moment of inertia is dominated by the contribution from the truss chords. The areas for the outrigger chords are displayed in Table 3-4, representing the total cross sectional area of the top and bottom chords in the outrigger truss system. The shear stiffness for the outrigger chords will not be modeled. The connectivity of the outrigger in each model will be discussed in Chapter 4. The steel properties are shown in Table 3-5. The cost of steel used for the cost of structure calculation is \$244.20/ft<sup>3</sup>. This calculation is discussed in Chapter 6.

**Table 3-4: Cross Sectional Areas of Outrigger Chords**

stories	ft <sup>2</sup>
1 to 10	1.7
11 to 20	1.7
21 to 30	1.7
31 to 40	1.6
41 to 50	1.4
51 to 60	1.3
61 to 70	1.1
71 to 80	1.1
81 to 90	1.1

**Table 3-5: Steel Properties**

strength	50	ksi
modulus	29000	ksi
density	0.49	kcf

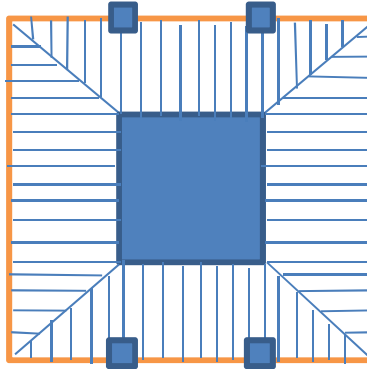
### **3.2.4 Flooring System**

The flooring system at each story consists of slabs and beams. It is assumed that the flooring system is simply supported by the core and megacolumns. This means that shear forces due to gravity loads are transmitted to the core and megacolumns, but moments are not. Furthermore, it is assumed that the flooring system has infinite axial stiffness so that lateral loads at each story are transmitted to the core and megacolumns.

### **3.3 Vertical Loads**

The flooring system for each story was designed according to Figure 3-3. This floor plan layout is similar to others designed for CMO skyscrapers. The three types of beams designed include the diagonal beams extending to each of the four corners; the exterior beams around the

perimeter and the shorter cross span beams perpendicular to the exterior beams. The dead load calculation is shown in Table 3-6.



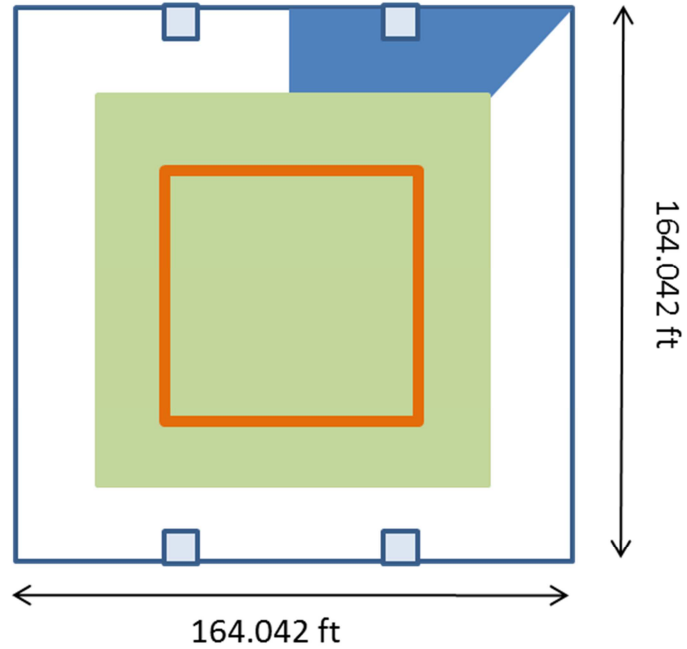
**Figure 3-3: Floor Plan Layout of Beams**

**Table 3-6: Floor Dead Load Calculation**

Slab plus deck	72.5	psf
Steel Beam Framing	16.18	psf
Mechanical	4	psf
Ceiling	6	psf
Partition	20	psf
<b>Total Floor DL=</b>	<b>118.68</b>	psf

The total distributed weight of the steel beams in the floor plan is 16.18 psf. The largest portion of the dead load is the slab plus metal deck. This 72.5 psf dead load assumes that the deck is 7.5 inches.

Figure 3-4 shows the tributary area for one megacolumn in the darkened upper right hand corner. The tributary area for the core is shown as the dark square in the middle of the plan view extending out half way between each core wall and the megacolumns on the exterior.



**Figure 3-4: Tributary Area for Core and 1 Megacolumn**

Values for tributary areas are listed in Table 3-7. The total tributary area should add up to the total floor plan area of the building. This calculation check is performed in the last row of Table 3-7. The total is the summation of eight megacolumn tributary areas and the core tributary area. The total value of 26909.74 ft<sup>2</sup> equals the total floor plan area, or 164.042 ft. squared.

**Table 3-7: Tributary Area Calculation**

	Tributary area	
megacolumn	1576.74	ft <sup>2</sup>
core	14295.82	ft <sup>2</sup>
<b>Total</b>	<b>26909.74</b>	ft <sup>2</sup>

The floor live load is taken as an assumed value of 100 psf. The combined dead and live load is listed in the first row of Table 3-8. This value is multiplied by either megacolumn or core tributary area to get the vertical point loads on the megacolumns and core.

**Table 3-8: Combined Dead and Live Point Loads**

	Loads/ Floor	
total	0.21868	ksf
megacolumn	344.8	k
core	3126.22	k

Point loads are placed at each story along the height of the megacolumns and core representing this dead and live load from the floor. The weight of exterior cladding on the building also is added to the point loads on the megacolumns along with self-weight of the megacolumn. The exterior tributary area for the megacolumn is used to calculate this cladding weight for each floor. This calculation is shown in Table 3-9 by multiplying tributary width by height of half floor above and below.

**Table 3-9: Cladding Weight Calculation**

cladding	0.0267	ksf
height	14.58151	ft
width	82.021	ft
weight	31.93293	k

Self-weight of the core and megacolumns are calculated by multiplying respective cross sectional areas by the story height and density of concrete given in Table 3-1. The cross sectional areas for the core are taken from the thicknesses in Table 3-2 multiplied by the

perimeter of the core wall. The cross sectional areas for the megacolumns are taken from Table 3-3. The values for self-weight are shown in Table 3-10.

**Table 3-10: Self Weight of Core and Megacolumn**

stories	col wt (k)	core (k)
1 to 10	17.43	4341.52
11 to 20	15.32	4202.25
21 to 30	13.13	3618.02
31 to 40	10.93	2912.80
41 to 50	8.74	2271.75
51 to 60	8.74	1655.89
61 to 70	8.74	1051.55
71 to 80	6.56	643.63
81 to 90	6.54	643.63

The total vertical point loads for each megacolumn and the core are shown in Table 3-11. These values represent the addition of dead, live, cladding and self-weight loads. The last row in the table represents a smaller roof load applied to the top floor.

**Table 3-11: Vertical Point loads**

levels	Core (k)	Megcol (k)
1 to 10	7467.75	394.16
11 to 20	7328.48	392.06
21 to 30	6744.25	389.86
31 to 40	6039.02	387.67
41 to 50	5397.98	385.48
51 to 60	4782.11	385.48
61 to 70	4177.77	385.48
71 to 80	3769.86	383.30
81 to 89	3769.86	383.28
90	3126.22	376.74

The self-weight of outriggers at respective floors are calculated by taking the cross sectional area of the outrigger chords given in Table 3-4 multiplied by their length and the density of steel given in Table 3-5. These loads are split between the core and megacolumns where the outriggers are connected.

### 3.4 Lateral Loads

#### 3.4.1 Wind Load

Wind load is developed according to the following equations and (ASCE 7-10).

Considering Bernoulli's Equation:

$$\frac{v^2}{2g} + \frac{p}{\gamma} + y = \text{constant} \quad (3-1)$$

where

- state 1: wind before it hits building,  $p = 0$
- state 2: air after it hits building,  $v = 0$
- Both states have same elevation  $y$

This equation degenerates to the following:

$$p = \frac{\gamma}{2g} v^2 = 0.00256v^2 \quad (3-2)$$

where

- $\gamma = 0.0766 \text{ lb/ft}^3 = \text{density of air}$
- $g = 32.2 \text{ ft/s}^2$
- $p = \text{pressure in psf}$
- $v = \text{velocity in mph}$

Applying the following equation to calculate the exposure coefficient:

$$K_z = 2.01 \left( \frac{h}{h_g} \right)^{2/\alpha} \quad (3-3)$$

where

$K_z$  = velocity pressure exposure coefficient evaluated at height  $z$

$h$  = height

The coefficients  $\alpha$  and  $h_g$  taken from Table 3-12:

**Table 3-12: Exposure Coefficients**

Exposure	$\alpha$	$h_g$ (ft)
Closely spaced obstructions	7	1200
Scattered obstructions	9.5	900
Open terrain, unobstructed	11.5	700

$$p = 0.00256K_z v^2 \quad (3-4)$$

The design wind speed is taken as 123 mph with an alpha value of 9.5 and an  $h_g$  value of 900 ft. The lateral wind forces are shown for some stories in Table 3-13. For the wind analysis, each force is placed on the exterior columns and then transferred to the core by the infinitely stiff floor members. More detail on this load transfer is described in Chapter 4.

**Table 3-13: Lateral Wind Forces**

story	wind force (kips)
1	78.20236
2	90.48875
3	98.55223
4	104.7055
5	109.7416
6	114.0358
7	117.7973
8	121.1558
9	124.1975
10	126.9832
81	197.2451
82	197.7553
83	198.2605
84	198.7611
85	199.2569
86	199.7481
87	200.2349
88	200.7172
89	201.1953
90	201.6691



### 3.4.2 Seismic Load

Calculations of loads from seismic forces are developed in a spreadsheet according to the equivalent lateral force method (ASCE 7-05). In this method the seismic weight at each story is the dead load only, shown for some stories in Table 3-14.

**Table 3-14: Dead Load Weights**

story	dead load (kips)
1	6816.7407
2	6816.7407
3	6816.7407
4	6816.7407
5	6816.7407
6	6816.7407
7	6816.7407
8	6816.7407
9	6816.7407
10	7093.7054
81	2612.0245
82	2612.0245
83	2612.0245
84	2612.0245
85	2612.0245
86	2612.0245
87	2612.0245
88	2612.0245
89	2612.0245
90	2785.1481

The seismic base shear,  $V$ , in a given direction is determined using the following equations:

$$V = C_s W \quad (3-5)$$

where

$C_s$  = the seismic response coefficient determined in accordance with ASCE Section 9.5.5.2.1

$W$  = the total dead load and applicable portions of other loads as indicated in Section 9.5.3.

Vertical distribution of the base shear is determined by:

$$F_x = C_{vx}V \quad (3-6)$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^n w_i h_i^k} \quad (3-7)$$

where

$F_x$  = lateral force

$C_{vx}$  = vertical distribution factor

$V$  = total design lateral force or base shear

$w_i, w_x$  = the portion of the total gravity load of the structure located or assigned to level  $i$  or  $x$ .

$h_i, h_x$  = the height from the base to Level  $i$  or  $x$

$k$  = an exponent related to the structure period as follows:

for structures having a period of 0.5 sec or less,  $k = 1$  (linear)

for structures having a period of 2.5 sec or more,  $k = 2$  (parabolic)

for structures having a period between 0.5 sec and 2.5 sec.,  $k$  shall be 2 or shall be determined by linear interpolation between 1 and 2

The  $k$  exponent was taken as 2 for a long period building. The value for  $C_s$  was calculated using the equation below. The design acceleration is assumed to be 0.2g, understanding this is an assumption for a long period building. This was done for simplification purposes in order to obtain a second load case for the analysis. The ductility factor is taken as 6 and the importance factor taken as 1. The lateral seismic forces for some stories are shown in Table 3-15.

$$C_s = \frac{SDS}{\left(\frac{R}{I}\right)} \quad (3-8)$$

where

$SDS$  = the design spectral response acceleration parameter in the short period range as determined from Section 11.4.4

$R$  = the response modification factor taken from table in Section 12.2

$I$  = the occupancy importance factor determined in accordance with Section 11.5.1

For the seismic analysis the lateral loads are placed at each story on the exterior columns and then transferred to the core by the infinitely stiff floors members. More detail on this load transfer is described in Chapter 4.

**Table 3-15: Lateral Seismic Forces**

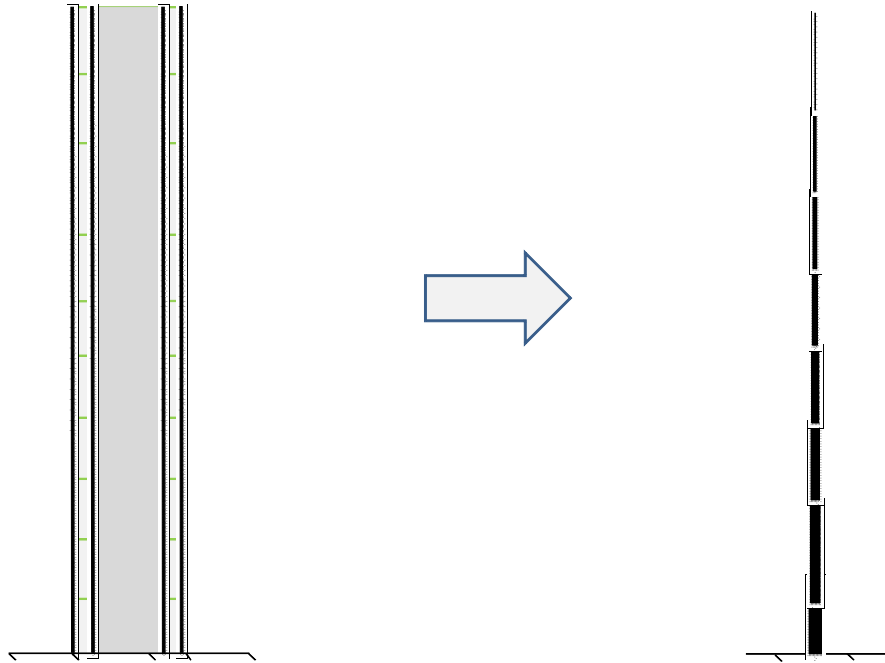
story	seis force (kips)
1	0.12
2	0.46
3	1.04
4	1.85
5	2.89
6	4.17
7	5.67
8	7.41
9	9.38
10	12.05
81	291.11
82	298.34
83	305.67
84	313.07
85	320.57
86	328.16
87	335.84
88	343.60
89	351.46
90	383.22

## **4 Analysis Methods**

The three analysis methods in this research were chosen in order to give models in one, two, and three dimensions. Intuitively, the accuracy would increase with each increase in dimension. This chapter will review how each method is developed for the example CMO skyscraper described in Chapter 3.

### **4.1 1D Stick Method**

This method is the simplest way for determining the structural response of a tall building under loading. The skyscraper is transformed into a single vertical cantilever member whose section properties vary with height. A tall slender building like a skyscraper will deflect under horizontal loading similarly to the way wheat in a field might bend in the wind. By superimposing a three dimensional building into a thin slender member, the calculations are simplified to one dimension for determining the deflection of this single cantilevered member under a load. The skyscraper is shown in the left in Figure 4-1. There is an interior core, vertical mega-columns, and horizontal outriggers connecting the core and mega-columns. The skyscraper is transformed to the vertical cantilever shown on the right in Figure 4-1. This simplification allows the analysis of the vertical cantilever to be performed in a spreadsheet.



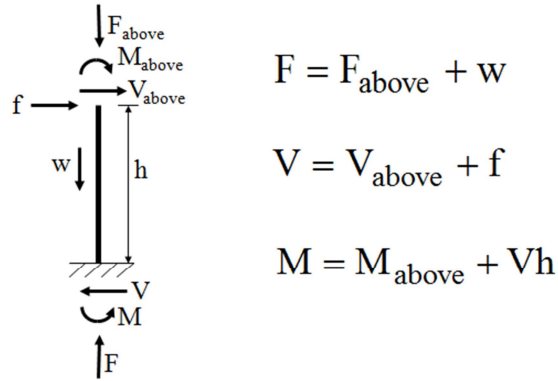
**Figure 4-1: Transformation of Skyscraper to Vertical Cantilever**

#### **4.1.1 Story Statics**

Figure 4-2 shows the equations for axial force, shear force, and moment equations at each individual story. These are calculated from the top story to the bottom story in the spreadsheet.

#### **4.1.2 Story Deflections**

The flexural deflections and rotations are shown in Figure 4-3 for a single story. Shear deformation is insignificant in tall slender skyscrapers and is not modeled in this method. These are calculated from the bottom story to the top story in the spreadsheet.

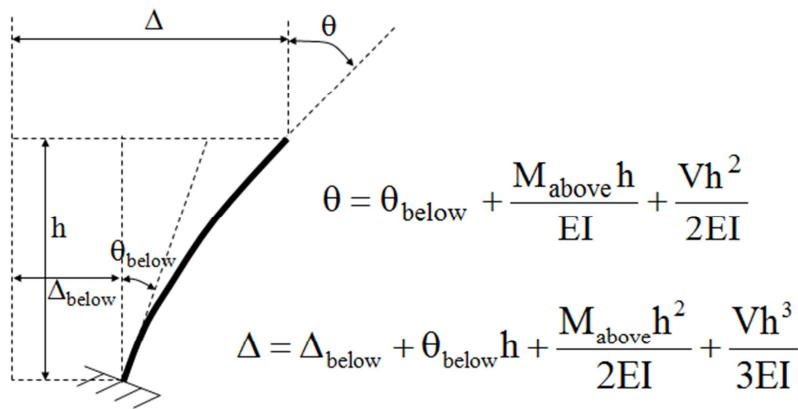


$$F = F_{\text{above}} + w$$

$$V = V_{\text{above}} + f$$

$$M = M_{\text{above}} + Vh$$

**Figure 4-2: Statics for One Story**



$$\theta = \theta_{\text{below}} + \frac{M_{\text{above}} h}{EI} + \frac{Vh^2}{2EI}$$

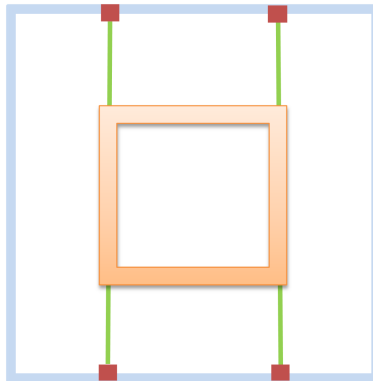
$$\Delta = \Delta_{\text{below}} + \theta_{\text{below}} h + \frac{M_{\text{above}} h^2}{2EI} + \frac{Vh^3}{3EI}$$

**Figure 4-3: Deflections for One Story**

### 4.1.3 Moment of Inertia and Outrigger Modification

In order to transform the three dimensional skyscraper into a one dimensional cantilever, an equivalent moment of inertia needs to be calculated. The cross sectional area of the megacolumns and core will decrease vertically up the building. A moment of inertia is calculated for each story based on this variation in cross sectional area.

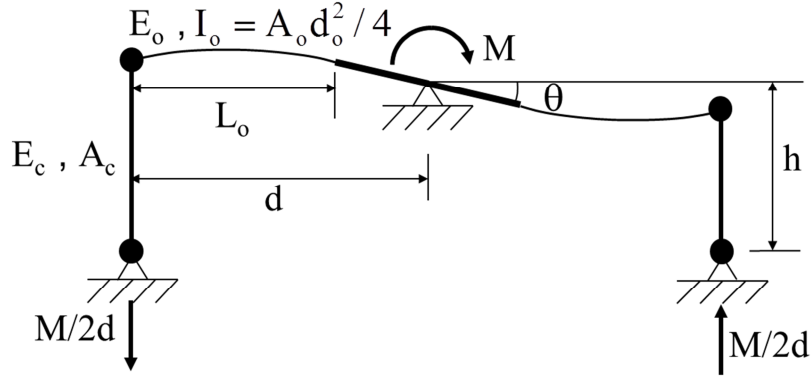
Applying the parallel axis theorem to a plan view of the skyscraper, the equivalent moment of inertia can be calculated. The plan view is reproduced in Figure 4-4 for the skyscraper. The elements that are used in the moment of inertia calculation are the megacolumns and core walls.



**Figure 4-4: Plan View of Skyscraper**

The megacolumn moment of inertia is calculated separately in the spreadsheet from the core walls. Mega-columns have an additional factor in the calculation of their moments of inertia. This factor is called the outrigger modification factor. This factor takes into account the outrigger and megacolumn interaction when the building deflects. The modification factor,  $\gamma$  is developed from Figure 4-5.

In Figure 4-5,  $h$  is the vertical distance between outriggers (ten stories in the example CMO skyscraper).  $A_c$  is the cross sectional area of one megacolumn,  $A_o$  is the cross sectional area of both outrigger chords,  $d_o$  is the outrigger truss depth (one story in the example CMO skyscraper), and  $d$  is the distance from the center of the core to the megacolumn.  $E_c$  is the modulus of elasticity of concrete, and  $E_o$  is the modulus of elasticity of steel.



**Figure 4-5: Outrigger Modification Factor**

Equation 4-1 is a compatibility equation derived from Figure 4-5 that equates the axial deformation in the megacolumn to the vertical deflection in the outrigger. This is used to get the relation between moment  $M$  and rotation  $\Theta$  shown in Equation 4-2.

$$\frac{M}{2d} \left( \frac{h}{E_c A_c} \right) = \theta d - \frac{M}{2d} \left( \frac{L_o^3}{3E_o I_o} \right) \quad (4-1)$$

$$\theta = \frac{Mh}{E_c I} \quad (4-2)$$

Equation 4-3 gives the equivalent moment of inertia  $I$  that accounts for stiffnesses of megacolumns, core, and outriggers.  $I_{core}$  is the moment of inertia of the core and  $I_o$  is the moment of inertia of the outrigger.

$$I = I_{core} + 2A_c d^2 \left( \frac{1}{1+\gamma} \right) \quad (4-3)$$

The outrigger modification factor  $\gamma$  seen in Equation 4-3 is defined in Equation 4-4. Equation 4-4 is derived from Equations 4-1 and 4-2 and from Figure 4-5.

$$\gamma = \frac{L_o^3 E_c A_c}{3h E_o I_o} = \frac{4L_o^3 E_c A_c}{3h d_o^2 E_o A_o} \quad (4-4)$$



Note that as outrigger stiffness ( $E_o I_o$ ) increases,  $\gamma$  goes to zero, and full megacolumn area contributes to I. As outrigger stiffness decreases,  $\gamma$  goes to infinity, and megacolumn area does not contribute at all to I.

#### 4.1.4 Spreadsheet Implementation

The stick analysis is performed using a spreadsheet with each row representing a story. The drift and rotation is calculated at each story under wind and seismic loads. The deflections are then summed down the column for a total deflection.

Safety factors are calculated for the skyscraper. The safety factors for wind and seismic stress, buckling, along with wind and seismic drift represent the analysis portion of the stick analysis. Each safety factor is graphed versus relative height to determine if the building's design is sufficient and which safety factor is governing. The safety factor equations are shown below for stress, buckling, and drift. The moment M in the first equation represents either the moment due to wind or seismic loads. The  $\Delta_{inter-story}$  in the third equation for drift, represents inter-story drift due to wind or seismic loads.

$$SF_{stress} = \frac{F_y}{\left(\frac{F_{axial}}{A}\right) + \left(\frac{My}{I}\right)} \quad (4-5)$$

$$SF_{buckling} = \frac{\pi^2 EI}{4(H_{total} - H_{relative})^2 F_{axial}} \quad (4-6)$$

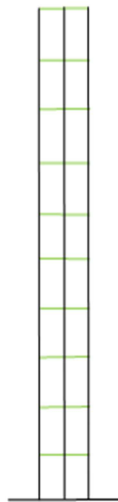
$$SF_{drift} = \frac{H_{story}}{\Delta_{allowable} \Delta_{inter-story}} \quad (4-7)$$

## 4.2 2D Frame Method

In this analysis method the skyscraper is modeled as a two dimensional frame. Horizontal outrigger members and floor members are now included in this model. This method models the interior square core as a single frame element.

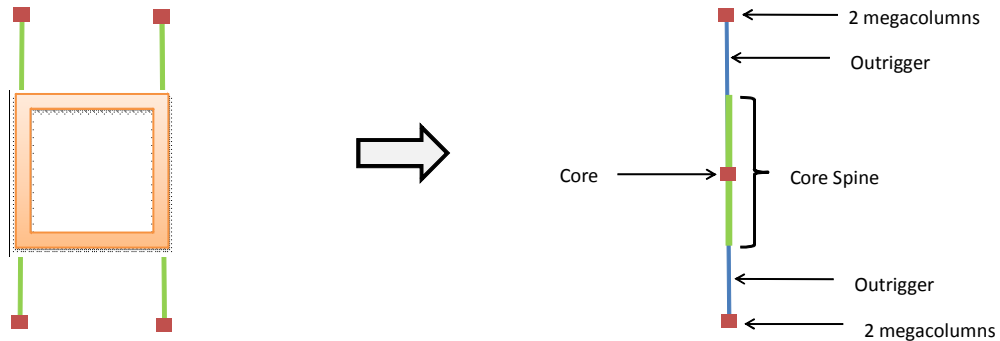
### 4.2.1 Frame Model

The frame in two dimensions is shown in Figure 4-6. A single vertical element in the center models the box core. The exterior vertical elements, one on each side of the core model two exterior columns each. Horizontal elements model the outriggers. Although the horizontal floor elements are not shown in between the outriggers in Figure 4-6, one is placed at each story in between those levels where outriggers are present. Each frame element has 6 degrees of freedom with member end release capability to create hinged connections.



**Figure 4-6: 2D Frame Model**

The plan view is shown in the left of Figure 4-7. Note that only four megacolumns are included in this model. The 2D frame model is shown on the right in Figure 4-7.



**Figure 4-7: Plan of 2D Frame Model**

#### 4.2.2 The Core Spine

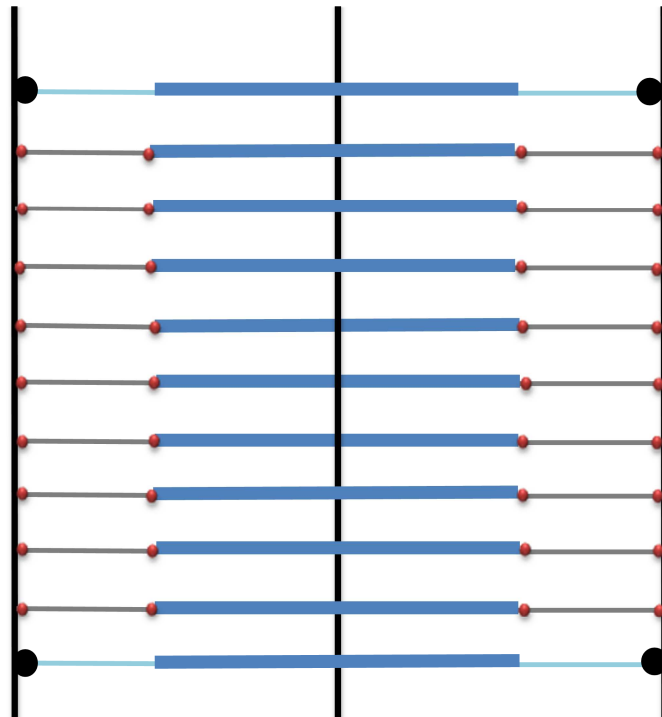
Note that in Figure 4-7, the megacolumns are connected to the core with an outrigger element and a core spine element in series. The length of the core spine element is half the width of the core. It has infinite flexural and axial stiffness ( $I=A=\infty$ ). It is rigidly connected to the core and to the outrigger, and the outrigger is hinge connected to the megacolumn. The core spine element models the finite width of the core. An elevation view is shown in Figure 4-8.

Figure 4-8 shows the floor elements at each story. These elements have infinite axial stiffness ( $A=\infty$ ) and zero flexural stiffness ( $I=0$ ). Shear deformation in the outriggers is not taken into account in this 2D model.

### 4.2.3 2D Frame Program

Each element stiffness matrix and force vector contributes to the formation of system stiffness matrix  $K$  and force vector  $F$ . The equation  $KU = F$  is then solved for the system displacement vector  $U$ . The system displacements are used to find individual element displacements and stresses.

The input file for the linear plane frame analysis program includes joint coordinates, connectivity of joints, loads, and element section properties. Numbering of the joints and members is kept in a spreadsheet in order to organize member groups and connectivity of members within the building.

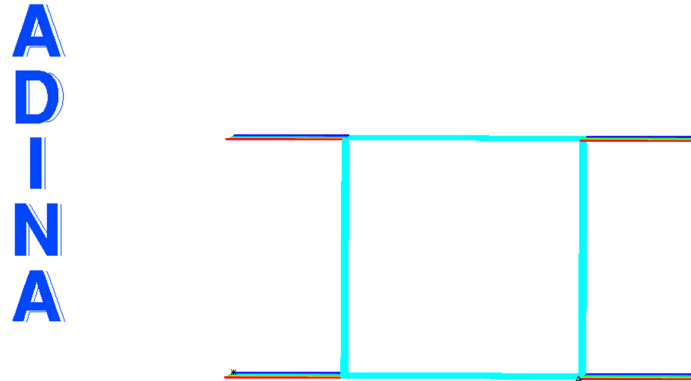


**Figure 4-8: Elevation of 2D Frame Model**

The section table for the input file includes the cross sectional area, modulus, moment of inertia, connectivity (pinned or fixed), and safety factors for buckling and stress for each member. These sections for each member are the same as those used in 1D stick analysis. The gravity point loads are placed at each joint and the lateral loads of wind and seismic are analyzed separately. The lateral loads placed at each story on the megacolumns are transferred to the core through the floor member and infinite core spine member.

### 4.3 3D Finite Element Method

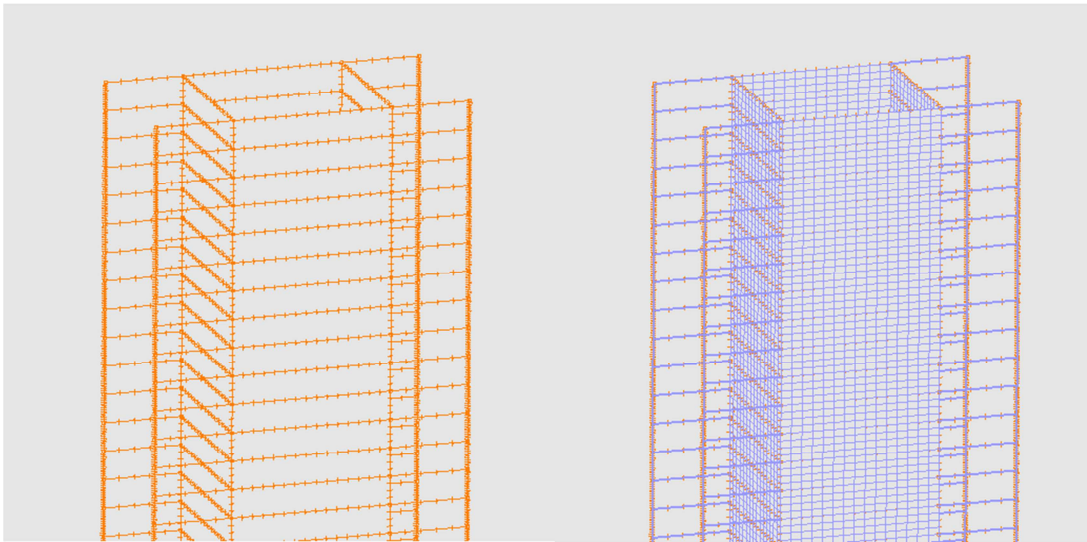
The final linear analysis procedure uses ADINA, a commercial finite element software package. The model of the skyscraper in plan view is shown in Figure 4-9, exported from ADINA.



**Figure 4-9: Plan View of Skyscraper Model in ADINA**

The main difference between 2D frame analysis and 3D finite element analysis is that core walls are modeled with shell elements. This also allows for some shear deformation which was neglected in the previous models. The megacolumns, outriggers, and axial stiff floor

members are all modeled in the same way as the 2D frame analysis, as frame elements. These members each have 12 degrees of freedom. The core spine no longer needs to be included for the 3D analysis. The megacolumns consist of 5388 elements. The outriggers consist of 180 elements. The axial stiff floors consist of 1620 elements. Cross sectional areas were defined for the changing megacolumn areas moving vertically up the building as defined in Chapter 3. Cross sectional areas were also defined for the outriggers in Chapter 3. The concrete core walls were modeled as shell elements. The concrete core consists of 21,600 4-node quad shell elements. The finite element model can be seen in Figure 4-10. In Figure 4-11 each member type is represented by color. Figure 4-12 shows the close up view of the 4-node quad shell elements of the concrete core.

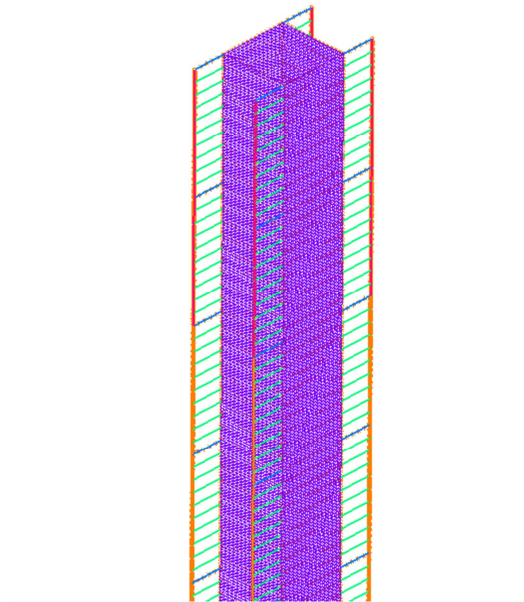


**Figure 4-10: Zoomed in View of Frame (left) and Meshed Frame (right)**

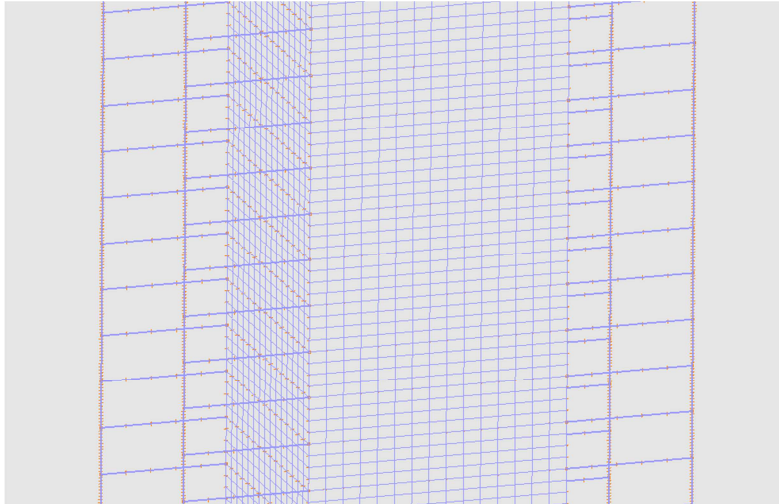
Horizontal loads placed on the building at each floor were the same as in the previous two analysis methods. The model is now three dimensional, requiring the loads to be split in half

at each floor. This can be seen in the Figure 4-13. This method provides a tributary area type distribution of the loads to the two mega-columns.

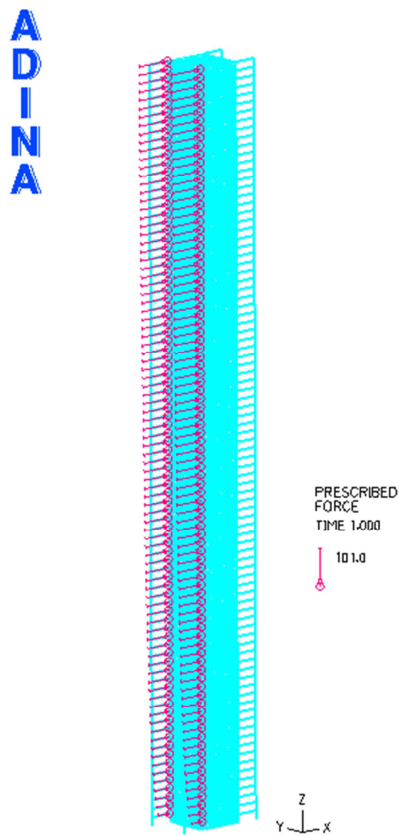
To reduce the number of system degrees of freedom, translation in the global y direction and rotations about the global x and z axes were assumed to be zero (lateral forces act in the global x direction). The total number of nodes in the model is 100,776. The total number of elements is 28,788. The total number of degrees of freedom is 3 (the global x and z translation and the global y rotation).



**Figure 4-11: Colored Member Types**



**Figure 4-12: Zoomed in View of Shell Elements in the Core Walls**



**Figure 4-13: Lateral Loads on the 3D Frame from ADINA**





## 5 Optimization Methods

This chapter explains the methods used for optimizing the sizes of each member in the skyscraper. Sections include megacolumn, core, outrigger, and floor members. Geometry and topology are unchanged within each optimization technique.

### 5.1 Trial and Error Method

The procedure for trial and error was performed within the spreadsheet for 1D stick analysis. There is a design tab within the worksheet where sizes can be chosen for each group of stories in the building for megacolumn areas, core thicknesses, and outrigger areas. As each of these sizes change, they individually effect how the building responds to wind and seismic loading. Table 5-1 shows design values for core thicknesses and areas of mega-columns and outriggers.

**Table 5-1: Design Variables for Trial and Error**

<b>Design Variables</b>			
stories	wall t	column A	outrig A
	m	m <sup>2</sup>	m <sup>2</sup>
1 to 10	2.20	0.8	0.16
11 to 20	2.12	0.7	0.16
21 to 30	1.80	0.6	0.16
31 to 40	1.43	0.5	0.15
41 to 50	1.10	0.4	0.13
51 to 60	0.79	0.4	0.12
61 to 70	0.50	0.4	0.10
71 to 80	0.30	0.3	0.10
81 to 90	0.30	0.3	0.10

Safety factors for wind stress and seismic stress are set at 2 and 1.7, respectively. Safety factors for wind and seismic drift are set to 1. The goal is to minimize the cost of the building. The optimization is accomplished by iterating to decrease member sizes while still maintaining required safety factors. A decrease in a steel outrigger size will affect the cost of the building more than a smaller size of a concrete member because of the higher price of steel. The outrigger and megacolumn were first held constant while optimizing the core wall thickness. Next, others were held constant while changing megacolumn area, and outrigger area.

## 5.2 Optimality Criteria Method

This optimization method uses a gradient based algorithm. The 2D frame analysis procedure is used in this optimization. Drift constraints in the program depend on multiple members in two dimensions. The program finds the best design by changing multiple members at one time in order to meet drift constraints and also avoid stress/buckling violations. Essentially, the program is minimizing cost, a continuous function, while meeting design criteria.

### 5.2.1 Kuhn-Tucker Conditions

When an optimization problem involves constraints, the Lagrangian function in Equation 5-1 must be minimized. The design variable vector  $x$  is a list of the megacolumn, outrigger, and core sizes.

$$L(x, \lambda) = f(x) + \sum_{i=1}^m \lambda_i g_i(x) \quad (5-1)$$

It assumes the drift constraints  $g_i$  for each story  $I$  are formulated as actual drift minus allowable drift so they are satisfied when  $g_i$  is less than zero. It also assumes all drift constraints are binding at the optimum. The constraint violation is multiplied by the penalty  $\lambda_i$  factor, also

known as the Lagrange multiplier. To ensure that this violation is minimized the Lagrange multiplier must be greater than or equal to zero:

$$\lambda_i^* \geq 0 \quad i = 1 \text{ to } m = \text{number of stories} \quad (5-2)$$

The Lagrangian is a function of both the design variables and the Lagrange multipliers. The necessary condition for optimality is that the gradient of the Lagrangian be zero with respect to both the design variables and the Lagrange multipliers. Differentiating Equation 5-1 with respect to Lagrange multipliers gives:

$$g_i(x^*) = 0 \quad \text{for } i = 1, \dots, m \quad (5-3)$$

Differentiating with respect to design variables gives:

$$\nabla f(x^*) + \sum_{i=1}^m \lambda_i^* \nabla g_i(x^*) = 0 \quad (5-4)$$

Equations 5-3 and 5-4 are the Kuhn Tucker conditions of optimality if one assumed drift constraints are binding.

An adjoint sensitivity analysis is performed where the derivatives or gradients in Equation 5-4 are calculated. Adjoint sensitivity analysis is equivalent to the principle of virtual work where a unit horizontal force is placed at each story to get the gradient of the corresponding horizontal displacement.

## 5.2.2 Algorithm

The optimality criteria algorithm resizes the design variables according to the following two equations derived from the Kuhn Tucker conditions:

$$x_i^{k+1} = x_i^k \left( \frac{-\sum_{j=1}^p \lambda_j^k \left( \frac{\partial g_j}{\partial x_i} \right)^k}{\left( \frac{\partial f}{\partial x_i} \right)^k} \right) \left( \frac{1}{\eta} \right) + x_i^k \left( 1 - \frac{1}{\eta} \right) \quad \text{for } i = 1 \text{ to } n \quad (5-5)$$

$$\sum_{j=1}^p \lambda_j^k \left( \sum_{i=1}^q \left( \frac{\partial g_l}{\partial x_i} \right)^k \left( \frac{\partial g_j}{\partial x_i} \right)^k \frac{x_i^k}{\left( \frac{\partial f}{\partial x_i} \right)^k} \right) = \eta g_l^k - \sum_{i=1}^q \left( \frac{\partial g_l}{\partial x_i} \right)^k x_i^k \quad \text{for } l = 1 \text{ to } p \quad (5-6)$$

$\eta$  is usually taken as 2. Superscripts refer to iteration numbers. Equation 5-5 is used to get new values for design variables and Equation 5-6 is solved to get updated values of Lagrange Multipliers.

After determining new values for design variables (member sizes), stress constraints are checked. If any of these constraints are violated, the corresponding member is resized by multiplying by the ratio of actual stress over allowable stress (reciprocal of safety factor).

### 5.3 Genetic Algorithm Method

A genetic algorithm mimics evolution. In nature, two parents will create offspring, and the survival of the fittest occurs between all offspring and parents. The genetic algorithm actually has proven to be quite robust for finding optimal designs. With the help of the genetic algorithm used in this research, member sizes were optimized in the example CMO skyscraper with a fixed geometry and topology. This method of optimization will be utilized for both the 1D stick model and the 2D frame model.

Chromosomes must first be created to represent designs in a starting generation. The chromosome is made up of genes, with one gene for each design variable (member size). Figure 5-1 shows a typical chromosome.

5	12	14	18	23	21	23	27	30	40	41	45	50	56	62	67	69	69	73	74	83
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

**Figure 5-1: Typical Chromosome**

Each gene in the chromosome for the 1D stick model represents a continuous integer bound within the ranges given in Table 5-2. Each gene in the chromosome for the 2D frame model has an integer value corresponding to a row in a section table. The section table is a list of cross-sectional properties (I, A, etc.) for different member sizes. These member sizes are used for megacolumns, outriggers, and core. The values for the 1D stick model and 2D frame model, whether continuous or discrete, allow the mega-columns to fluctuate cross sectional areas along with the core and outriggers. Each chromosome in the starting generation represents a complete design of the skyscraper. The first nine genes in the chromosome represent the nine megacolumn sizes. The second nine genes represent the core sizes, and the last group of genes represents the infinite core, floor member, and outrigger sizes. A fitness value is calculated for each design or chromosome. The fitness determines cost and effectiveness in meeting drift and stress constraints. In other words, fitness explains how good the design is. When any of the constraints are violated, the fitness is penalized.

**Table 5-2: Member Area Ranges for Continuous Values**

	m <sup>2</sup>		in <sup>2</sup>	
	min	max	min	max
Core	20.00	230.00	31000	356500
Megacolumn	0.20	0.90	310	1395
Outrigger	0.03	0.50	47	775

A group of starting chromosomes, or parents, is randomly generated and a tournament is created. The tournament takes the ‘size of tournament’ number of parents randomly from the starting generation and compares their fitnesses against one another. The best parent is chosen as the mother. Another tournament of parents is randomly generated, and the best parent is the father. A random probability is compared with the crossover probability. If the random probability is smaller than the crossover probability, crossover is performed. If not, the mother and father are copied over directly as children. The genetic algorithm uses blend crossover for the 1D stick model and uniform crossover for the 2D frame model to create new children. Blend crossover allows children designs to receive random values anywhere in between the values of the mother and father. A random number between one and zero is generated for each gene in the chromosome. If  $x_1$  is the mother value and  $x_2$  is the father value, then the children values  $y_1$  and  $y_2$  are:

1.  $y_1 = (r)x_1 + (1 - r)x_2$
2.  $y_2 = (1 - r)x_1 + (r)x_2$

With uniform crossover, a random number between one and zero is generated for each of the genes in the chromosome. For a particular gene, if  $x_1$  is the value from the mother design and  $x_2$  is the value from the father design, then the values  $y_1$  and  $y_2$  for the children designs are:

1. *if*  $r \leq 0.5$      $y_1 = x_2$      $y_2 = x_1$
2. *if*  $r \geq 0.5$      $y_1 = x_1$      $y_2 = x_2$

This process is performed again and again, until a generation of new children is formed. In addition to crossover, mutation will also randomly occur within the children. A random probability is chosen gene by gene, and this probability is compared with the chosen mutation probability. If the random probability is smaller than the mutation probability, the value in the

gene will be replaced by another random integer. This introduces some new variation within the children designs that was not initially present in the starting population of designs. The mutation used in the genetic algorithm for both models was dynamic, meaning that the further into the generations the algorithm moves, the amount of mutation occurring diminishes.

All fitnesses are then measured, and parents are allowed to compete against the children. The most fit parents and children are then taken as the next generation. The process of parents competing against their children is called elitism.

After a specified number of generations are developed, the top designs are taken as the optimal designs from the genetic algorithm. Probabilities for crossover and mutation, along with the generation size and number of iterations can be manipulated to achieve the quickest convergence to an optimized group of solutions. The parameters used in the genetic algorithm for 1D stick are shown in Table 5-3. The parameters used in the genetic algorithm for 2D frame are shown in Table 5-4. The genetic algorithm using the 1D stick model will be compared with the trial and error optimization. The genetic algorithm using the 2D frame model will be compared with the optimality criteria optimization. For each of these comparisons cost is being minimized.



**Table 5-3: Genetic Algorithm Parameters 1D Stick Model**

generation size	100
number of generations	1000
tournament size	5
crossover probability	0.6
mutation probability	0.1

**Table 5-4: Genetic Algorithm Parameters 2D Frame Model**

generation size	50
number of generations	30
tournament size	10
crossover probability	0.6
mutation probability	0.1

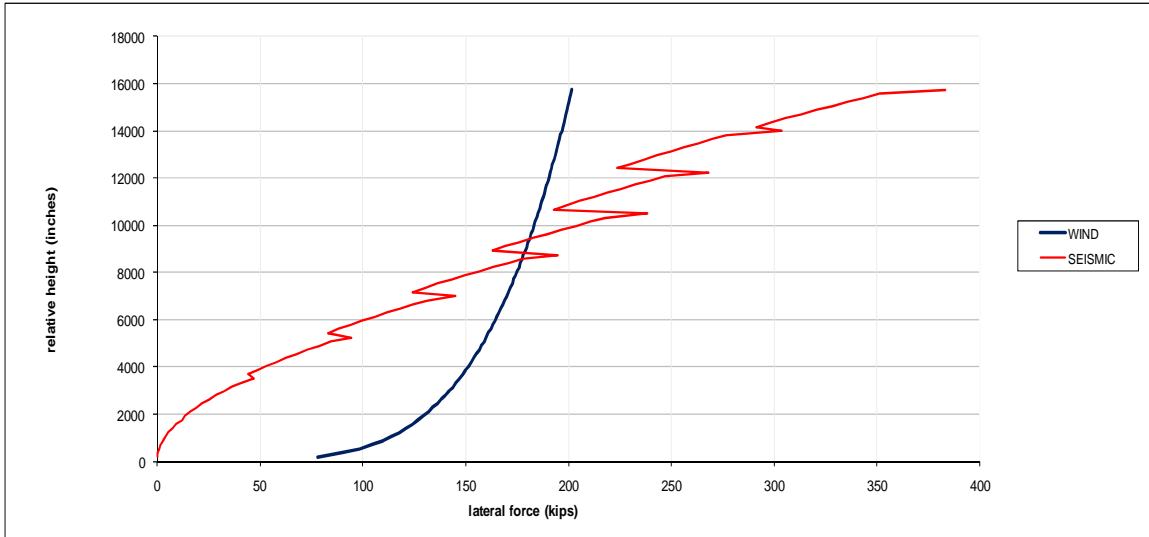
## **6 Results**

Comparisons of the analysis and optimization methods were made. The results will compare deflection values in analysis because deflection is the controlling criterion in the design of skyscrapers.

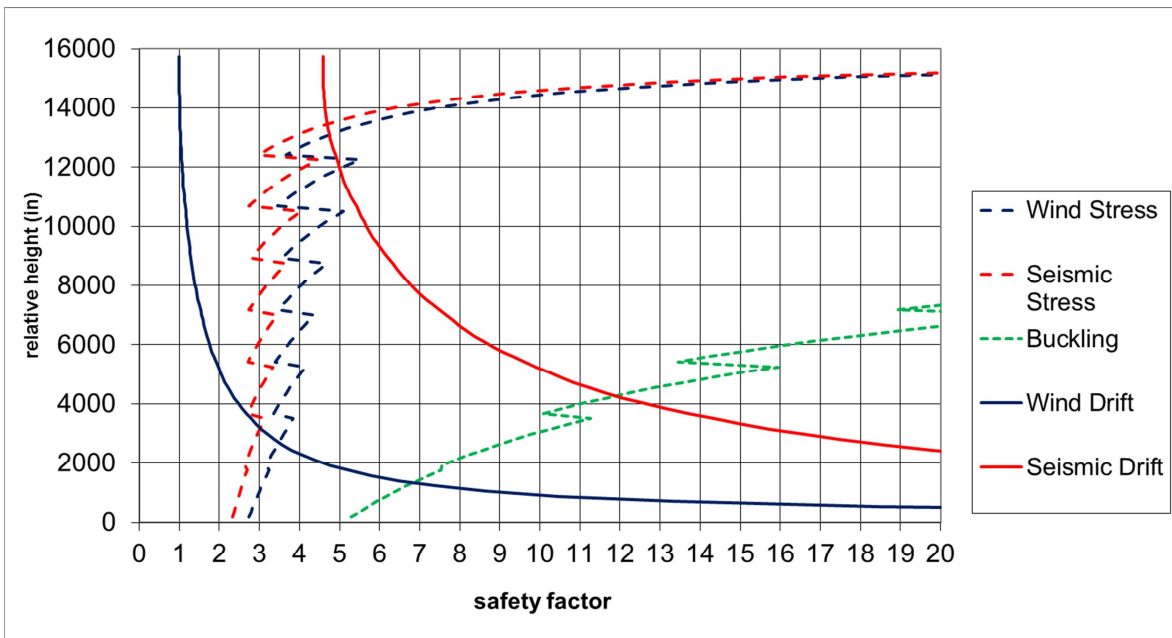
### **6.1 Analysis**

Figure 6-1 shows the wind and seismic forces developed from the spreadsheet. Each lateral force represents a point on the graph and then connected by a straight line to show the changes in weight and force with each story. Notice that the seismic force jumps by the outrigger weight every ten stories. The maximum lateral force for wind is 202 kips, and the maximum lateral force for seismic is 383 kips.

In Figure 6-2, the safety factors from 1D stick analysis are plotted vs. height. Note that seismic stress controls at the bottom and wind drift controls at the top. The allowable wind drift safety factor is 1, the allowable seismic stress safety factor is 1.7. This design is feasible and very close to optimal.



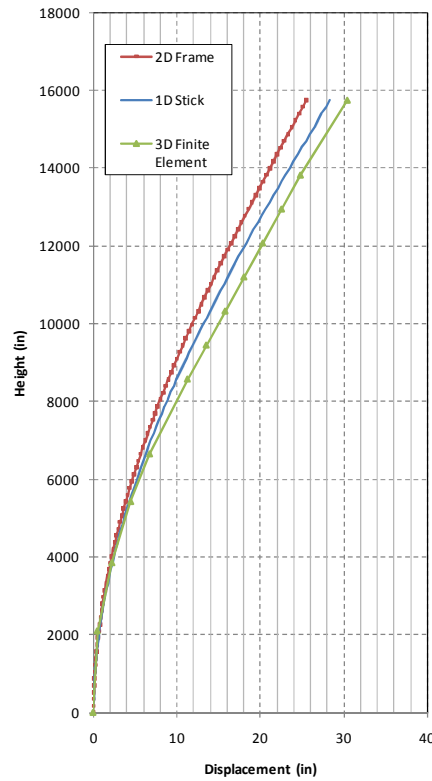
**Figure 6-1: Lateral Force versus Height**



**Figure 6-2: Safety Factor Analysis**

Figure 6-3 shows the deflection of the skyscraper under wind loading. All analyses are shown in this graph. The values from the 2D frame and 1D stick analyses are very close to those

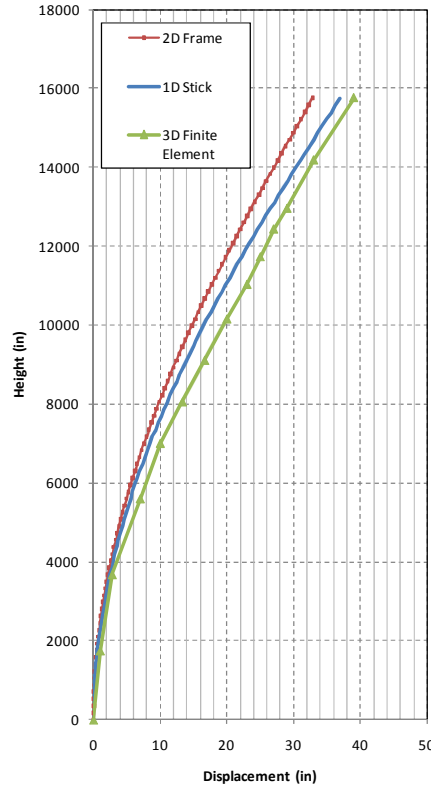
of the 3D finite element analysis. The 1D stick analysis is a more conservative analysis with a maximum deflection of 28.7 inches versus the 2D frame analysis with a maximum deflection of 25.5 inches.



**Figure 6-3: Wind Displacement**

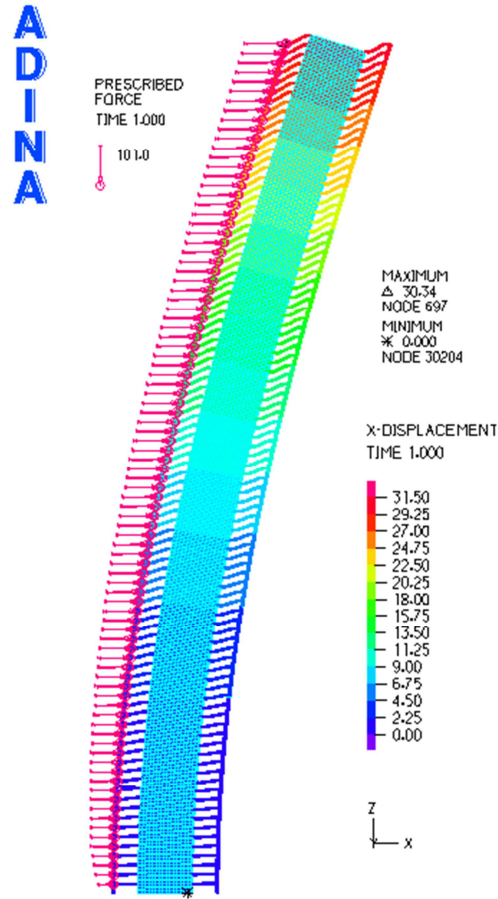
Figure 6-4 shows the deflection of the skyscraper under seismic loading. All analyses are shown in this graph as well. The 2D frame analysis and 1D stick analysis have comparable results to the 3D finite element analysis, once again proving these analyses are both successful in approximating the skyscraper's structural response. The 1D stick analysis is more conservative

with a maximum deflection of 36.9 inches, while the 2D frame analysis has a maximum deflection of 32.9 inches.



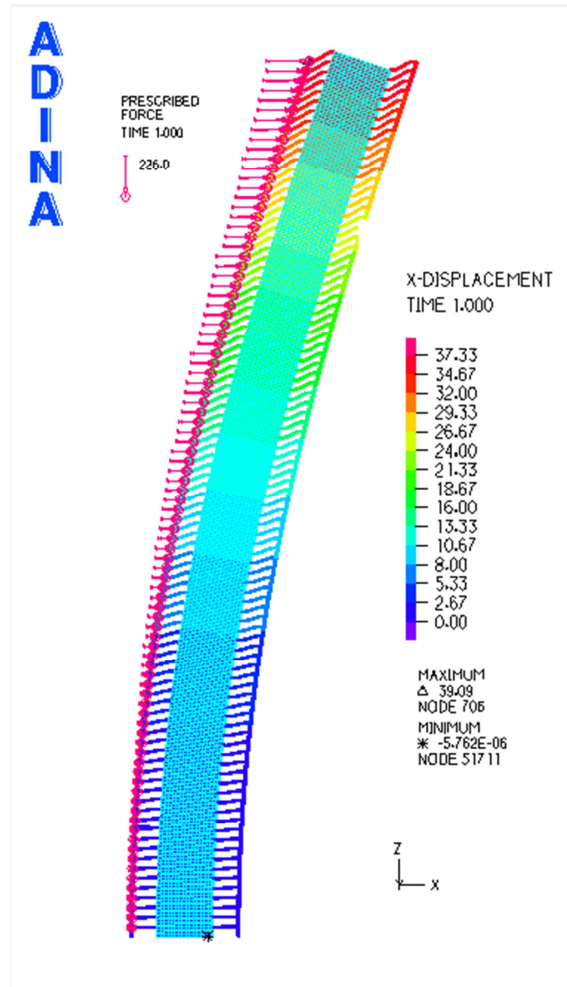
**Figure 6-4: Seismic Displacement**

Figure 6-5 shows the deflected building under the wind loads from the 3D finite element analysis. The legend for the coloring of the members shows the amount of deflection occurring in the skyscraper. The maximum displacement occurs at the top of the exterior left megacolumn. This value is 30.34 inches. This value of lateral deflection is close to the values calculated from the 1D stick and 2D frame analyses. The maximum value for the 1D stick analysis was 28.7 inches. The maximum deflected value for the 2D frame analysis was 25.5.



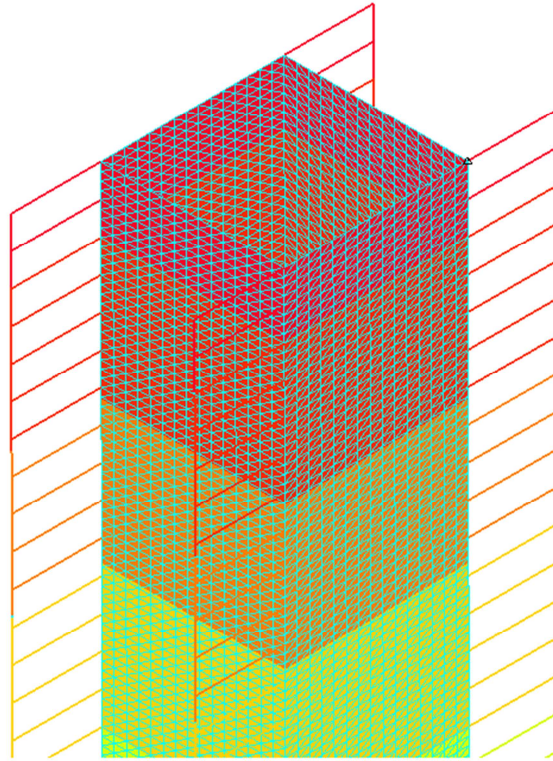
**Figure 6-5: Wind Deflection from 3D Finite Element**

Figure 6-6 below shows a similar output from the 3D finite element analysis for the seismic loading of the building. The maximum deflection is 39.1 inches. The finite element analysis proves to be the most conservative and the highest calculated value of deflection for the example CMO skyscraper. The 1D stick analysis provides the closest approximation to this result.



**Figure 6-6: Seismic Deflections from 3D Finite Element**

Figure 6-7 shows the deflection in the core in 3D finite element analysis that could not be seen in the previous figures because of the fineness of the mesh within the core. Notice that the deflection seen in the megacolumns and outriggers (by color) is also distributed into the core in the same magnitude (color). This shows the accuracy of the system working as a whole.



**Figure 6-7: Deflection in Core**

## **6.2 Optimization**

The optimization results for the 1D stick model are shown in Figure 6-8. The cost is shown below each design. The cost is calculated by multiplying cost of material by volume of material for steel and concrete respectively. Each of the wind drift safety factors are above and close to two for these feasible designs. A good design is considered to have decreasing size in members up the building which can be seen in Figure 6-8. The genetic algorithm produced a design of structure that was 13% cheaper than the trial and error optimization design. The time for optimizing by the trial and error method was close to ten minutes while the genetic algorithm using stick model was instantaneous. This proves to be a much more efficient method for



optimization, especially as design problems become larger. Trial and error becomes significantly more strenuous and difficult as the size of structure increases in size.

		Trial and Error		Genetic Algorithm	
		Area		Area	
		in <sup>2</sup>	m <sup>2</sup>	in <sup>2</sup>	m <sup>2</sup>
MegaColumn		1247	0.80	1334	0.86
		1097	0.71	1216	0.78
		939	0.61	1208	0.78
		782	0.50	1247	0.80
		625	0.40	1237	0.80
		625	0.40	985	0.64
		625	0.40	795	0.51
		470	0.30	528	0.34
		468	0.30	386	0.25
		341000	220	331579	214
Core		328600	212	311483	201
		279000	180	252266	163
		221650	143	212124	137
		170500	110	170198	110
		122450	79	144378	93
		77500	50	118644	77
		46500	30	66785	43
		46500	30	34549	22
		465	0.3	166	0.107
		465	0.3	145	0.094
Outrigger		465	0.3	163	0.105
		465	0.3	167	0.108
		465	0.3	160	0.103
		465	0.3	123	0.079
		388	0.25	87	0.056
		388	0.25	76	0.049
		388	0.25	51	0.033

<b>Cost</b>	<b>\$14,947,677</b>	<b>Cost</b>	<b>\$13,022,598</b>
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**Figure 6-8: Optimized Designs for 1D Stick Model**

The optimization results for the 2D frame model are shown in Figure 6-9. The cost of each design is shown at the bottom of the figure calculated the same as in the 1D stick model results. The wind drift safety factors are above and close to 2 as well proving to be feasible designs. The genetic algorithm produced a design that was 10% cheaper than the design produced by the optimality criteria method. When more than one outrigger size was used in the

optimality criteria method, oscillation occurred. In order for the optimality criteria method to converge, only one outrigger size could be chosen. This shows the deficiency of the optimality criteria method when compared with the genetic algorithm for the 2D frame model. The genetic algorithm uses larger megacolumn and core sizes in order to allow a smaller size of outriggers. The genetic algorithm has the capability of searching over a larger number of designs, resulting in the least expensive design. The time required for optimization by the genetic algorithm was 30 minutes for the 2D frame model. This was greater than the optimality criteria method which was 15 minutes.

		Optimality Criteria		Genetic Algorithm	
		Area		Area	
		in <sup>2</sup>	m <sup>2</sup>	in <sup>2</sup>	m <sup>2</sup>
MegaColumn		1395	0.90	1364	0.88
		1240	0.80	1147	0.74
		1085	0.70	992	0.64
		930	0.60	899	0.58
		775	0.50	744	0.48
		697	0.45	744	0.48
		620	0.40	713	0.46
		542	0.35	589	0.38
		418	0.27	496	0.32
Core		372000	240	328600	212
		372000	240	328600	212
		306900	198	291400	188
		234050	151	217000	140
		172050	111	189100	122
		113150	73	114700	74
		62000	40	86800	56
		31000	20	68200	44
		31000	20	40300	26
Outrigger		465	0.3	310	0.2
		465	0.3	310	0.2
		465	0.3	264	0.17
		465	0.3	202	0.13
		465	0.3	155	0.1
		465	0.3	109	0.07
		465	0.3	109	0.07
		465	0.3	47	0.03
		465	0.3	47	0.03
		<b>Cost</b>	<b>\$15,385,622</b>	<b>Cost</b>	<b>\$13,786,421</b>

**Figure 6-9: Optimized Designs for 2D Frame Model**



## **7 Conclusions**

### **7.1 Analysis**

The 3D finite element analysis is regarded as the most accurate analysis method of the three methods compared in this thesis. The 1D stick analysis showed an average error of 5.5% while the 2D frame analysis showed an average error of 15.9% when compared with the 3D finite element analysis. This larger error is due to the core spine modeling method. The infinitely stiff core spine members at each floor create additional stiffness for the structure causing this error. The 1D stick analysis is the best approximate analysis for the CMO skyscraper shown by the work in this thesis. In addition, when taking into account the time taken for the CMO skyscraper model to be created in the 3D finite element analysis, the 1D stick analysis becomes even more appealing. The computation time taken for the 3D finite element analysis was 20 minutes while the 1D stick analysis was instantaneous. This research shows that when approximate analysis is required, the 1D stick method is the best in estimating structural response.

### **7.2 Optimization**

Considering the optimization methods for the 1D stick model, the genetic algorithm method proved to be the best in minimizing cost and meeting constraints. The genetic algorithm was instantaneous in run time. Even with the generation number set to 1000, the genetic

algorithm took less than 5 seconds to run. The genetic algorithm method also provided a structure that was 13% less expensive than the trial and error optimized design.

For the 2D frame model the genetic algorithm again was better than the optimality criteria method for the CMO skyscraper because it provided the cheapest design. The genetic algorithm was also the best method due to the optimality criteria's non-convergence and constant size for outriggers. The execution time for the optimality criteria method was 15 minutes and for genetic algorithm 30 minutes. Although the genetic algorithm took more time to run, it proved to be the best method for optimizing the CMO skyscraper because it found a feasible optimum that was 10% less expensive.

All methods for 1D stick and 2D frame provided feasible designs, meeting constraint requirements and developed designs that had decreasing member sizes up the structure (except in the single outrigger size case for optimality criteria). Considering the purpose of this research for future design of the Greenplex, the genetic algorithm combined with the 1D stick model is the recommended method for finding the optimum design. When additional buildings and bridges will be added to the structure, the genetic algorithm using the 1D stick model will be capable of searching the design space for an optimum in a much shorter amount of time than the 2D frame model. The research has also shown that the genetic algorithm provides the cheapest cost of structure.

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