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Contextualized Motivation Theory (CMT): Intellectual Passion, Mathematical Need, Social Responsibility, and Personal Agency in Learning Mathematics

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ABSTRACT

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Student motivation has long been a concern of mathematics educators. Here, I characterize motivation, defined as an individual's desire to act in particular ways, through analysis of students' extended, collaborative problem solving efforts. Grounded in a longitudinal research project in calculus learning and teaching, Contextualized Motivation Theory (CMT) offers a means for understanding the complexities of student motivations in mathematics learning. Students in this study chose to act upon various intellectual-mathematical motivations and social-personal motivations, existing simultaneously, within a supporting "web" of motivations. Students exhibited intellectual passion in persisting beyond obtaining correct answers to build understandings of mathematical ideas. CMT positions personal agency as the active power in intellectual passion, foregrounds mathematical need as a kernel of students' problem solving industry, characterizes the social nature of motivation, and encompasses conceptually driven conditions that foster student engagement in mathematics learning.

Keywords: motivation, agency, collaboration, problem solving

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

"He who is not alive to the subtleties of student desires and student motivation—indeed, he who is not thrilled and intrigued by them—has little likelihood of being a good teacher" (Davis, 1955, p. 134). Teachers' perceptions about student motivation have been shown to influence classroom activities and lesson plans (Middleton, 1995). Based on these findings, Middleton (1995) suggested that teachers and teacher educators need to better understand student motivations in learning mathematics. Some theorists have also "suggested that motivation is the key to enhancing learning. Many teachers would agree" (Middleton & Midgley, 2002, p. 374).

Little is actually known about student motivation in the mathematics classroom, especially student motivation to understand mathematics. The National Council of Teachers of Mathematics (NCTM) has recognized the desire and need for more information about student motivation in the mathematics classroom. In 2011, the NCTM yearbook will comprise manuscripts that explore motivation as an important element in student learning.

As a teacher and researcher, I am intrigued by the persistence students display in mathematics problem solving when they are invited to work together on carefully designed problems without instruction on solution procedures. This thesis presents a qualitative analysis of student motivation based on data collected from a university experimental calculus class. Additionally, Contextualized Motivation Theory (CMT) is introduced to illuminate some of the complexities and nuances of motivation in learning that are manifest through students' mathematical actions.

CHAPTER 2: THEORETICAL PERSPECTIVE

My perspective on motivation in learning is grounded in the exercise of personal agency (Walter & Gerson, 2007). In this perspective, the individual is an "active performer who makes purposeful choices in constructing mathematical knowledge" (Walter & Gerson, 2007, p. 208). With personal agency playing a central function in the classroom, one can detect student motivations because one can see student decision making in action.

Agency and Personal Causation

As was mentioned before, personal agency involves students making purposeful choices (Walter and Gerson, 2007). In his agency in social cognitive theory, Bandura (1989) commented that most human behavior is purposive and "regulated by forethought" (p. 1179). He also defines agency as "the power to originate actions for given purposes" (Bandura, 1997, p. 3)

If personal agency is to play an important role in the mathematics classroom, students must feel they are the originators of their own mathematical actions and that they have the power to make and act upon decisions to bring about change (Bandura, 1989). Such intentional choice making to effect change has been termed "personal causation" (deCharms, 1984). Personal causation is an essential part of my view of personal agency.

Without a sense of personal causation, agency is limited. "When …the person has not caused the change—some other agent or object has interfered with personal causation and the person feels that he or she is a pawn" (deCharms, 1984, p. 276). When personal causation is interfered with, individuals can become passive—losing both appetites and desires (Marcel, 2003). As will be discussed in the next sections, teachers, other students, and the subject matter are all agents that can affect a student's personal causation—positively or negatively.

However, it is important to note that although choices are purposeful, “Motivation, like much of our mind, is only partially available to introspection” (Hannula, 2006, p.166). Sometimes individuals are so entrenched in routines that most choices are made without explicit reflection (Martin, 2004). In essence, students choose to act based on a set of desires they possess, but the student may not be aware of which desire they are acting upon at a given time.

Agency and the Teacher’s Role

Students’ exercise of personal agency can be enhanced or hindered by teacher actions in the classroom (Boaler, 2003; deCharms, 1984; Walter & Gerson, 2007). In order to enable students to be powerful personal agents in their own learning, teachers should provide appropriate structure, support, and challenge for students during mathematical problem solving (Elmore, 2005).

Giving structure to students in the mathematics classroom can be done through task wording or teacher guidance. Tasks with closed questions that lead students to particular solution paths are highly structured tasks. When a teacher probes students by asking them questions, suggests directions for students to explore, or tells students to perform a certain mathematical action, they are providing structure for their students. Too much or too little structure limits personal agency and thus limits student motivation (Boaler, 2003; deCharms, 1984). This is due to the fact that “structure is always both enabling and constraining" (Giddens, 1984, p. 169). If a student is not given any structure at all, they can become confused—not knowing which choices would lead to the desired goal. Agency is constrained in this instance because, in order for agency to be successful, individuals must have knowledge of different options to pursue. On the other hand, if the teacher gives too much structure, students no longer have a range of choices upon which to act—thus making them to feel like pawns (Ryan & Grolnick, 1986).

Supporting the student by respecting them, listening to them, and helping them in their creative mathematical efforts is fundamental in the teaching of mathematics. In order to enhance the exercise of agency, students must be treated as adults who are actively engaged in the learning process. According to Speiser and Walter (1996),

> Treating students as adults means, fundamentally, to help open ways for them to learn and reason critically. The history of science flows through complex, many-sided conversations, told through further conversations, heard through acts of personal engagement and imagination. Our integrity as teachers flows in part from how we see our students in this dialogue, which includes our conversations with them….Each crack someone falls through represents, not just potentially but actually, a voice…which we need to hear, a person whose experience and growth should matter to us. (p. 371)

Finally, it is crucial that students be challenged in their mathematical work, but not so much that they become discouraged (see Meyer, Turner, & Spencer, 1997; Stein, Smith, Henningsen, & Silver, 2000). Challenging problems require students to think about, analyze, and reason through the problem or task at hand in order to come to a conclusion about how to use mathematical ideas to invent solutions. Challenge is often measured by the cognitive demand required by the task—the higher the cognitive demand, the higher the challenge. Stigler and Heibert (1999) distinguished between three types of thinking required for various mathematics problems: “practice routine procedures, apply concepts or procedures in new situations, and invent something new or analyze situations in new ways” (p. 70). Stein, Grover, and Henningsen (1996) similarly categorized tasks into four levels by increasing cognitive demand: “memorization tasks,” “procedures without connections,” “procedures with connections,” and “doing mathematics” (p. 472). “Doing mathematics” means students must use “formulas, algorithms, or procedures with connection to concepts, understanding, or meaning” (Stein, et al. 1996, p. 467). Tasks that require high cognitive demand are optimally challenging when accompanied by sufficient structure. Conversely, consistently presenting students with problems that involve practicing routine procedures fails to encourage students to choose to become active problem solvers.

Agency and Social Systems

One metaphor describes the relationship of agency and social systems as a room. In this room, an individual is bound by the social system, or walls, in the room. However, inside the walls, "he or she is able to move around at whim" (Giddens, 1984, p. 174). The tension between the metaphor and the actual agency and social systems relationship is that the metaphor does not take into account the responsibility that comes with agency. Personal agency is an individual matter, but the consequences of enacted agency affect others. Personal agency involves “the responsibility to act with mindful awareness of others” (Walter & Gerson, 2007, p. 205). In contrast to the metaphor of moving around “at whim,” I see the actors inside the walls of the room participating in a “dance of agency” (Pickering, 1995). Teachers and students participate in this dance with each other (Boaler, 2003) and with the subject matter (Pickering, 1995) as they engage in meaningful mathematical discourse. It is easy to imagine how teachers and students could participate in a “dance of agency”. The teachers could use their agency to guide students in the right direction and students could choose to make mathematical decisions (dance steps) based on the structure and guidance provided by the teacher and suggested by the type of mathematics (music) that is involved.

Mathematics as a discipline does not have the same kind of agency that humans possess, but Pickering (1995) has described what is meant by the “agency of a discipline”.

> It is, I shall say, the agency of a discipline—elementary algebra, for example—that leads us through a series of manipulations within an established conceptual system. The notion of discipline as a performative agent might seem odd to those accustomed to thinking of discipline as a constraint upon human agency, but I want to recognize that discipline is productive. There could be no conceptual practice without the kind of discipline at issue; there could be only marks on paper. (p. 115)

Agency and Motivation

The grounding principle of personal agency does not imply that external motivation does not exist, but rather that the learner purposefully chooses to act upon some motivations and chooses not to act upon other motivations (Patterson, Grenny, McMillan & Switzler, 2005). This contrasts with perspectives that neglect choice or suggest that motivations completely determine actions (Deckers, 2001). Individuals, as agents, “act for themselves” instead of being “acted upon” (2 Nephi 2:26, The Book of Mormon).

From the Book of Mormon we also learn the following truth: “there is an opposition in all things” (2 Nephi 2:11). There are always conflicting choices upon which one can act. For example, one could choose to participate in understanding mathematics or choose not to. Lehi continues to explain, “Wherefore, the Lord God gave unto man that he should act for himself. Wherefore, man could not act for himself save it should be that he was enticed by one or the other” (2 Nephi 2:15-16). When a person is “enticed” by one choice over another, that person has a desire to act in a certain way. In other words, they are motivated to make that choice. Lehi explained that without being enticed, we would not be able to act. I believe the word “enticed” could respectfully be replaced by the word “motivated” without changing the meaning of the scripture. Thus the scripture would read, “Wherefore, man could not act for himself save it should be that he was [motivated] by one or the other” (2 Nephi 2:15). One can conclude then, that agency could not exist without motivation. Also, it would not make sense to have motivation without agency.

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Capacity to Develop Desires

I agree with Dewey (1913) that there is no such thing as an uninterested child and extend this view by suggesting that there is no such thing as an unmotivated student (Maslow, 1943; Middleton & Spanias, 1999). Each student has and acts upon motivations that ground personal action with respect to mathematical engagement. Furthermore, students have the capability to develop new motivations. Framed by the perspective of agency, *motivation* may be generally defined as an individual's desire to act in particular ways (Weiner, 1992). Harel (2008) mentioned how students can develop the desire to be puzzled:

> Humans—all humans—possess the capacity to develop a *desire* [italics added] to be puzzled and to learn to carry out mental acts to solve the puzzles they create. Individual differences in this capacity, though present, do not reflect innate capacities that cannot be modified through adequate experience. (p. 894)

Because all humans are able to develop new desires, motivation should not be seen as static. An individual's desires can and do change over time. In this thesis, it is shown how a group of students act on desires (motivations) to understand mathematics. Other groups of students may not possess similar desires, yet, through "adequate experience" such students are capable of developing similar desires to understand mathematics.

CHAPTER 3: LITERATURE REVIEW

This chapter provides a literature review that is organized into seven thematic sections: Definitions of Motivation, Theories of Motivation, Intrinsic vs. Extrinsic Motivation, Findings and Significant Statements of Motivation Literature in Mathematics Education, Deficiencies in the Literature, Learning Mathematics with Understanding, and The Cat Task. The first four sections build relationships between extant literature within each theme. The fifth section, Deficiencies in the Literature, articulates the cumulative deficits in existing literature that this study is designed to explore. The two final sections, Learning Mathematics with Understanding and The Cat Task, give essential background information to help situate the research question and methods for this thesis.

Definitions of Motivation

The literature in the field of motivation has been described as full of "fuzzy but powerful constructs" (Pintrich, 1994, p. 139). One of those fuzzy constructs is the word motivation itself. This is partly due to the fact that motivation researchers have used over 140 different definitions of motivation for their studies (Kleinginna and Kleinginna, 1981). Pinder (1984) explained the lack of consensus and cohesion,

> It is only a slight exaggeration to say that there have been almost as many definitions of motivation offered over the years as there have been thinkers who have considered the nature of human behavior….Some writers view motivation from a strictly physiological perspective, while others view human beings as primarily hedonistic, and explain most of human behavior as goal oriented, seeking to gain pleasure and avoid pain. Others stress the rationality of humans, and consider human behavior to be the result of conscious choice processes. Some thinkers stress unconscious or subconscious factors. (p. 7)

One definition stated, "to be motivated means *to be moved* to do something" (Ryan & Deci, 2000, p. 54). Many dictionaries, textbooks, and researchers defined motivation in a similar manner. For example, Denhardt, Denhardt, and Aristigueta (2008) defined motivation as "what causes people to behave as they do" (p. 146). These definitions of motivation were limiting because they were often interpreted to mean that something, or someone, forced one to act in a certain way. Such interpretation rejected personal agency as a fundamental aspect of human actions.

Huitt (2001) combined many other definitions of motivation to come up with the following definition: "Motivation is an internal state or condition (sometimes described as a need, desire, or want) that serves to activate or energize behavior and give it direction (see Kleinginna and Kleinginna, 1981)" (p. 1). Though there is a plethora of definitions of motivation, many educational researchers chose not to define motivation in their papers. Murphy and Alexander (2000) conducted a review of motivation studies in education and found that only four percent of the articles defined the word motivation explicitly (see p. 33). When motivation is not defined (e.g. Francisco, 2005; Mayer, 1998) clarity and meaning are compromised.

How researchers choose to define motivation, whether they state the definition or not, significantly influences how they interpret and analyze data. For example, Hannula (2004) defined motivation as "a potential to direct behavior through the mechanisms that control emotion. This potential is structured through needs and goals" (p. 9). Hannula (2006) used his definition in analysis of the motivational structure of one particular student, Frank. Hannula's findings were based on identification of Frank's needs and goals that were evidenced by his behavior and interpretations of his cognition and emotions. Further details about Hannula's definition and research will be elaborated upon later.

For the purposes of this study, motivation is defined as an individual's desire to act in particular ways (Weiner, 1992) and framed by the perspective of agency. Notice in this definition, motivation is conceptualized as a desire instead of a need, want, or goal. The word “desire” was chosen intentionally to encapsulate needs, wants, and goals. Though not contained in the definition of motivation for this study, I will also refer to the *powers* of student motivation in the mathematics classroom. Powers may be defined as “capacit[ies] for action or performance” (Porter, 1913, p. 1122) or “tendencies in action” (Dewey, 1913, p. 62). The educational philosopher John Dewey (1913) said motive “is the name for the end or aim in respect to its hold on *action*, its power to *move*” (p. 60). Examining the *powers* of student motivation will help researchers and educators more fully recognize and foster the capacities of students’ desires.

As will be further discussed, there are many definitions and theories of motivation. However, the purpose of this study is to add a new perspective on motivation in the mathematics classroom by developing a contextualized motivation theory grounded in the mathematical activities of agentive calculus students.

Theories of Motivation

Just as no single definition of motivation is suitable for all contexts, there is no widely accepted single standard theory on motivation. Numerous theories and models of motivation permeate the literature. Here I discuss the following major types of motivational theories: behavioral theories, attribution theories, need theories, goal theories, and personal construct theories.

Behavioral Theories

Behavioral theories of learning attempt to explain the motivations behind an individual’s actions. Behavioral theorists focus on the effects of rewards and punishments on students’ behaviors. In Pavlov’s (1928) classical conditioning, individuals are viewed as acting reflexively in response to stimuli. The relationship between stimulus and response can be enhanced through enforcement. In operant conditioning (Skinner, 1938), when a student's behavior is reinforced, or rewarded, the student will be motivated to continue that behavior. The reward is seen as an incentive for acceptable behavior. Conversely, if a behavior is punished, the student will be motivated to decrease the behavior. For example, if a student is given praise after completing a math problem, the student would be seen to do math problems in the future. On the other hand, if the student is given no praise or is reprimanded for getting a wrong answer, the student would have less motivation to do work. Behavioral theories on motivation are limiting because they assume humans act in a certain way because they automatically respond to stimuli or incentives—personal choices and intentions are neglected.

Behavioral theories dominated the literature on motivation for the first half of the 20th century (Bindra, 1968; Hull, 1943; Pavlov, 1928; Rescorla & Solomon, 1967; Skinner, 1938; Skinner, 1953; Spence, 1960; Watson, 1913). They are not extremely popular in research today because researchers no longer accept the mechanistic views associated with the theories. However, "newer reformulations of these theories have focused on the potential conflict between an individual's perceived necessity for success and perceived necessity for avoiding failure" (Middleton & Spanias, 1999, p. 68).

Attribution Theory

Mid-century, researchers began to question whether behaviorist theories were sufficient to account for motivations in learning. Attribution theorists attempted to explain what factors students perceived to contribute to success or failure (Heider, 1958; Weiner, 1972). In Middleton and Spanias's (1999) review of motivation literature, attribution theory was found to be the theoretical orientation most widely held by mathematics education researchers. It has also been called the "dominant contemporary theory of motivation" (Graham and Weiner, 1996, p. 72).

Weiner (1972, 1974) was the main researcher who applied attribution theory to education. In his view, students' perceptions of success or failure are dependent on three different components: locus, stability, and controllability. Locus describes where causality originates. If a student feels success or failure is due to an outside factor, the locus is external. Otherwise, the locus is internal–attributable to factors for which an individual is responsible. Stability describes whether or not causes change over time. Finally, controllability explains whether students have control over the factors that attribute to success or failure. For example, ability is an attribution that has an internal locus and no control. Effort, on the other hand, is an attribution with an internal locus that one can control. An example of an attribution with an external locus is luck. Luck also has no controllability (Huitt, 2001).

The principle idea of attribution theory is that if students believe achievement depends on factors that they can control, they will be more motivated to learn and will achieve more (Pintrich, 2004). In one study, the researchers found that "overall…students are confident in their ability, feel they try hard, and see achievement as connected to effort" (Sullivan, Tobias, & McDonough, 2006, p. 89). On the other hand, if students think their failures are due to factors that they cannot control, such as a lack of ability, students tend to fail. Learned helplessness (Dweck, 1986) is a condition that arises when a student has experienced so much failure that he or she believes that success is not possible.

Need Theories

Often referred to as Maslow's Hierarchy of Needs, Maslow's (1943) theory of motivation is still one of the most influential and well-known theories in the field today (Huitt, 2004; Koltko-Rivera, 2006). Maslow argued that there was a lack of a well-founded theory on motivation. Maslow's theory took into account modern understandings about motivation and avoided adopting a model that was based on animal behavior. Generally depicted as a pyramid, Maslow’s (1943) theory contains two groups of needs: deficiency needs and growth needs. Figure 1 shows one interpretation of Maslow’s hierarchy of needs.

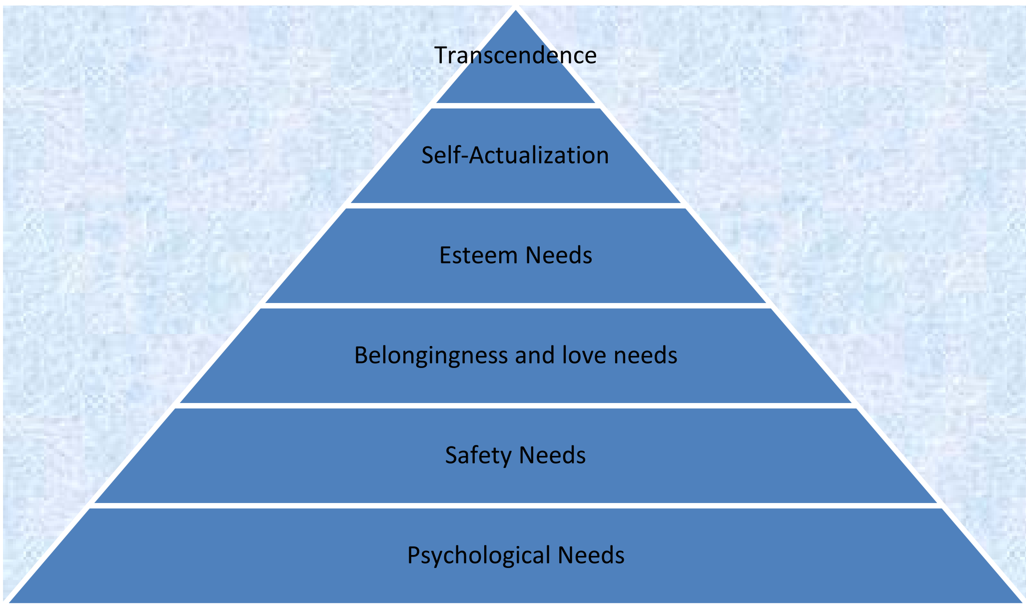

Figure 1: Maslow's Hierarchy of Needs

The bottom four needs in this pyramid represent the deficiency needs (Maslow, 1943). Deficiency needs, beginning from the bottom, are:

- Physiological needs: basics of life, such as food and water
- The safety needs: stay away from dangers
- Belongingness and love needs: affectionate relationships whether for friends or for family
- Esteem needs: self respect and esteem for oneself and others

Maslow (1943) suggested the needs be arranged in a hierarchy because lower needs have to be met before the higher ones are considered.

> It is quite true that man lives by bread alone—when there is no bread. But what happens to man’s desires when there *is* plenty of bread and when his belly is chronically filled? *At once other (and “higher”) needs emerge* and these, rather than physiological hungers, dominate the organism. And when these in turn are satisfied, again new (and still “higher”) needs emerge and so on. This is what we mean by saying that the basic human needs are organized into a hierarchy of relative prepotency. (p. 375)

In his first conceptualization of the theory, Maslow (1943) only listed one growth need, self-actualization. Self-actualization represents an individual's need to do and be what he is capable of doing and being—to realize one's potential. Later, Maslow (1971) described self-transcendence as a category of needs even higher than self-actualization (see Koltko-Rivera, 2006). Self-transcendent people go beyond meeting their own needs and consider those of others and of society. Two additional needs not included in the pyramid, the need to know or to understand and aesthetic needs, were recently included in Maslow's theory (Maslow & Lowery, 1998; Huitt, 2004). Aesthetic needs include the need for beauty and order.

Maslow (1943) noted one shortcoming of his theory was that some people go backwards in the hierarchy. For example, there are martyrs, people who would sacrifice food, safety, and all else for what they believed to be a higher cause.

Following Maslow's lead, other theorists suggested humans have categories of basic needs. McClelland (1975) claimed people acquire needs as they experience life. These needs include: the need for achievement, the need for affiliation, and the need for power. Ryan and Deci's (2000b) Self-Determination Theory suggests students possess three innate needs: the need for autonomy, competence, and relatedness.

Goal Theories

Achievement goal theories deal with the cognitive and affective components of students' behaviors (Ames, 1992; Covington, 2000; Dweck, 1986; Dweck & Leggett, 1988). Goal theorists focus on people's perceptions and interpretations as well as patterns of self-regulation (Middleton & Spanias, 1999).

Two opposing orientations permeate the educational research on goal theory. In goal theory, a student is thought to have either a learning goal orientation or a performance goal orientation (Dweck, 1986; Dweck & Leggett, 1988). Learning goal orientation has also been labeled a mastery goal (Ames & Archer, 1988) or task-involvement goal (Maehr & Nicholls, 1980) orientation. If a student possesses a learning (mastery/task) goal orientation, they value the importance of the skill to be learned and believe success comes from controllable factors, such as hard work and effort (Ames & Archer, 1988; Weiner, 1979). Such a student also sees difficulties as challenges to be faced instead of insurmountable feats (Francisco, 2005). On the other hand, if a student has a performance or ego-involvement (Maehr & Nicholls, 1980) goal orientation, they value outperforming other students and believe that success depends on uncontrollable factors such as ability and self-worth instead of hard work and effort (Dweck, 1986). These students tend to avoid challenging tasks (Dweck & Leggett, 1988) and rely heavily on short-term learning strategies such as memorizing (Dweck, 1986).

Middleton and Midgley (2002) said a learning (mastery/task) goal orientation "clearly represents the most beneficial form of motivation" (p. 375). Much research has also confirmed that a mastery orientation leads to better academic and motivational outcomes than a performance orientation (Dweck, 1986; Dweck & Leggett, 1988; Elliot & Dweck, 1988; Midgley & Urdan, 2001; Pintrich, Roeser, & DeGroot, 1994; Stipek, 1997).

Ames (1992) identified certain classroom structures that impact whether or not students adopt a mastery goal orientation. They included, "design of tasks and learning activities, evaluation practices and use of rewards, and distribution of authority or responsibility" (p. 263). In order for students to develop a mastery goal orientation, tasks and activities must be challenging, meaningful and diverse (Brophy, 1987; Nicholls, 1989). Evaluation practices are most beneficial when social comparison is not often employed (Ames, 1992). Rewards tend to increase a mastery orientation only when they are based on effort instead of seen as a bribe

(Brophy, 1987; Deci & Ryan, 1985). Finally, when students have autonomy in the classroom and are involved in decision making, they are also more likely to have a mastery goal orientation (e.g., Ryan & Grolnick, 1986).

Although learning and performance goal orientations are generally seen as mutually exclusive, some researchers are beginning to find that an individual can simultaneously hold learning and performance goals. These goals can also be supportive of each other (Dweck, 2002; Hannula, 2004; Harkness, D'Ambrosio, & Morrone, 2007). As part of a larger study, Hannula (2002) interviewed two middle school students, Maria and Laura. Hannula found that both girls simultaneously held a learning goal and a performance goal for doing mathematics. Due to this observation and a theoretical background of self-regulated learning, Hannula (2002, 2004, 2006) theorized students could have a variety of personal needs and goals, not just those few outlined by research. For one student, competence and social status were both perceived as needs, but social status was of more value to the student. Likewise the student had various goals, but the goal to perform was more important than the goals to understand and avoid failures.

Hannula (2004) suggested each student held their own individualized needs-goals structure. Furthermore, Hannula surmised that emotions were important indicators of motivation. Hannula (2006) also said, "Motivation cannot be directly observed," it is only manifest in "affect, cognition, and behavior" (p. 175). Thus, by combining these theoretical beliefs and the data shared earlier, Hannula (2006) conceptualized a new definition of motivation: Motivation is "a potential to direct behavior through the mechanisms that control emotion. This potential is structured through needs and goals" (p. 9).

Dowson and McInerney (2003) also recognized that students possess multiple goals. Through classroom observations and individual interviews, the researchers identified nine different social and academic goals held by middle school students in general. The academic goals were categorized as mastery, performance, and work avoidance goals. The social goals included: social affiliation, social approval, social responsibility, social status, and social concern. Dowson and McInerney suggested that students can possess more than one goal at a time and that motivational orientations "comprise a much more complex and dynamic system than has been acknowledge[d] in the [goal orientation] literature" (p. 109). Nuttin (1984) described why research about students' personal goals could be problematic, "the multiplicity of goals sought by man represents a chaotic puzzle that is inconsistent with the simplicity sought by science" (p. 83). Such a chaotic puzzle becomes even more difficult when it is realized that each individual has a unique set of personal goals that vary over time.

Personal Construct Theories

Personal construct psychology was developed by Kelly (1955). Kelly believed people are scientists in that they construct knowledge about their world through experience and use that knowledge to predict future events. Individuals build up their own personal theories about the world and act accordingly. These theories are filled with personal constructs. Constructs are ideas of reality forged by observation or experience that consist of two extremes—such as sad and happy. As individuals go throughout life, they build ideas of what sad people are like and what happy people are like and then place everyone they meet somewhere along the sad/happy continuum. Individuals go through a similar process for every personal construct they hold.

Studies in education using personal construct theories have found that motivations are highly individual (Dweck & Molden, 2005; Hannula, 2006; Middleton, 1995; Owens, 1987). One of the limitations of the usefulness of these theories for educational practices is that if motivation is so individual in nature, it would be onerous for a teacher to try to identify and cater to all the different personal constructs of students.

Intrinsic vs. Extrinsic Motivation

Studies across theoretical orientations recognize the dichotomy between two types of motivation: intrinsic and extrinsic (Corpus, McClintic-Gilbert, & Hayenga, 2009; Harter, 1980; Murphy and Alexander, 2000; Renninger, 2000; Ryan & Deci, 2000). Intrinsic motivation is where “a task is performed because it is rewarding within itself not because of a reward to be earned as a consequence” and extrinsic motivation is “performing a task to get something outside of the activity itself” (Whang & Hancock, 1994, p. 306).

Intrinsic behavior was first noticed while observing animal behavior (Ryan & Deci, 2000). Researchers discovered that animals would engage in playful and exploratory behaviors even in the absence of reinforcement. Although the origins of intrinsic and extrinsic motivations in the classroom are unclear, these constructs can be explained by elements of Weiner's (1972) attribution theory, Bandura's (1977) conception of self-efficacy and other prominent motivational theories. In behavioral theories, intrinsic motivation is evident where the action itself is the reward. Students are extrinsically motivated when they are given an incentive to do their work. In attribution theories, intrinsic and extrinsic motivations are described by the locus of control. If the locus is internal, then the motivation is said to be internal. If the locus is external, then motivation is extrinsic. In a goal theory orientation, a learning (or mastery) goal is associated with intrinsic motivation and a performance goal with extrinsic motivation.

Generally, positive connotations are often thought to be associated with intrinsic motivation and negative connotations with extrinsic motivation. Some researchers found that students who are intrinsically motivated spend more time engaged in the activity, learn better, and enjoy the activity more than those who are motivated extrinsically (e.g. Lepper, 1988). Despite this, Cameron, Banko, and Pierce (2001) discovered that providing extrinsic rewards, related to performance, can sometimes enhance intrinsic motivation. On the other hand, superfluous extrinsic rewards or extrinsic pressures have been found to decrease student's intrinsic motivation and academic involvement in the future (Condry, 1977; Deci, 1975; Lepper & Greene, 1978).

In order to give more credibility to the notion of extrinsic motivation, Ryan and Deci (2000) revised the general definitions of intrinsic and extrinsic motivation. For Ryan and Deci, intrinsic motivation is "doing something because it is inherently interesting or enjoyable" and extrinsic motivation is "doing something because it leads to a separable outcome" (p. 55). Using these definitions, Ryan and Deci claim there are various levels of extrinsic motivation, some of which are self-regulated. In other words, a person can be extrinsically motivated without feeling pressure to act in a certain way. For example, a student might be motivated to participate in mathematics because it will help them gain a better career. Though such a reason would be considered an extrinsic motivation, it is not necessarily a bad one. Also, since certain activities and subjects are not inherently interesting to a child, Ryan and Deci maintained the view that extrinsic motivation is necessary. The quest for educators then becomes "how to motivate students to value and self-regulate such activities… without external pressure" (p. 60). Although Ryan and Deci shed a more positive light on extrinsic motivation, their view still maintains a dichotomy of motivations—either an individual is intrinsically motivated or extrinsically motivated to act in a certain way.

Commonly held distinctions between intrinsic and extrinsic motivations may be insufficient to enlighten our understandings of student motivations in learning mathematics or to appropriately shape pedagogical decisions. Lin, McKeachie, and Kim (2003) suggested that "intrinsic and extrinsic motivation, rather than being at opposite ends of a single dimension, may be much more complex in their relationships with one another and other variables affecting student achievement" (p. 253). Partitioning motivation as inherently intrinsic or extrinsic may foster inadequate and potentially erroneous views of lived experiences and choices by learners in classrooms.

Findings and Significant Statements of Motivation Literature in Mathematics Education

In order to inform pedagogical decisions to help students learn, many mathematics teachers seek to understand what motivates their students. Middleton (1995) asked both middle school students and teachers about their beliefs as to what makes mathematics intrinsically motivating for students and found that teacher's beliefs played a big role in the types of activities they provided in the classroom. Also, the better teachers were at anticipating their students' motivational beliefs, the better they were at providing an environment that fostered intrinsic motivation. Middleton concluded, "Overall, teachers were poor at predicting their students' motivational constructs" (p. 276). It seems that as teachers, we need to learn more about student motivations in learning mathematics. Unfortunately, in mathematics education, "motivation has not been a popular topic of study lately" (Hannula, 2006, p. 165). Those who do write about motivation in the mathematics classroom tend to focus on motivational constructs such as ethnicity, interest, engagement, affect, self-regulated learning, and ethnicity.

Ethnicity and Motivation

Many of the studies reviewed with respect to motivation in mathematics centered on motivational differences across ethnicities, especially between Asians and Caucasians (Chen & Stevenson, 1995; Lee, Tinsley, & Bobko, 2003; Leung, 2002; Rao, Moely, & Sachas, 2000;

Salili & Lai 2003; Stevens, Olivarez, Lan, & Tallent-Runnels, 2004; Treisman, 1992; Whang & Hancock, 1994). International comparisons of mathematics achievement have consistently shown Asians outperform their American counterparts (Chen & Stevenson, 1995; Stevenson & Stigler, 1992; Wang & Lin, 2005). Due to these findings, some researchers sought to determine factors that contribute to greater success for Asian students—including motivational factors. Many factors were familial, cultural, and social. According to Chen and Stevenson (1995), Asian students are held to higher standards by their parents to achieve, they study more diligently, and have less outside interferences to compete with their time.

Although students in Asia showed superiority in standardized tests, Chinese students have been reported to have lower self-efficacy than Caucasian American students (Leung, 2002; Whang & Hancock, 1994). One might conclude, therefore, that Chinese students may not feel as confident in their abilities to do well in mathematics or that they had a more accurate perception of their abilities than did Americans. Another interpretation may conclude that some American students reported over-confidence in their abilities because they did not want to be considered less intelligent than their peers.

Another difference between Asian students and Caucasians was found in their attribution beliefs. In general, Asian students generally believed that controllable factors, such as hard work and effort, undergird success. On the other hand, Caucasians attributed their success to external factors such as having innate ability or having a good teacher (Chen & Stevenson, 1995; Salili & Lai, 2003; Stevenson & Stigler, 1992, Yan & Gaier, 1994). Despite these findings, other research suggested there is no difference in attribution across ethnicities–students across the world believe effort is important to achieve in mathematics (Elliott & Bemperchat, 2002; Leung, 2002).

Lack of success in minorities has been studied as well. For example, Treisman (1992) wanted to know what kept Blacks, Hispanics, Native Americans, and rural Caucasians from performing as well as the other students in the University of California, Berkeley calculus classes. At first, part of the hypothesis was that these minority students were not as motivated to achieve as were their Asian and affluent White counterparts. However, after completion of the study, a different conclusion was drawn. Treisman wrote, “These kids [minorities] were motivated! Unfortunately, we had been mistaking ‘disorientation’ for lack of motivation” (p. 366). In a comparison of Black students and Chinese students, it was found that the Chinese students spent more time working together on calculus problems. After implementing a course that was designed to help students realize the importance of working together, vast improvements were made. In fact, “Black and Latino participants…substantially outperformed not only their minority peers, but their White and Asian classmates as well” (Treisman, 1992, p. 369).

Interest and Motivation

According to Dewey (1913), learning based on intrinsic interest is qualitatively superior to coercive learning. Furthermore, Dewey postulated that emphasis on items peripheral to an object/task in order to make it more interesting do not work. He said, “When things have to be made interesting, it is because interest itself is wanting. Moreover, the phrase is a misnomer. The thing, the object, is no more interesting than it was before. The appeal is simply made to the child’s love of something else” (p. 11). Dewey believed researchers needed to go beyond the task and the subject to see motivation. A reason must be found in the person, apart from the subject matter, to give the lesson a moving force.

Interests are often assumed to be "intrinsic motivational determinants of academic achievement" (Koaler, Baumert, & Schnabel, 2001, p. 448). Over 100 studies have documented a positive correlation between interest and achievement (see Schiefele, Krapp, & Winteler, 1992). In one particular study, 7th and 10th students in Germany were asked about their interest in math and the value math had for their lives. Students who reported more interest in mathematics tended to go on to take higher level classes in mathematics and had greater test scores (Koaler, Baumert, & Schnabel, 2001).

Engagement and Motivation

Engagement in mathematics is a major part of current motivation literature. In a traditional perspective (see McMahon & Portelli, 2004), the definition of engagement is discussed in behavioral terms. "Engaged students attend their classes, try reasonably hard to do well in them, complete the homework they are assigned, and don't cheat" (Steinberg, 1996, p. 67). Some researchers and many educators wonder what motivates students to engage in mathematics in this traditional sense. Sullivan, Tobias, and McDonough (2006) found that some students might purposefully choose not to participate during mathematics classes. When pulled out of class to work on mathematics with the researchers, students exerted effort the entire time. In contrast, the same students would not engage during their normal mathematics classroom. Social pressures were evident in students' interview responses that it was not "cool" or "popular" to be good at math and that it was easy to "pass" mathematics courses without trying. Thus, Sullivan, et al. claimed that motivation "may be as much a product of group or cultural factors as individual goals" (p. 91).

Williams and Ivey (2001) found the engagement pattern of one student, Bryan, to be particularly fascinating. In observing Bryan at the first of the year in his beginning algebra class, the researchers noted, “Bryan’s face registered no interest, no involvement, virtually no expression” (p. 83). However, during the first group work of the year, Bryan’s engagement changed drastically. He was smiling, answering questions, and solving the problems at hand. Then a few days later, he was back to his old behaviors of apathy and impassiveness. Williams and Ivey examined several current theories of motivation, including goal orientations, to try to elucidate Bryan’s changing engagement. Nevertheless, “none of the motivational frameworks discussed…is sufficient by itself to explain Bryan’s attitudes” (p. 92). The researchers suggested that current motivation theories “focus on the ‘what’ or the ‘why’ but have forgotten the ‘who’” (p. 96). Such findings suggest the need for a theory of motivation grounded in actual student actions in the mathematics classroom.

Affect and Motivation

Affect is often discussed as a component of motivation. Affect has been defined as “a feeling or emotion as distinguished from cognition, thought or action” (Huitt, 2003). Studies on affect in the mathematics classroom focused on attitudes, beliefs, emotions (McLeod, 1994), values, ethics, and morals (DeBellis & Goldin, 1997).

Part of the research on affect in psychology revolves around emotions. Davis (1955) said, “The human mind…is devastatingly subject to the influence of the emotions” and that this influence is a central “problem of good teaching” (p. 133). Meyer and Turner (2002) discussed the historical separation of motivation and emotion in extant research. Meyer and Turner felt emotions are “important mediators of motivated actions to approach or avoid learning rather than merely as outcomes” (Meyer & Turner, 2002, p. 110). In other words, emotions are not just results of actions; they help individuals decide what to act upon. In addition to needs and goals, Hannula (2006) also looked at the importance of emotions for motivation. He suggested that “emotions are the most direct link to motivation” (p. 167). Hannula studied a student’s behavior during problem solving, his responses to an Online Motivation Questionnaire (OMQ), responses to a Mathematics Related Beliefs Questionnaire (MRBQ), and student interviews. The student, Frank, was found to have a need for competence and a need to please his teacher. Franks goals included: doing well in mathematics, solving the problem at hand, being fluent, and mastering the content. When Frank was able to have his needs fulfilled by accomplishing his goals, he was happy and this emotion allowed him to be more motivated to do mathematics.

Another portion of affect literature is dedicated to student’s self-efficacy beliefs. Self-efficacy refers to a student’s assessment of their potential to accomplish a certain task or succeed in a particular subject matter (Bandura, 1997; Mayer, 1998; Schunk, 1991). Researchers found that students tend to learn from an early age that success in the classroom is valued in society and the more they perceive they will succeed, the more they will want to achieve (McClelland, 1965; Stipek, 1984). When students had high self-efficacy, they were more motivated and achieved more in the mathematics classroom (Stevens, Olivarez, Lan, & Tallent-Runnels, 2004; Schunk & Hanson, 1985).

In order to measure mathematics self-efficacy in Texas high school students, Stevens et al. (2004) employed an instrument formed by Pajares and Graham (1999). Using an 8-point Likert-scale, students were asked to rate their confidence in solving specific problems. The instructions stated: “Suppose that you were asked the following math questions in a multiple choice test tomorrow. Please indicate how confident you are that you will give the correct answer to each question” (p. 212). The researchers also measured motivation using a scale of intrinsic versus extrinsic motivation. Stevens, et al. (2004) found that “self-efficacy predicts motivational orientation and mathematics performance” (p. 208) and that self-efficacy strongly correlated with whether or not students continued to take additional mathematics courses. Furthermore, no distinction was found in the motivational systems across ethnicity—Hispanic and Caucasian students.

Self-regulated Learning and Motivation

Self-regulated learning "refers to the self-directive processes and self-beliefs that enable learners to transform their mental abilities, such as verbal aptitude, into an academic performance skill, such as writing" (Zimmerman, 2008, p. 166). Students who used more self-regulatory strategies have been found to attain greater academic performance and acquire higher grades than students who used little self-regulatory strategies (Pintrich, Smith, Garcia, & McKeachie, 1993). It is also of interest to note that students' awareness of choice is an important element of self-regulated learning (Winne & Perry, 2000).

Boekaerts (1999) presented a three layer model of self-regulated learning. The three layers, starting with the inner layer, were: regulation of processing modes, regulation of the learning processes, and regulation of the self (p. 449). Motivation, or directing one's behavior, is discussed as part of the second layer, regulation of the learning processes. Schmitz and Wiese (2006) wanted to see how self-regulated learning training affected, or was related to, motivation. University engineering students answered questionnaires and surveys about aspects of self-regulated learning and motivation. The conclusion was that "students who received self-regulatory training displayed significant improvements in the following questionnaire measures: intrinsic studying motivation, self-efficacy, effort, attention, self-motivation" (Zimmerman, 2008, p. 174). In other words, when students learned how to regulate their own learning, they displayed more interest in the subject, felt more confident in their mathematical abilities, and exerted more effort in the mathematics classroom.

Other Motivation Findings in Mathematics Classrooms

Francisco (2005) conducted a longitudinal study with a group of five high school students that sought to describe students' views on mathematical learning. Clinical interviews were conducted with each of the students. Five themes emerged from students responses to these interviews, one of which was motivation. One student said "doing interesting stuff" and "being into what's going on" helped him "do math all day" (p. 63). Other sources of motivation in mathematics included interesting tasks, social environment, opportunity to discover, using objects, knowing why, and helping others. Francisco (2005) said his findings "challenge the prevalent view that motivation is only a task-related issue" (p. 67).

Middleton and Spanias (1999) reviewed the findings in research on motivation in mathematics and found some consistencies in the results as well as some deficiencies (which will be reported on later). The following were five commonalities in the findings:

> First, findings across theoretical orientations indicate that students' perceptions of success in mathematics are highly influential in forming their motivational attitudes.
> Second, motivations toward mathematics are developed early, are highly stable over time, and are influenced greatly by teacher actions and attitudes.
> Third, providing opportunities for students to develop intrinsic motivation in mathematics is generally superior to providing extrinsic incentives for achievement.
> Fourth, inequities exist in the ways in which some groups of students in mathematics classes have been taught to view mathematics.
> Last, and most important, achievement motivation in mathematics, though stable, can be affected through careful instructional design. (pp. 79-82)

Deficiencies in the Literature

The following paragraphs summarize deficiencies found in extant mathematics education motivation literature.

Definitions of Motivation

There is no common definition of motivation in studies dealing with motivation. Some researchers do not even define motivation, hoping it would be apparent to the readers. Many who do state a definition do not mention students' agency as a central feature of motivation. If one believes that personal agency is essential to purposeful decision making, then it follows that students would be able to choose from among motives and decide which to act upon.

Theories of Motivation

Studies on important related topics such as engagement, interest (Renninger, 2000), affect (Zan, Brown, Evans, & Hannula, 2006), self-efficacy (Bandura, 1997), competence (Urdan & Turner, 2005), achievement goals (Urdan & Midgley, 2003) or motivational beliefs (Middleton, 1995) do not provide a level of detail sufficient to reveal the complexities of motivations exhibited by students during mathematical argumentation, collaborative problem solving, and learning mathematics with understanding in actual classrooms. Hannula (2006) suggested the lack of detail is, "perhaps inevitable, given that the authors' approaches aim to measure predefined aspects of motivation, not to describe it" (p. 166).

Motivation research studies in mathematics education tend to be atheoretical, lacking a theoretical basis to support claims or predictions for learning and teaching (Middleton & Spanias, 1999). For example, atheoretical studies may compile lists of motivations (such as wanting to win the teacher's approval, wanting to develop self-confidence, and wanting to contribute a new idea) to reveal that the "diversity and depth of these phenomena [are] never exhausted, however long and carefully the teacher may observe the student's motives" (Davis, 1955, p. 134). Hence, atheoretical lists do little to help us understand the dynamic powers of student motivations in learning mathematics with understanding. In contrast, this study builds theoretical constructs from which inferences concerning student motivations emerge.

Measures of Motivation

Traditional measures of student motivation have been criticized as being "static, in the sense that a student's momentary response to a test task is viewed as an indicator of his or her position on some scale representing a certain underlying trait" (Järvelä, Salonen, & Lepola, 2002, p. 210). These traditional methods of measuring motivation tend to label students with particular motivational traits without observing their behavior in the classroom. Research evidence has shown that how students behave in the classroom is not always consistent with student's self-reported intentions and desires. Actual enacted behavior depends largely on specific learning contexts (Boekaerts, 1996). Thus, the On-line Motivation Questionnaire (OMQ), a self-report measure of motivation, was developed to assess student's "situation-specific motivational beliefs (appraisals) and emotions" (Boekaerts, 2002, p. 80). Although the OMQ helps teachers and researchers understand how students feel about specific tasks, it does not explain student's motivations for enacted mathematical behaviors during the task.

Murphy and Alexander (2000) remarked, "one assumption seemingly underlying a segment of [motivation] research is that individual's motives, needs, or goals are explicit knowledge that can be reflected upon and communicated to others" (p. 38). Due to this assumption, most mathematics education studies "used self-report measures as indices of motivation without actually looking at and listening to children who are engaged in mathematical activity. The potential biases associated with self report measures of attitude have been clearly delineated" (Middleton & Spanias, 1999, p. 83).

Because most motivation theories have been "developed by psychologists and tested outside classroom contexts" (Urdan & Turner, 2005, p. 297), Urdan and Turner expressly point to a need for "inductive, grounded studies of motivation in classrooms" (p. 313). Järvelä et al. (2002) also suggested motivation be dynamically assessed in natural learning environments. Corpus, McClintic-Gilbert, and Hayenga (2009) utilized a "domain-general framework of motivation" (p. 164) to document within-year changes in students' intrinsic and extrinsic motivations and suggested that "Future domain-specific and person-centered approaches" (p. 164) are needed to help us understand the complexities of student motivation in mathematics learning in actual classroom settings.

Other Deficiencies

Middleton and Spanias (1999) reported some other deficiencies in extant research literature on motivation. For one, motivation is sometimes reported as an ancillary affect in studies designed to examine other factors (see Francisco, 2005). Another deficiency was that most research data are gathered under models of mathematics instruction that are not driven by students' conceptual development (Middleton & Spanias, 1999). This thesis was intentionally designed to identify student motivations for understanding mathematics in a conceptually based classroom setting.

Learning Mathematics *w*ith Understanding

According to Harel (2008), an essential feature of learning mathematics is the necessity principle. The necessity principle states, "For students to learn the mathematics we intend to teach them, they must have a need for it, where 'need' here refers to intellectual need" (p. 900). Although Harel makes a distinction between the necessity principle and motivation in his work, it is suggested in this thesis that recognizing a mathematical need encourages students to gain greater conceptual understanding of mathematics.

NCTM posits that conceptual understanding plays a critical role in learning and building mathematical proficiency (NCTM, 2000). "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000 p. 20). Students need to learn with understanding not only for gaining procedural fluency but also for establishing competency in mathematical reasoning, argumentation, and problem solving. Substantial evidence suggested that "[s]tudents will be served well by school mathematics programs that enhance their natural desire to understand" (NCTM, 2000, p. 21). Given my definition of motivation, students' *natural desires* to understand may reasonably be interpreted to mean students' *motivations* to understand.

Teachers rarely have opportunities to gain insight into students' motivations for learning mathematics with understanding. However, I suggest that careful consideration of motivation as the *powers*, or "tendencies in action" (Dewey, 1913, p. 62), for learning mathematics with understanding and "the ways in which these can be carried forward" (Dewey, 1913, p. 62) may comprise a fundamental fulcrum upon which classroom practices pivot. In other words, it would be beneficial if teachers could learn to realize the capacities various motivations hold to empower students to understand mathematics in the classroom. Then teachers could strive to foster such desires and encourage students to strive to develop them. This thesis aims to give more insight into the powers of student motivation to understand mathematics.

The Cat Task

According to Speiser, Walter, and Maher (2003), "For many students, to learn calculus with understanding poses special challenges" (p. 3). The task students worked on during this study is called the Cat Task. For the Cat Task, students were given a series of 24 photos taken of a cat in motion (Muybridge, 1887/1957). Students are asked to find how fast the cat was moving when photographed in two different frames, 10 and 20 (see methods section for more details). The Cat Task, also known as the Catwalk Task, has been successfully used in many calculus classes (Speiser & Walter, 1996; Speiser, Walter & Maher, 2003), including a calculus class of dance majors (Speiser & Walter, 1994) and a graduate mathematics education course for current and prospective teachers (Case, 2008; Rasmussen, 2008; Speiser & Walter, 2008). The goal of the Cat Task is to "motivate the derivative as a rate of change" (Speiser & Walter, 1994, p. 137) and "confront the basic concepts underlying calculus with the data that calculus is supposed to help us understand" (Speiser & Walter, 1996, p. 351). As derivative is a fundamental aspect of calculus, it is imperative students gain more than a procedural fluency in calculating derivative—they should understand the mathematical need for derivative and understand how derivatives are used in real-world situations. Another reason the Catwalk was used in the calculus classes was because it required students to make sense of real-world, discrete motion data as opposed to continuous functions. As Speiser and Walter (1994) pointed out, "The traditional calculus course views functions, rather than data, as the primary objects of study. The traditional pitfall is to think of a function as defined by a formula" (p. 150). With the Catwalk, "Data, not just functions, move to center stage" (p. 151).

Rationale for the Research Question

As described in the preceding review on extant literature, little is known about student motivation in the mathematics classroom. In viewing video data from an honors calculus classroom, I noted students often persisted in problem solving to build mathematics understanding. The deficits in current research and my observations of the honors calculus students led me to ask and search for an answer to the following question:

What insights might be gained about the powers and nuances of student motivation by studying the actions of students as they engage in mathematics problem solving in an inquiry-based calculus classroom?

This thesis will address this question by careful articulation of analyses of data and the resulting emergence of Contextualized Motivation Theory (CMT) in an effort to contribute to the body of research in motivation in mathematics education.

CHAPTER 4: METHODS

This qualitative study, using grounded theory techniques, was based on data collected from a teaching experiment at a large private university in the western United States. Two mathematics education professors taught experimental honors calculus courses at the university for three semesters. The first two semesters focused on building student understanding of topics in Calculus I whereas the third semester focused on Calculus II. Topics in Calculus I included: limits, continuity, the derivative and applications, extrema, the definite integral, fundamental theorem of calculus, and L'Hopital's rule. Calculus II focused on techniques and applications of integration, sequences, series, convergence tests, power series, parametric equations, and polar coordinates. Typically, the enrollment in each honors class was between 20 to 25 students. A team of graduate and undergraduate mathematics education majors helped gather data for the larger study. All of my data, excepting a follow up survey, were collected in the second semester of the Calculus I course, Fall 2006.

In designing this study, an honors calculus class was intentionally chosen because honors students are typically considered motivated to learn. There may be many other instances where students are less motivated. However, exploring the case in which students are motivated allowed for an analysis of data in order to discern and categorize these students' various desires to understand mathematics. The findings can then be employed to help those students, who may be less motivated, develop their own desires (Harel, 2008) to understand mathematics.

Classroom Atmosphere

The Honors Calculus I class met for two hours a day three times per week throughout the semester. The room was set up so students sat at tables, but the students also could see the whiteboards if they turned around their chairs. All materials in the classroom were available for students to use at their convenience. Examples of such materials were: graphing calculators, graph paper, rulers, overhead projector and sheets, white board and markers, big poster paper, and computers. During each class session, students worked in collaborative settings, in groups of 4-6 people, on open-ended mathematics tasks with limited teacher lecture.

Tasks were carefully designed and selected to elicit mathematical need and engage students in building fundamental mathematics through problem solving. Solving the tasks required the development of invention by students of the central tools of calculus. Students often presented their intermediate work as well as final solutions in whole class discussions. Homework assignments included final write-ups of student-developed solutions for each task and for student-posed extension problems as well as textbook exercises selected from the homework list for all university calculus sections assigned by the mathematics department. Formative and summative assessments were given periodically throughout the semester, as well as the same mathematics department final taken by all students enrolled in any Calculus I course at the university.

From the beginning of the semester, students were encouraged to explain their thinking and to provide compelling arguments for their mathematical actions. Students worked together to negotiate the meaning of the problems and the paths that should be taken to make progress. The instructors did not tell students how to proceed to solve any of the presented tasks. Instead, the teacher's role was to facilitate students' growth of mathematical understanding by listening carefully to student discourse, observing students' problem solving activity, and occasionally asking probing questions to prompt student explanation and reflection on their work. Instructors "made pedagogical decisions," including design or selection of tasks, "based on how students were framing or structuring and solving problems" (Walter & Hart, 2009, p. 164). If students asked if they were done or had the right answer, the instructors would respond with statements and questions such as: "Tell me what you have done," "What do you think?" "Help me understand your thinking," "Have you thought about the advantages and disadvantages of using the model you did?" and "What do you understand about the relationship between velocity and acceleration?" In this way, teachers placed the responsibility on the students to determine when they have understood the problem well enough and have come to reasonable conclusions.

Forms of Data Collected

Table 1 gives a timeline for data collection. I will elaborate on each form of data in the next few paragraphs.

Table 1

Timeline of data collected

Type of data	When collected	Who collected
1. Student Introduction Survey	Beginning of Fall 2006 semester, before class started	Principal researchers/ teachers of the larger calculus teaching experiment
2. Video data of class 3. Field notes	During every class period in the Fall 2006 semester	Graduate and undergraduate research team members
4. Student work: homework, write-ups, assessments	During the Fall 2006 semester when they were due or given	Teachers of calculus class
5. Transcripts of video data	After the course ended Winter 2007-Winter 2008	Graduate and undergraduate research team members, including myself
6. Follow-up survey	Winter 2008 semester	Research team

Before the course began, each student filled out a student introduction survey. In this survey, students listed their class standing, current or anticipated major, all of their previous math classes in high school and college, and scores on standardized tests (ACT, SAT, AP Calculus). In addition, students answered sixteen open-ended questions regarding their perspectives about mathematics learning and teaching (Student Introduction Survey, Appendix A). For example, one question asked, "What are the purposes of mathematics?" Another asked, "What do you find least appealing about mathematics? Why?"

Each class period during the semester, members of the research team videotaped the class. One camera focused on students and student inscriptions as they collaboratively worked on mathematics tasks, and another camera focused on student presentations or whole class discussions. Each video was transcribed. Verbatim transcripts were linked with video time codes. The transcripts were created line-by-line, with each new line and time code corresponding to a turn, meaning a change from one speaker to the next. Timecodes were used to help identify the passage of time between utterances as well as help researchers and readers identify certain episodes in the data. Transcripts were also annotated with interpretative phrases, and checked for accuracy by research team members.

After video transcription, particularly compelling episodes were identified within sessions grouped according to the tasks upon which students worked. Often, work on one task extended over several two-hour class sessions. Also, during each class, other team members took field notes to characterize student interactions and class activities. Researchers' field notes, students' homework, and exams were available for analysis. Student background information and pre- and post-course mathematical beliefs surveys contributed to the data corpus.

All the previously mentioned data were collected prior to my joining the research team. In addition to analyzing that data, a follow-up survey was used to support the grounded theory. The follow-up survey (Appendix D), written collaboratively by our research team, was sent out to each student from the class in Winter 2008, a little over a year after the class ended. The survey asked students what they thought about the class. I authored one question that was of particular interest to me: "During our honors calculus class, we often noticed that students would work to go beyond just finding a correct answer. When you did this, why? When you did not do this, why not?" This question was asked to see what motivated students to understand the mathematics. I did not ask students, "What motivates you to do mathematics?" Students have many different ideas about motivation, which are not necessarily consistent with my definition. I sought to collect data that would help describe student motivation in mathematics learning without using the actual word to elicit student responses.

The follow-up survey is a form of self-reported data about motivation. As previously stated, self-reported data is not always the best measure of motivation (Hannula, 2006). There are also potential biases associated with self-reported measures (Gall, Borg, & Gall, 1996; Pintrich & Schunk, 1996). Therefore, this additional piece of data will serve as secondary data to support analysis of student desires to understand mathematics.

Participants

Students voluntarily enrolled and self-selected into honors calculus classes, usually not knowing beforehand that the section in which this study was conducted was part of a teaching experiment. After the first semester of the project, some students knew about the class by word of mouth and made the decision to join the class. The determining factor of participation was that each student agreed to be a part of the research project.

This study reports on the mathematical actions of a group of six students seated at the focus table. At the beginning of the semester, the principal researchers chose a table to videotape so that student progress could be captured and carefully followed throughout the semester. Table 2 illustrates background information for each student.

Table 2

Background participant information

Participant's name	Year in school	Anticipated major	Last mathematics courses taken
Justin	Junior	Some type of Engineering	AP Calculus AB AP Statistics
Riley	Freshman	Undeclared	Pre-Calculus
Andrew	Senior	International Relations	Pre-Calculus
Daniel	Sophomore	Actuarial Science	AP Calculus AB Statistics (HS)
Derrick	Freshman	Mechanical Engineering	AP Calculus AB
Kacy	Freshman	Mathematics	AP Calculus AB

As explained earlier, these students took a survey about mathematics learning and teaching (Student Introduction Survey, Appendix A) at the beginning of the semester. One of the questions on the survey asked: "What do you feel are the responsibilities of a student in this course?" Initial responses to this question indicate that students expected to come to class every day to receive a lecture on calculus. Five of the six participants emphasized that it was important for students to come to class and pay attention to the teacher. Kacy stressed that students needed to be prepared for class so "they are not holding back the rest of the class."

When asked "What do you like most about mathematics?" the majority of students commented on there being definite processes or answers to math problems. Derrick's answer encapsulates the sentiments of the others. He said, "You know what you have to do to get the answer [in mathematics] and you will know if you are right or wrong. In a similar vein, Justin said, "the solutions to problems in math are definite and exact."

Another question on the survey stated "List three necessary qualities of an excellent mathematics learner." Responses to this question varied. Some of the qualities listed were: memory, focus, comprehension, organization, patience, humility, diligence, self-discipline, attention to detail, and application skills. Daniel also chose "desire to learn" as one of the necessary qualities of a mathematics learner. He said, "Having a desire to learn is important because it is the first step to learning anything. If a student has desires to understand the mathematics, they will work towards that goal."

Task

During the calculus course, students worked on many carefully selected tasks designed to help them build up essential conceptual understandings of the critical ideas of calculus. The analysis presented here is primarily of video data collected at the beginning of the semester while students were working on the cat task (Speiser & Walter, 1994) [1]. Walter and Hart (2009) described the reasoning for selecting the Cat Task for the study:

> Sessions during which students worked on the Cat Task were selected for presentation and analysis here, in part, because the task is designed to elicit student mathematics wherein the motivations of the students might be other than showing that they can recognize and follow a procedure or demonstrate technical skills. In this way, students are presented with opportunities to enact personal agency in mathematical choice for problem solving in an unfamiliar context. We wanted to know what motivates students as problem solvers at the initial points in their experiences in a particular class before formation of motivational patterns (Corpus, et al., 2009) or emphases by teachers translate into student endorsement of those emphases (Urdan & Turner, 2005). (p. 165)

[1] The cat task was initially created by Bob Speiser to challenge honors calculus students in creating meaning for the derivative as it relates to motion.

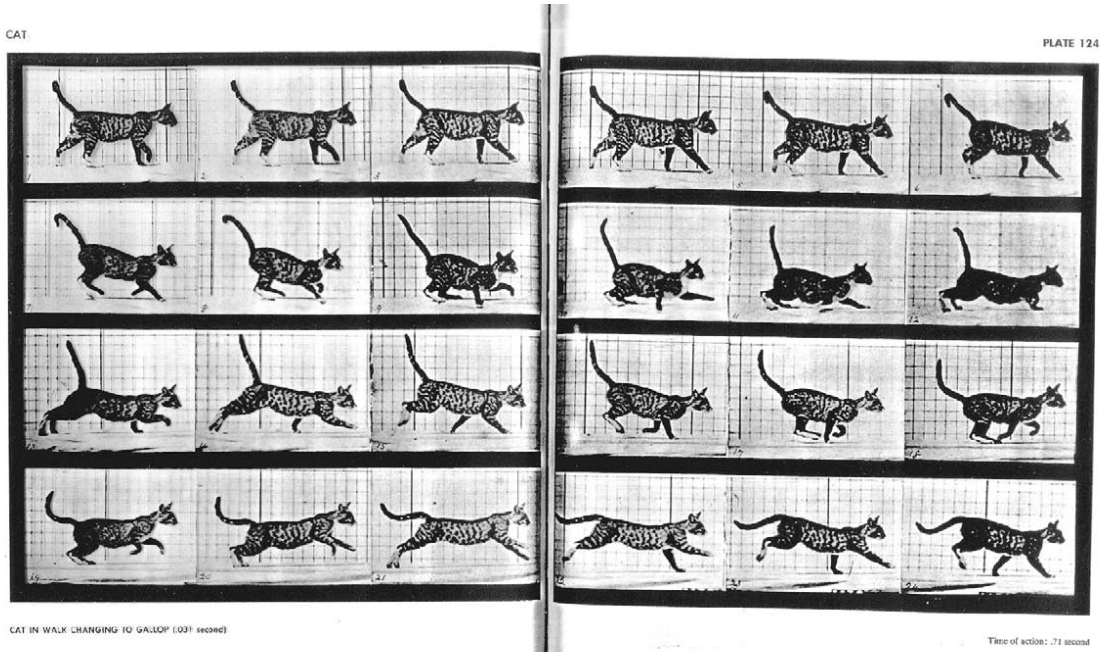

Figure 2: Series of photos (Muybridge, 1887/1957) given to the students as part of the Cat Task

The Cat Task involves a series of photos of a cat going from a walk to a gallop in front of a grid (Figure 2). The 24 photos, or frames, were taken at intervals of 0.031 seconds (Muybridge, 1887/1957). The gridlines are 5 centimeters apart, with a dark vertical gridline every tenth line. Students are invited to find how fast the cat was moving when photographed in frame 10 and in frame 20. The task was chosen to elicit the intellectual need for derivative as a mathematical tool in problem solving and to prompt students' conceptual development of the derivative without teacher lecture.

Prior to the cat task, students had not received instruction in this class on derivatives. Calculus ideas that the students had built up before this task include: making and interpreting distance and time graphs and computing limits. During the Desert Motion task (diSessa, Hammer, Sherin, & Kolpakowski, 1991), students also developed some basic understandings of displacement, velocity, and acceleration.

Constructing a Grounded Theory

A grounded theory approach (Charmaz, 2006; Corbin & Strauss, 2008) is appropriate for building contextual theory about motivation in learning through analysis of video data of students' mathematical problem solving. In this approach, "observed student powers and tendencies in action were analyzed to provide theoretical indices of motivation" (Walter & Hart, 2009, p. 164).

In particular, the data analysis procedure used for this study combines the grounded theory process described by Charmaz (2006) and a model for analyzing videotape data developed by Powell, Francisco, and Maher (2003). The process is outlined below and details will be given in the following sections.

- Data collection
- Developing a research question
- Viewing attentively the video data
- Identifying critical events
- Coding
- Memo writing
- Theoretical sampling
- Theoretical sorting and diagramming concepts
- Composing narrative

It is important to note that although steps are outlined in a specific order, previous steps were often revisited to strengthen the analysis. Charmaz (2006) described the grounded theory process by saying, "We start with gathering data and end by writing our analysis and reflecting on the entire process. In practice, however, the research process is not so linear" (p. 10). Refining analysis and revisiting data were recurring and important parts of each step of the process outlined above.

Data Collection and Research Question

The collection of data was elaborated upon earlier in this chapter and the development of the research question was explained in Chapter 3.

Viewing Attentively the Video Data

The primary data for this study were the classroom videos and verbatim transcripts of the videos. Integration of student write-ups of solutions to mathematics tasks, student introduction survey, and follow-up survey into the analysis will be explained later. Video data allowed me to look at and listen to students who are engaged in mathematical activity and use their actual mathematical experiences to build theory. In grounded theory, “data form the foundation of our theory and our analysis of these data generates the concepts we construct” (Charmaz, 2006). Thus, self-report is not the only index of motivation.

The videos of student problem solving on the cat task were viewed in their entirety several times as well as videos of students working on the task preceding the cat task. In this way, I became well acquainted with the environment of the class as well as with the students’ mathematics. Factual, time-coded descriptions of each video were written to construct a timeline of the classroom video data. For example, twelve minutes into the fifth class period, the following description was written:

> 12:35—Justin said it would be more precise to find the slope at the point 10 than to do the average between the two points in order to find the speed of the cat at frame 10. Justin suggested that someone try to figure out the slope. This episode is right before the group is going to present their work in progress to the class. The group decides to graph their cubic function on the calculator instead of drawing the graph on overheads.

During initial viewing of the video, video was watched and descriptions were written without "intentionally imposing a specific analytical lens" (Powell, et al., 2003, p. 415). Such a timeline made it easy to quickly locate particular video content and to situate each event relative to other events.

Identifying Critical Events

In order to describe and understand students' motives, I needed to characterize what students were doing, what choices they were making, and evidence of possible reasons for their activities. After several viewings of the videos, salient episodes of classroom happenings were selected as *critical events*. Critical events "demonstrate a significant or contrasting change from previous understanding, a conceptual leap from earlier understanding….[a critical event] may be any event that is somehow significant to a study's research agenda" (Powell et al., 2003, p. 416-417). At first, events for this study were deemed critical when students chose to engage in doing mathematics in any form during the Cat Task. After initial coding was completed, critical events became limited to episodes during work on the Cat Task when students chose to seek an understanding of the mathematics. Open codes such as questioning, persisting, explaining, and extending mathematical activity beyond answering explicit questions posed in the task, served as indicators of student motivation for learning mathematics with understanding.

Using Transana (Woods & Fassnacht, 2007), clips of the critical events were created to allow in depth analysis of small segments of video data. Transana also has features that allow researchers to create keywords and codes that link to specific clips. The coding that is essential to and emergent during the grounded theory process is described in the next section.

Coding

Open and focused coding (Charmaz, 2006) of selected classroom episodes supported the development of Contextualized Motivation Theory (CMT). Open coding is “breaking data apart and delineating concepts to stand for blocks of raw data. At the same time, one is qualifying those concepts in terms of their properties and dimensions” (Corbin & Strauss, 2008, p. 195). Open codes at the beginning of data analysis were developed to locate and note videotaped activities and student utterances. Open coding was done line-by-line for each class period in which students worked on the Cat Task. As might be expected, a single statement by a student was often coded with multiple open codes. Open codes reflected student language and activity. For example, when Daniel said, “I still don’t know the derivative of ‘Q’ though or what ‘Q’ is,” the transcript was memoed with open codes such as “don’t know” and “still” to note verbatim linguistic activity as possible indicators of motivation. Other open codes for that statement included derivative, formal notation, and looking at text to note mathematics content or students’ mathematical work or inquiry. Gestures, inscriptions, facial expressions, and emotions were also noted during open coding. For example, when a student spontaneously raised both arms and excitedly said, “got it” the open code “got it” noted emotion. Open coding also noted the indicators of student motivation for learning mathematics with understanding as described in the previous section.

During each phase of analysis, constant comparisons were made between data, categories, and concepts. This is known as a constant comparative method (Charmaz, 2006; Corbin & Strauss, 2008). In this phase, critical events were compared with each other and similarities were noted among several open codes. Open codes with common themes were clustered to create focused codes (Coding Organization, Appendix B). Focused codes are theoretical constructs from which inferences emerge regarding student motivations for learning mathematics with understanding. For example, one focused code was *precision*. It was observed that when students desired precision in problem solving, they were also striving to gain better understandings of the mathematics. Some open codes that were clustered in the focused code *precision* were mathematical activities, such as re-measuring distances, and verbatim phrases including “most accurate”, “as close as possible”, “more precise”, and “little more perfect”.

Focused codes within context were characterized according to intellectual-mathematical motivations and social-personal motivations. Intellectual-mathematical motivations are focused codes that note intellectual desires of students to understand mathematics. *Precision* is one example of an intellectual-mathematical motivation. Students also exhibited social-personal motivations. Social motivations include desires to belong to a community and to help others. For example, a social motivation in building understanding of mathematics is *a desire to communicate effectively* within a community of learners. Personal motivations are seen as desires to invest, define, or evaluate self. Personal motivations might include *self-investment* or *enjoyment*.

Memo writing

Writing of initial and focused codes was accompanied by memo writing. Powell et al. (2003) describes this process: “In our model, as researchers watch, describe, code, and otherwise attend to their video data they continually write in a notebook…about their emerging and evolving theoretic, analytic, and interpretive ideas” (p. 414). Whereas the codes were written in few words and in the same language the students used, memos were not similarly restricted. Insights, observations, and connections among events in the video as well as among the other forms of collected data were documented. These memos were in my own words and at times included connections to existing literature.

The following memo was written while watching video from the second hour of the ninth day of class, over a period of about 8 minutes.

> 11:23-19:55—The group is trying to figure out how to find a tangent line at a point which corresponds to a corner on a piecewise graph without using their calculator. The fact that the group knows how to find tangent lines using their calculator, but decides to use another way, is indicative that the students sought to understand the mathematics. At this point in the conversation, Derrick finds the equation of the tangent line in the book. Daniel looks in the book too and says: "Can you make any sense out of that?" Daniel poses a question to the group and the group decides to respond to the question. This happens a lot with this group. Someone asks a question or says they don't understand a concept and then the group works to answer the question. Derrick then says it [the equation of the tangent line] is close to the equation of the derivative. Sense making seems to be important to these students at this point in time. Also, here Derrick is trying to relate new concepts to his previous knowledge. After this, the group discusses what happens if you jump off a moving thing that is swirling in a circle, such as a merry-go-round. Perhaps students think the mathematics would make more sense if they could relate it to something they already understand. The group discusses which way they would fall off a merry-go-round and decide one would fly off linearly perpendicular to the circle. Derrick says: "Oh yeah! That makes sense!" Again, the word "sense" comes into the conversation. Andrew then says: "So we can draw a curve through those three points [meaning three discrete points representing the position of the cat at frames 9, 10, and 11] and find the tangent line through point 10." Here it can be seen that Andrew is able to apply the discussion of the playground equipment to what the group is trying to do mathematically in the Cat Task.

Memos and focused codes helped me to develop theoretical categories for motivations. After theoretical categories were initially developed, advanced memos helped illuminate connections among the focused codes. At this point, conceptual categories started to become more refined as the theory was emerging. The subsequent advanced memo, written from the same piece of video data used above (Day 9, Hour 2, 11:23-19:55), illustrates the use of focused codes and emerging theory in memo writing:

> The indication that these students are learning mathematics with understanding is their persistence in extending mathematical activity beyond answering explicit questions posed in the task. The students have a *desire to make sense* of tangent lines and how they relate

> to the Cat Task. This desire seems to be intellectual in nature. Other motivation researchers would say that the individuals in the group have a "learning" orientation instead of a "performance" orientation. Simply classifying it as a learning orientation does not adequately capture the complexities of what is going on here because we already know these students are motivated to learn and understand the mathematics. Furthermore, the students want to make sense of the mathematics as a community of learners. They seem to also desire to *build shared meaning* within the group. Finally, it is interesting that students rely on their knowledge from the playground to help them understand tangent lines. During the Cat Task, students have also related their knowledge of lasers, physics, and running track to help them understand the mathematics that emerged while working on the task. Because students often refer to other knowledge, I think they have a *desire to make connections* between their previous knowledge and new things they learn. Piaget talked about something similar—assimilation.

Theoretical Sampling

Theoretical sampling was done to further develop the theoretical categories. Theoretical sampling may be defined as "a method of data collection based on concepts derived from data" (Corbin & Strauss, 2008, p. 144). The follow-up survey (Appendix D) was one form of theoretical sampling. After I developed some theoretical categories, such as intellectual/mathematical motivations, the follow-up survey was sent out to help further develop these categories by using student responses to an open-ended question. Student write-ups of the cat task and the student introduction survey were two other pieces of data under the category of theoretical sampling. Even though these data were previously collected, they were analyzed after basic categories were developed from the video data.

The follow-up survey, student write-ups, and the student introduction survey were all analyzed using the same steps as the video analysis. In other words, identifying critical events, coding, and memo writing were done for these three pieces of data just as was done with the video data.

Theoretical Sorting and Diagramming Concepts

Memos were written on note cards with corresponding theoretical categories as their titles. The memos were sorted according to their titles and compared to other memos to discern relationships among various categories. Sorting memos and diagramming concepts were often done simultaneously. Whereas sorting memos gives us ideas, diagrams can also "offer concrete images of our ideas" (Charmaz, 2006, p. 117). Many different diagrams were constructed during analysis. For an example of one of the early diagrams made, see Appendix C (Analytic Diagram). During this step of the analysis, interpretive insights, observations, and diagrams served to build the narrative and refine important relationships within the structuring of the emerging theory. Corbin and Strauss (2008) stated, "When an analyst actually sits down to write a memo or do a diagram, a certain degree of analysis occurs. The very act of writing memos and doing diagrams forces the analyst to think about the data" (p. 118).

Composing Narrative

After all previously mentioned steps in the analytic process, a first draft of the findings was written. However, writing did not stop the refining process. I constantly revisited each piece of data and strengthened codes, concepts, diagrams, and theoretical categories. Many subsequent drafts ensued before the paper, and the theory, was finalized. In the following section, analysis is presented according to theoretical categories of CMT—classroom episodes are not always in chronological order.

CHAPTER 5: DATA AND ANALYSIS

I present typical transcript excerpts from classroom video to characterize student motivations for learning mathematics with understanding during problem solving and to demonstrate the grounded development of CMT. For clarity in analysis, interpretation, and theory building, transcript excerpts are formatted in columns to include, from left to right: video time codes; name of student speaker; verbatim utterances with bracketed annotations and bolded emphases; and open and focused codes. Italicized codes represent focused codes. Focused codes that are both italicized and bolded characterize student motivations. At first, open coding is displayed to illustrate how open coding was conducted. Open codes are then omitted in later transcripts in order to more prominently emphasize student motivations. Refer to Appendix F to see open codes for all transcripts (Coded Transcripts, Appendix F). Time codes identify the day and hour, (D-H), and the minutes and seconds, M:S, when each student began speaking.

The focus is primarily on a group of six students: Justin, Riley, Daniel, Derrick, Kacy, and Andrew. During the interval students worked on the Cat Task, these six students were in two different groups. Group 1 included Daniel, Andrew, Justin, and Riley. After a few days, some students switched groups so that Daniel, Andrew, Justin, Kacy, and Derrick were together in Group 2.

Intellectual-Mathematical Motivations

When presented with the Cat Task, students in the calculus class began to make sense of the problem. Much time was spent on measuring the position of the cat in each frame. The students in Group 1 determined the distances the cat moved in each of the 24 frames by using the gridlines in the photographs to measure the position of the cat's nose in each frame (Figure 3). In his final write-up Justin explained how the group started thinking about the problem. He wrote, "To start we began analyzing the photographs. We decided that all measurements will be made from the cat's nose. We decided that that nose represents the front of the cat and any flexing of the cat's body won't cause any confusion in measuring. Measurements began using the first visible solid line in frame 1 as 0 cm."

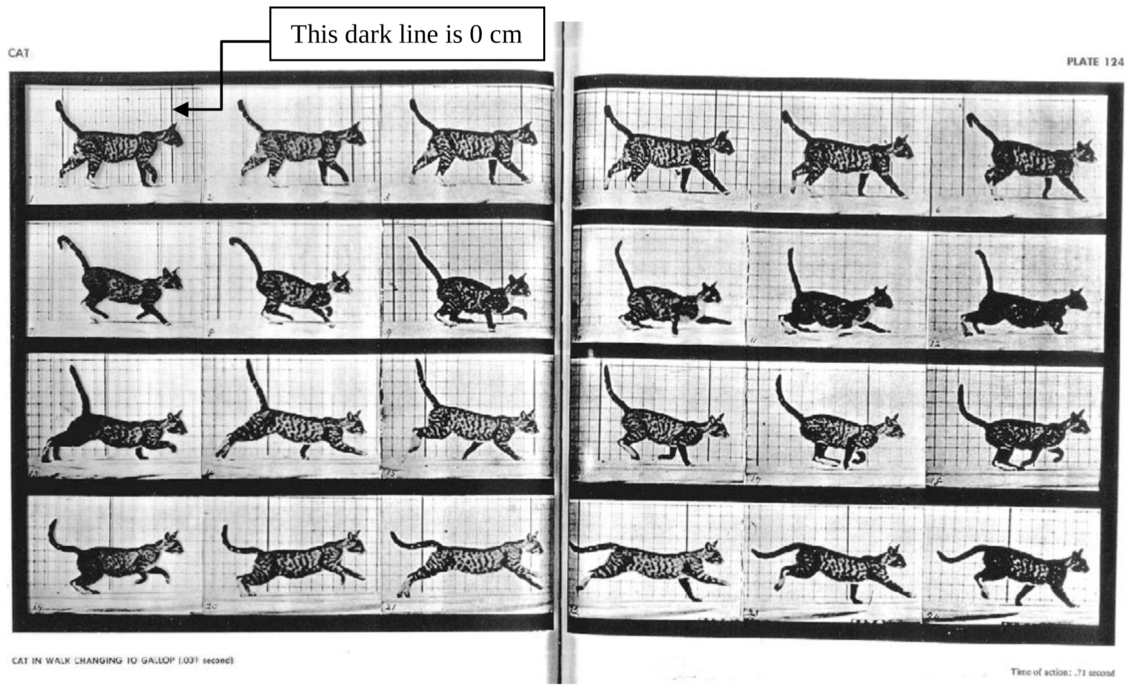

Figure 3: The cat photos again (Muybridge, 1887/1957). The arrow indicates the dark gridline from which students began measuring the cat's position.

The following table (Table 3) shows the measurements of the cat's position obtained through group effort.

Table 3

Group 1 measurements for the cat's position

Frame	Time (seconds)	Distance (cm)
1	0	5
2	0.031	5.5
3	0.062	7.5
4	0.093	9
5	0.124	10
6	0.155	11
7	0.186	11.5
8	0.217	12
9	0.248	13
10	0.279	15
11	0.310	25
12	0.341	26
13	0.372	36.5
14	0.403	45
15	0.434	52.5
16	0.465	59.5
17	0.496	65.5
18	0.527	75
19	0.558	85
20	0.589	95
21	0.620	109
22	0.651	115
23	0.682	125
24	0.713	136.5

Note. Distance was measured starting from the first dark gridline of the cat photographs.

After making a table of measurements, Group 1 plotted points to obtain a graph representing the cat's position (Figure 4).

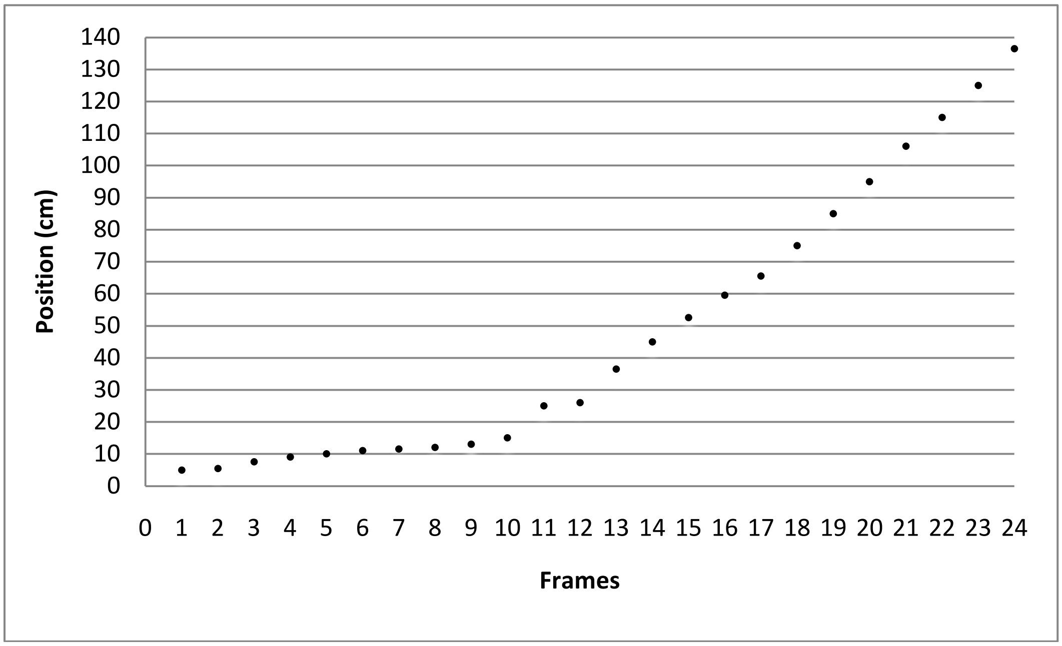

Figure 4: Recreated student graph representing the cat's position for each frame

Prior to the first episode presented here, students calculated an average rate of change between frames 9 and 10 by dividing the distance (2 cm) the cat moved by the time interval (0.031 sec) between frames to determine that the cat was moving 64.52 centimeters per second at frame 10 and chose this as an initial value for the rate of change at frame 10.

Then, Justin suggested the possibility of a three-step solution approach: 1) plot every frame number and corresponding distance as ordered pairs on a graphing calculator, 2) use the regression capabilities of the calculator to find a differential function to fit the data points, and 3) use the derivative function on the calculator to predict the instantaneous velocity of the cat at any frame.

At this point it is clear that Group 1 knew about the word derivative and knew that it could be found on the calculator. However, all of the students in the group seemed unsure about their understanding of the derivative (3-2, 26:44-27:13). Daniel voiced his uncertainty to the group (Figure 5, 26:44).

Timecode	Speaker	Transcript	Codes (*Focused)*
(3-2) 26:44	Daniel	I still don't know the derivative of "Q" though, or what "Q" is. [Looks at p. 76 in book-recreated below] $Q'(a) = \lim_{b \to a} \frac{Q(b)-Q(a)}{b-a}$	-Still don't know -Derivative -*Persistence* in understanding derivative and notations ***-Poses problem*** ***-Desire to know***

Figure 5: Transcript. Daniel desires to know

Instead of acting on Justin's earlier suggestion of using the derivative function on the calculator, Daniel extended mathematical activity beyond answering explicit questions posed in the task when he posed a problem by saying that he still did not "know the derivative of Q though or what Q is" (26:44). Because Daniel said, "still don't know", I infer that he desired to persist in trying to conceptually understand the derivative and associated textbook notations. Since persistence is viewed as an indicator of motivation, in this instance (26:44), the focused code *desire to know* notes Daniel's motivation to satisfy a mathematical necessity to conceptually understand derivative. Furthermore, Daniel's expression of his *desire to know* was also interpreted as problem posing by the rest of his group (Figure 6, 26:49-26:52).

26:49	Riley	"Q" is the quantity of the function [Quietly, matter-of-fact]	-Response -***Respond to student posed problem*** -Reads book
26:52	Andrew	Thank-you [chuckles]…what does that mean? [Garbled words]…we're trying to figure out what the book's talking about. 'Cuz I mean, we already know how to get the average [average rate of change], and it took us like ten minutes to figure out what he [book author] was talking about.	-What does that mean? -***Desires meaning*** for book language and notation -***Emulate other's use of agency*** -Question for understanding -Figure out -Ten minutes -*Persistence* in understanding average rate of change -Book notation -***Build shared meaning*** -Already know

Figure 6: Transcript. Riley and Andrew responding to Daniel

Riley and Andrew responded to Daniel's problem posing by also looking at the textbook to see if they understood the notation used for derivative (26:49-26:52). "In the excerpt provided above, note that these students' discussion was a reflection of their choice to act on a student-posed extension of the Cat Task. The extension, what was Q and what was the derivative of Q, was posed by a student after the intellectual need for the derivative had been elicited" (Walter & Hart, 2009, p. 168). Hence, *respond to a student-posed problem* was often a motivation for students to learn mathematics with understanding. Riley read from the book that "Q" was "quantity". However, Andrew, questioning for understanding, responded, "What does that mean?" For Andrew, at this juncture, a *desire for meaning* was a motivation for understanding derivative and the book notation. As Andrew said, the group could already compute an average rate of change, but then spent "ten minutes" trying to make sense of the book notation. Again, student persistence in sense making is noted as an indication of motivation for building conceptual understanding of mathematics (26:52).

After students in Group 1 worked to build shared meaning for the notation in the book, they decided that their somewhat protracted work to find the average rate of change corresponded to how the book "was talking" (Figure 7, 27:09) about the average rate of change.

27:09	Justin	To find out we already **knew** what he was talking about.	-We -***Build shared meaning*** for average rate of change -Already knew -*Statement of understanding*
27:13	Andrew	Huuh—that's horrible. [Laughs, a moment's silence.]…ok, I'll try an' figure out what he's talking about.	-Laughing -*Displaying emotion* -Try an' figure out -*Persistence* in understanding derivative ***-Extend scope of ideas*** -*Trying to figure things out*

Figure 7: Transcript. Extend scope of ideas.

Students in Group 1 might have chosen to proceed by taking the derivative on the calculator, or to stop working and settle with average rate of change for their answer to the task-posed question about how fast the cat was moving. Nonetheless, they chose to endeavor to make sense of the definition of derivative in the textbook (27:13). Therefore, to *extend scope of ideas* related to average rate of change was a motivation for these students.

Figure 8: Students in Group 1 responding to a student posed problem

After a minute and a half, Andrew and Daniel were still trying to figure out "Q" and what it is. The group had not yet come to an understanding of derivative as described in the book. In striving to understand derivative, these students had a desire to *adapt to mathematical norms* described by experts, such as a textbook in the previous episode, and, as in the next episode, mathematics instructors. Daniel decided to ask one of the instructors about the notation in the book (Figure 9, 3-2, 30:25). At this point in the data presentation, open codes will be omitted as explained previously.

(3-2) 30:25	Daniel	We understand "Q" signifies like—we figured out this [equation for derivative] could help us to understand exactly, like how to get the instantaneous--	-*Statement of understanding* ***-Precision*** ***-Adapt to mathematical norms***
30:29	Instructor	The instantaneous…[Daniel: Yeah] So is your question about what is "Q"?	-*Teacher influence*
30:32	Daniel	Yeah, "Q" defined in this, I guess [pointing to the equation students were discussing in the book]. I'd like to know how to find that too [another equation on the same page], but basically, I needed to know what "Q" was.	***-Desire to know*** ***-Recognize mathematical need***

Figure 9: Transcript. Precision as a motivation.

When talking to the instructor, Daniel stated that they thought the equation for derivative in the book could help them be *precise*, to "understand exactly, like how to get the instantaneous velocity" (3-2, 30:25) of the cat. The group decided they had already found average rate of change to answer how fast the cat was moving at frame 10, but they were also "trying to think how much farther [they] could go" (3-2, 35:58). Students decided that the instantaneous velocity would give a better answer for how fast the cat was moving at a particular frame than average velocity would. Students looked to the book as an authoritative source because they wanted to "understand exactly" how to find the instantaneous velocity. Thus, *precision* was a motivation for these students. The use of the word "understand" is of note here because it indicates that Daniel desires to know more than a procedure to find instantaneous velocity. It is clear from previous discussions and work that he has figured out how to substitute numbers into the equation. Instead, Daniel voices his group's desire to expand their current understanding on the mathematics they were developing while working on the Cat Task.

In order to better understand the equation for derivative in the book, Group 1 then talked about derivatives, limits, and velocity. They did not know much about limits, so the group started to discuss something they were familiar with, driving on the freeway, and applied it to the cat situation (Figure 10, 3-2, 33:07).

(3-2) 33:07	Justin	It's like when you're driving in your car, and you're looking at your speedometer, you just look at your speedometer, it's saying 65 mph, so instantaneous velocity is at, zero [the interval of time is zero seconds], because, at that exact moment; but what we're trying to work out here is, we're looking at two different time frames, and say, 'k, we're goin' 60 miles an hour-this is the time frame [at this time frame]-we're going 63 miles an hour at this time frame, what do we do if we're somewhere right in the middle? 's kinda what we're looking at.	-*Relating to other experiences* ***-Desire to make connections*** -*Trying to figure out*

Figure 10: Transcript. Desire to make connections.

All members of the group looked beyond the required answer for the Cat Task and were searching for a greater understanding of how to find the instantaneous velocity between two established discreet points. Students had calculated the average rate of change to find the average velocity between frames 9 and 10 to be 64.52 cm/s and between frames 10 and 11 to be 322.58 cm/s. However, students were striving to find the velocity exactly at frame ten, between two significantly different average velocities. To visualize the situation, students used the position graph displayed earlier (Figure 4) zoomed in on frames 9-11 (Figure 11).

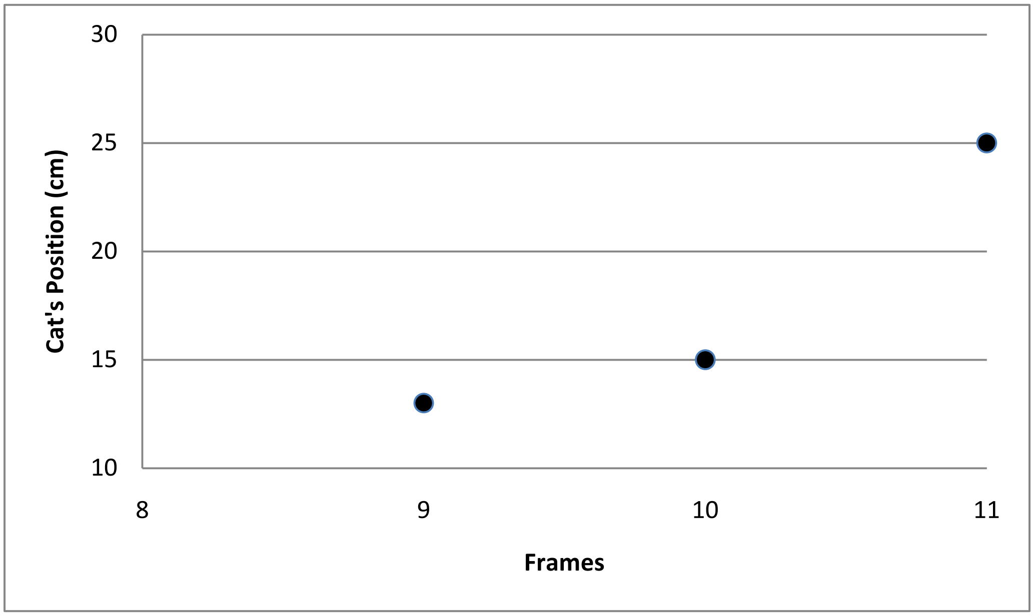

Figure 11: Graph of cat's position in frames 9, 10, and 11

At this point of the discussion (33:07), Justin compared the cat's movement to driving in a car. Justin argues that if you were in a car, you could tell the instantaneous velocity by looking at the speedometer at a specific time. But, Justin notes that the problem they have come up against in the Cat Task is more like knowing "we're goin' 60 miles an hour" during one time frame and "63 miles an hour" during another time frame, but wanting to know the speed "right in the middle" (3-2, 33:07). Because he related to other experiences, I infer that Justin had a *desire to make a connection* between his current knowledge and the task at hand. This *desire to make connections* was often a motivation for students to continue to make sense of the mathematics.

Prior to the next episode, Andrew asked, "how are we going to present this information" to the class? At this time, Group 1 did not have sufficient time to continue to explore their ideas on instantaneous velocity, so their preliminary answers for how fast the cat was moving at frame 10 and frame 20 were found by calculating the average rate of change from frames 9-10 and 19-

20, respectively. At frame 10, they decided the cat was moving at 64.516 cm/s and at frame 20, the cat was moving at 354.85 cm/s. However, Riley seems to have a concern about the obtained answers (Figure 12, 3-2, 45:28).

(3-2) 45:28	Riley	64.5 [cm/s] from 9 to 10 [their answer for average rate of change from frame 9 to 10]. The thing that bugs me on that one is from 10 to 11 I got 225 [225 cm/s-average rate of change from frame 10 to 11].	*-Inconsistency/cognitive conflict*
45:35	Justin	That's what- yeah, I was just looking at too. But that's just, it's the cat's accelerating really quick there.	***-Resolve inconsistencies/conflict***

Figure 12: Transcript. Students resolve inconsistencies.

Although Group 1 had an answer and were getting ready to present their results to the class, Riley was concerned about the answer they got for how fast the cat was moving at frame 10 (45:28). The group had calculated the rate of change from frames 9 to 10 to get their answer. However, Riley (and other members of the group) noticed that the rate of change from frames 10 to 11 was significantly different. There was an inconsistency among calculations which equated to a cognitive conflict for Riley. This inconsistency powered Riley and members of his group to look deeper into what was going on. Thus, a desire to *resolve inconsistencies and/or cognitive conflicts* became evident as motivations for these students to understand the mathematics. In essence, students were motivated to move from a state of disequilibrium to state of equilibrium (Harel, 2008).

As active learners, students continued to try to *extend the scope of ideas* associated with finding the cat's velocity at frame 10 by attempting to determine the slope of the tangent line, but they did not know how to situate the tangent line relative to a function that was nondifferentiable at frame 10. Upon completion of their work on the task, in his final write-up Daniel poses a question that highlights a mathematical necessity elicited by their work on the task, “The problem that arises is that one can take many tangent lines at .279 seconds-which is right?” Andrew, in his final write-up of the task, tied the group’s work back to the question about derivative that Daniel posed several days earlier (3-2, 26:44). Andrew summarized the groups’ *building of shared meaning* for derivative,

> As an extension to the project we explored tangent lines relative to finding the instantaneous velocity of the cat in frame 10. Although we were not able to find a way to find a tangent line that touches the curve of the line from points 9 to 11 we were able to solidify our idea that the average velocity obtained from frames 9 to 11 was as close to instantaneous velocity as were going to get. The idea of using the tangent line in accompaniment of a secant line is finding points on the curve of the line. If Q is the point that the tangent line touches the [curve and] P is the point that the secant line crosses the curve of the line. As Q and P are brought closer together the limit is made smaller and smaller. As the limit gets smaller the closer we get to the instantaneous velocity. The limit in this situation is the amount of time that has lapsed between the two points P and Q. In this project we were striving to bring the difference between points Q and P on the x-axis to be zero. The closest that we could bring these two points was 0.031 seconds.

Figure 13 represents Kacy’s drawing of Andrew’s explanation. (Note: Kacy’s points P and Q are opposite of Andrew’s description.)

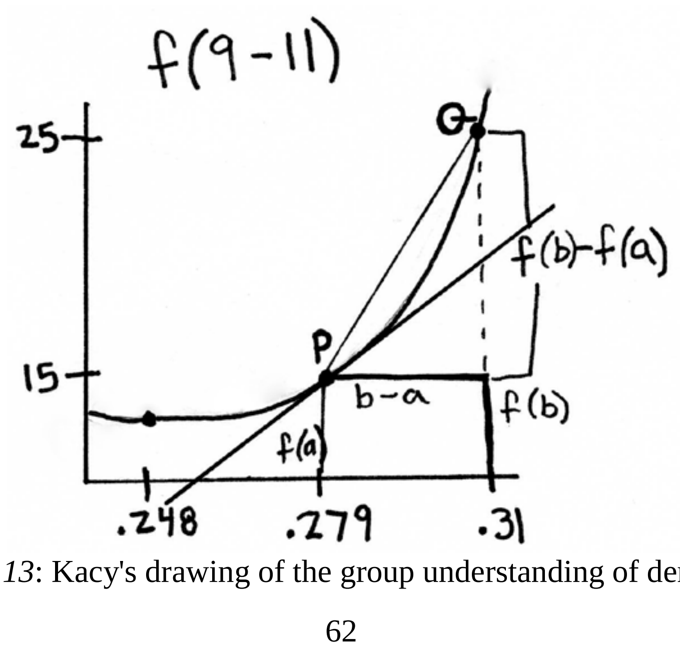

Figure 13: Kacy's drawing of the group understanding of derivative

Andrew's description of the group's motivation to *build shared meaning* will be discussed in the section on social-personal motivations. The focused code *recognize mathematical necessity* notes the intellectual-mathematical motivation that is fundamental to these students' building conceptual understanding of the definition of derivative and the role of tangent lines in finding instantaneous rate of change. Daniel posed a question, "which [tangent line] is right?" and Andrew shares the group response–they recognized the mathematical necessity of average rate of change imposed by discrete data, the mathematical necessity of a continuous, differentiable function over an interval including frame 10 in order to determine instantaneous velocity of the cat at frame 10, and the mathematical necessity of interpolation.

Confirming evidence shows that students chose to exhibit "intellectual passion" (Polanyi, 1958/1974, p. 142) in pursuit of clarity, meaning, and coherence in their mathematical work. Focused-code examples of intellectual passion–*desire to know, desire for meaning, recognize mathematical necessity, respond to a student-posed problem, adapt to mathematical norms, resolve inconsistencies and/or cognitive conflict, desire to make connections, extend the scope of ideas*, and *precision*–are all directly connected with the mathematics in which the students were engaged. Recognize mathematical necessity, desire meaning, desire to know, etc. are theoretical constructs for student motivations that emerged from open coding, focused theoretical coding and from the construction of stratified categories. Motivations of this nature were stratified as intellectual-mathematical motivations and make up one component of Contextualized Motivation Theory (CMT). Walter and Hart (2009) describe how the motivations in CMT are more than an inventory list:

> In contrast to providing an inventory list, our theoretical constructs resulted from analytic refinement of open and focused codes and our interpretive analysis demonstrates the

> relationships between these constructs. “An interpretive analysis invites the reader’s imaginative participation in related experiences through the *theoretical rendering* of the category...pure description, in contrast, invokes interest in, and often, identification with research participants’ stories” (Charmaz, 2006, p. 147). (pg.169)

Social-Personal Motivations

Social-personal motivations emerged similarly in the development of CMT. Social motivations include desires to belong to a community and to help others. Personal motivations include desires to invest, define, or evaluate self, which, if enacted, satisfy a personal need (Maslow, 1954), want, goal, or belief (Hannula, 2006; Deckers, 2001). However, in practice, trying to tease apart personal motivations from social motivations is similar to trying to tease apart the mathematics one learns from the contexts in which the mathematics is learned. For example, an individual’s “knowing is inherent in the growth and transformation of identities and it is located in relations among practitioners, their practice, the artifacts of that practice, and the social organization and political economy of communities of practice” (Lave & Wenger, 1991, p. 122).

In the episodes described above, students worked as communities of learners to *build shared meanings* for average rate of change and the derivative to answer the questions explicitly posed by the Cat Task and in response to student-posed problems. They collaboratively gathered measurement data to construct a collection of ordered pairs representing displacement of the cat. They calculated average rate of change and checked each other’s calculations. As evidenced by Andrew’s final write-up presented earlier, students succeeded to *build shared meanings* for the derivative. Within CMT, the focused code *building shared meaning* is a social-personal motivation.

Another social-personal motivation, closely tied to intellectual-mathematical motivations, was identified as *emulate others’ use of agency*. At times, students were seen to choose to persist in understanding the mathematics after they had seen one of their peers use their agency to seek understanding. Seeing and emulating choices made by others can be represented as a cycle (Figure 14) where choices are continually being acted upon.

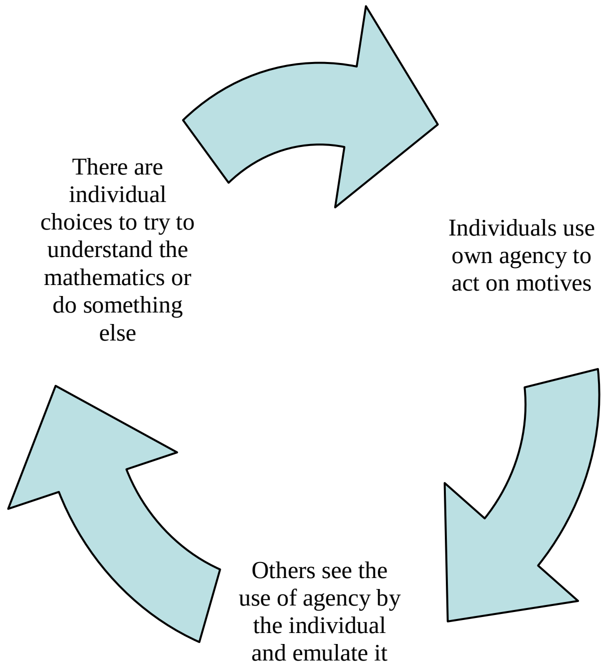

Figure 14: Agency emulation cycle

As an illustration of the motivation, *emulate others' use of agency*, I take the reader back to the first presented episode in this analysis (3-2, 26:44-27:13). In this instance, Daniel had a choice to try to understand derivative or just follow a procedure. Daniel expressed his *desire to know* more about derivative and chose between simultaneously existing intellectual-mathematical motivations. Riley, Justin, and Andrew (3-2, 26:52) *emulated Daniel's use of agency* by also choosing to learn more about derivative by *responding to a student-posed problem*. The group proceeded to critically examine notations in the book to extend their understandings of the derivative.

After Group 1 came up with their initial answers to the Cat Task, they were preparing to present their work-in-progress to the entire class. After a suggestion that the group get up and report their answers, Andrew used his agency to express a different desire (Figure 15, 3-2, 46:07).

(3-2) 46:07	Andrew	Well, we've got to present the information in a way that makes sense. [Daniel: Yeah, that's true] Ok, why don't we just make a table, you know, out of the rate of change from 9 to 10 and the rate of change from 19 to 20 and then that's how we display, you know the rate of change of the uh, centimeters.	***-Desire to communicate effectively*** ***-Consensus***

Figure 15: Transcript. Desire to communicate effectively.

Andrew said their group needed to "present the information in a way that makes sense" (3-2, 46:07). It is inferred that a *desire to communicate effectively* with peers was a social motivation for students when presenting their mathematical work either to the entire class or within a group.

Recall from before (3-2, 45:28) that Riley had a concern with finding how fast the cat was moving at frame 10 by calculating the average rate of changes from frames 9-10. Riley attempted to resolve his concern about the average velocity being so different from frames 9-10 and from frames 10-11 by subtracting the two average rates of change (225 cm/s minus 64.5 cm/s) to get 160.5 cm/s for the velocity at frame 10. Riley's ideas had been temporarily set aside in favor of preparing for the group presentation. However, in the next episode, Daniel decided to reintroduce and pursue Riley's concern (Figure 16, 3-2, 52:00).

(3-2) 51:57	Riley	About 354 [cm/s for frame 20] and 64.5 [cm/s for frame 10]. [Referring to the answers they will present to the class]	
52:00	Daniel	Yeah, 64.5 [cm/s] and I don't know though, that was kind of interesting what you [Riley] did though minusing the amount [subtracting velocities obtained from frames 9-10 and from frames 10-11]. I don't know if that gets closer.	***-Social responsibility and interaction***
52:11	Riley	That was the idea of at 11 o' clock you were moving 65 and at 12 o'clock you were moving 68. The difference is 3, like one subtracted from the other one.	***-Desire to make connections***

Figure 16: Transcript: Social responsibility and interaction.

Daniel chose to revisit Riley's idea by saying, "it was kind of interesting what you did" (52:00). Daniel acted upon a perceived *social responsibility* to consider other's ideas. Together with a *desire to communicate effectively*, the motivation of *social responsibility and interaction* proved to be one of the most salient of the social-personal motivations. This is also evident from analysis of answers given to a question given in the follow-up survey (Appendix D). The question asked: "During our honors calculus class, we often noticed that students would work to go beyond just finding a correct answer. When you did this, why? When you did not do this, why not?"

Part of Daniel's response included: "Other people's ideas also made me want to look deeper into different subjects." Justin said, "I remember many times not only would we try to find the correct answer, but we'd see if there were alternative methods in getting that answer. Since there were a number of us in our groups, each person thinks differently and therefore approaches each problem a different way. This leads to various ideas on how to solve it. It was helpful to view everyone's approaches." In their responses, Daniel and Justin voiced that a *social responsibility* to consider other's ideas led to desires to understand the mathematics and to explore alternative solution paths to the problem.

Andrew also perceived a *social responsibility* in collaborating with his classmates. In addition, a *desire to communicate effectively* and being able to participate in group interactions motivated him to persist in understanding the mathematics. Andrew said, "When I worked to go beyond finding an answer it was to participate in group discussions. I was interested in finding out what others had discovered, and they were interested seeing other ideas I had discovered on my own." Andrew's response also indicates that he was "interested" in the ideas that had been "discovered". This *enjoyment in learning* will be discussed in more detail later.

The next time the students had an opportunity to work on the Cat Task in class was on Day 5-two class periods later. There was some change in the group arrangement resulting in Group 2 together at the focus table: Derrick, Kacy, Andrew, Daniel, and Justin. Since the students formed a new group, they tried to understand what each other had done so far. Students in Group 2 were desirous to build *consensus* of meaning from work in previous groups so they shared their measurements and preliminary answers for the speed of the cat at frame 10. Derrick and Kacy got 161.29 cm/s and Group 1 (Andrew, Justin, Daniel) said they got 141.16 cm/s. The new group, Group 2, then started to think about the upcoming presentation they had to give on their work in progress.

During the second hour of class, Group 2 presented their findings to their peers. They explained how they determined the velocity for frames 10 and 20 by setting up an equation, velocity = change in distance divided by time, to get the answers discussed (Frame 10: 64.52

cm/s, Frame 20: 354.84). Then, they went on to talk about how they graphed the distance on the calculator, found an equation to match the data and took the derivative to get the velocity. The velocities that the group reported finding from using the derivative function on the calculator and the cubic regression are the following: Frame 10: 143.362 cm/s (3.2 mph) and Frame 20: 325.531 cm/s (7.28 mph). Then, members of Group 2 went back to their seats to discuss the problem. In the following episode, they discussed what they knew about the "r"-value they presented (Figure 17, 5-2, 31:21-31:47).

(5- 2) 31:21	Andrew	Make sure we're all on the same page.	***-Consensus*** ***-Build shared meaning***
31:26	Daniel	How do you do the, like "r"…	*-Posing problem*
31:29	Andrew	That's something like, I learned in my statistics class, [Daniel: 'k] 'cuz we use a lot of stat plots and all that stuff, and you have to find a line that uh, fits all the statistic plots when you're trying to find like y'know like define like trends or whatever [Daniel: Yeah] And so like "r" is just, can't remember how to find "r", I can't remember.	***-Desire to make connections***
31:47	Daniel	Wait-how did you find "r"?	*-Student posed problem*

Figure 17: Transcript. Group builds shared meaning.

When Andrew wanted to "make sure [the group was] all on the same page" (31:21), he sought to *build shared meaning* for mathematical ideas by coming to a *consensus* with his peers on presented work. In response to Andrew's statement, Daniel asked a question about the obtained "r" value (31:26). The cubic regression that the group found had an associated r value of approximately 0.996, representing a tight fit to the plotted data. Andrew shared the r value during the presentation but it was not previously discussed in their group. Since the group was working as a community of learners, they seemed to have a desire to come to a *consensus* on mathematical ideas in the Cat Task. This desire of *consensus* often led students to further explore and seek to understand the mathematics.

On the next day of class, Group 2 developed a velocity graph by plotting the change in distance for each consecutive pair of displacement, or position, data points (Velocity Graph, Appendix E). While they were working, Derrick mentioned how fast the time passed during class (Figure 18, 6-2, 50:02).

6-2 50:02	Derrick	This class seems to go by pretty fast [As they talk, everyone continues to work].	-***Self-investment*** *in focused effort*
50:05	Daniel	Yeah.	-***Consensus***
50:06	Derrick	That's a good thing.	-*Value judgment*
50:07	Kacy	It is.	-***Consensus***
50:08	Daniel	Cuz we're like learning and like, it's…entertaining I guess [rising intonation]?	-***Enjoyment in learning***
50:15	Derrick	Yeah, we're not just sitting here taking notes; we're actually like doing stuff	-***Active learning***

Figure 18: Transcript. Social and personal motivations.

In the preceding excerpt, Derrick said it was "a good thing" for the class to "go by pretty fast" (6-2, 50:02). When cognitive effort is sharply focused over time, awareness of temporal measurement is often lessened. Hence, the perception that time is passing quickly is heightened. Daniel credited his experience of that phenomenon to "learning" as "entertainment" (50:08). This idea of learning being fun was also described in several of the responses to the follow-up survey. Daniel said, "Everyday in coming to class has helped shape me and my opinion on what mathematics and in general, what learning can be. It can be something fun, engaging and ultimately a great learning experience." In the same follow-up survey, Justin commented, "It was a fun class. Can you believe I was actually excited to go to class? A math class none the less!"

Derrick also preferred to be actively engaged in “doing stuff” instead of “just sitting here taking notes” (50:15). There seemed to be consensus about personal motivations of *self-investment* with focused effort and *enjoyment in learning*, as well the intellectual-mathematical motivation for *active learning*.

CHAPTER 6: DISCUSSION OF FINDINGS

This chapter is dedicated to the introduction and elaboration of Contextualized Motivation Theory (CMT). In order to understand what CMT is, one must first understand what CMT is *not*. CMT is not designed to be a general theory of motivation—it does not explain the reasons for all of individuals' enacted behaviors. CMT does not describe student motivations to be *interested* in mathematics, student motivations to *achieve* good grades in mathematics, or student motivations to be *engaged* in the mathematics classroom. Finally, CMT does not attempt to explain instances in which students decided not to seek an understanding of the mathematics.

Contextual theories explain actions in terms of their circumstances. Hägerstrand (1984) explained, "Every action is situated in space and time and… its immediate outcome dependent on what is present and absent as help or hindrance where the events take place" (p. 377). CMT offers a lens for understanding the complexities of student motivations in mathematics learning within particular, contextual conditions. CMT states that student motivations for understanding mathematics fall under two major categories, intellectual-mathematical motivations and social-personal motivations. These motivations are manifested simultaneously within a supporting "web" of closely related contextual motivations. In seeking to understand mathematics, students choose to act upon one or more of their motivations at a time. Although students' motivational "webs" are individual in nature, groups of students can share motivations. Finally, the *powers* of motivations to understand mathematics are the same for all students. The subsequent paragraphs will serve to elaborate on the following constructs of CMT: Categories of Motivations, The Motivational Web, and Powers of Student Motivations.

Categories of Motivations

Complete analysis of extended student collaborations over several days grounded the development of Contextualized Motivation Theory (CMT). Here, from analysis of selected data, I note students' intellectual-mathematical motivations– recognize mathematical necessity, desire for meaning, desire to know, precision, extend scope of ideas, respond to student-posed problems, active learning, resolve inconsistencies and/or cognitive conflicts, adapt to mathematical norms, and desire to make connections. I also note students' social-personal motivations– build shared meaning, emulate others' use of agency, desire to communicate effectively, social responsibility and interaction, consensus, self-investment, and enjoyment in learning. These motivations are not intended to be comprehensive in nature—they do not encapsulate all motivations that a student may choose to act upon in a given situation.

Intellectual-mathematical motivations were the most frequently coded student motivations in this study—accounting for approximately 60 percent of the total motivations. Social-personal motivations made up the other 40 percent. At first, this finding suggested social-personal motivations were not as dominant in these students' motivational systems as intellectual-motivations. This conclusion may be valid since this thesis focused on motivations to understand the mathematics as opposed to just engaging in following mathematical procedures. However, when we remember the motivations were coded in situ—from classroom happenings, student write-ups, and student surveys—a different interpretation can be surmised. Students may possess many other social-personal desires such as the desire to please a parent or a desire to get a good job in the future. Such desires were not evident because the actual detailed "work" required to seek an understanding of mathematics during problems solving does not connect to the more removed motivational forces (parents, career) as do the intellectual/mathematical motivations. This finding has close ties to Harel's (2008) necessity principle. Recall that the necessity principle states that for students to learn mathematics, they must perceive an intellectual need for the mathematics. Thus, it would make sense that motivations to understand mathematics would be largely intellectual-mathematical in nature. The implications for such conclusions will be discussed in the following chapter.

Shah, Hall, and Leander (2009) advocate that individuals dynamically contextualize, manage, regulate, and prioritize choices of action. Indeed, in this study, students chose to act upon various intellectual-mathematical motivations and social-personal motivations, which are intricately intertwined with each other. For example, the social-personal motivation of *build shared meaning* may be closely related to and exist concurrently with the intellectual-mathematical motivations of *desire meaning* and *respond to student-posed problems*. The intellectual-mathematical motivation of *active learning* may be associated with a social-personal motivation of *emulating others' use of agency* (see Figure 19).

Intellectual-Mathematical ⟺	**Social-Personal**
Desire Meaning & Respond to Student-Posed Problems	Build Shared Meaning
Active Learning	Emulate others' use of agency

Figure 19: Relationship of intellectual-mathematical and social-personal motivations

The Motivational Web

Contextualized Motivation Theory would be hard to fully understand without relating it to something more familiar. For this reason, a spider web metaphor will be introduced to help illuminate some features of CMT. In CMT, the plethora of motivations one chooses to act upon can be represented metaphorically by a complex web (Figure 20) of related desires. Figure 20 provides a visual image of this web and the following paragraphs will address aspects of a spider web that can be compared to student motivation.

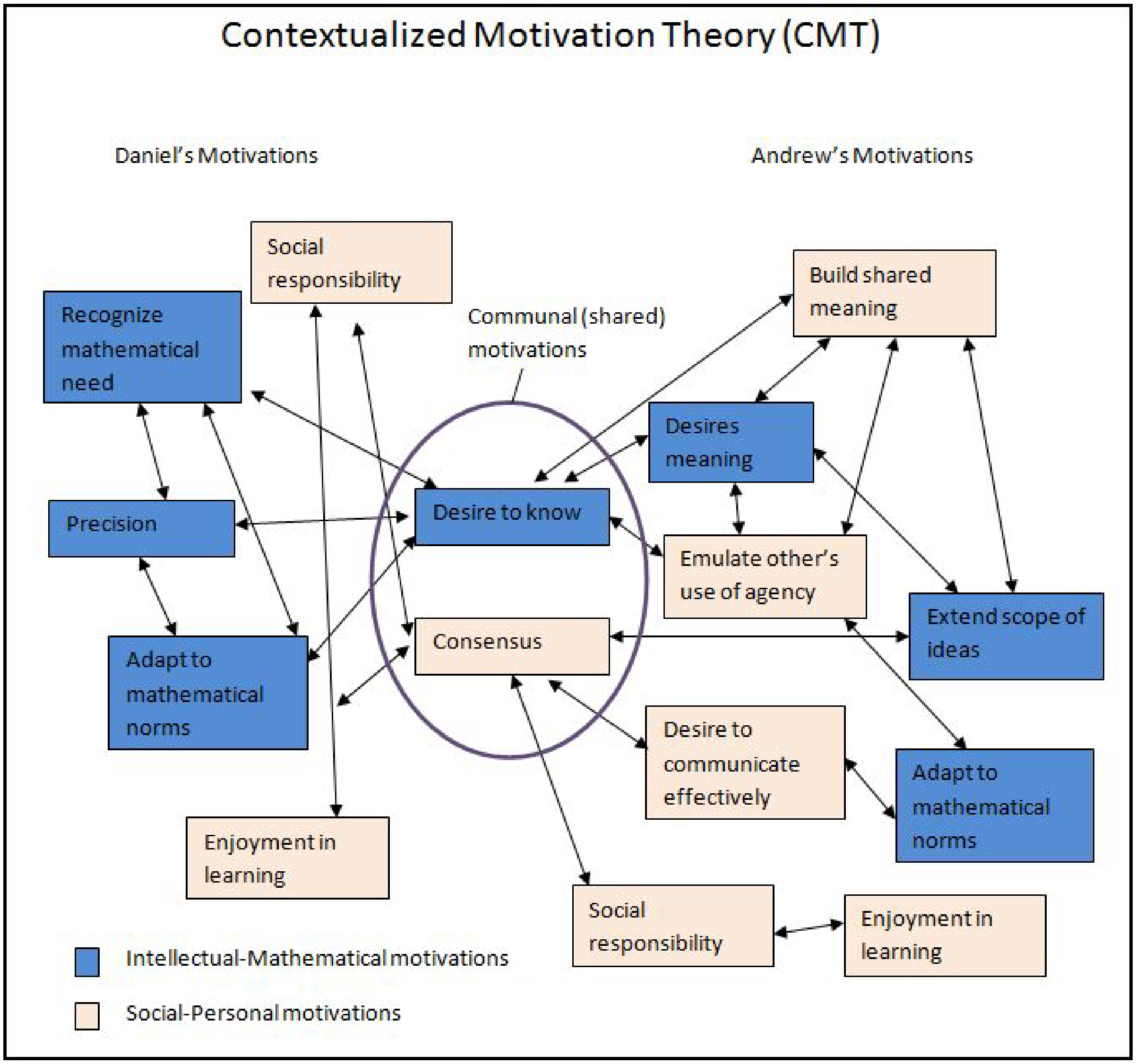

Figure 20: CMT's motivation web metaphor. This is a partial diagram of Daniel and Andrew's possible motivation web. The arrows represent connections among different motivations that were acted upon during the same choice. The motivations in the middle were used by Daniel and Andrew during the same time period, so they are communal motivations.

1. Every web is unique and communal webs can be built.

When spiders make their webs, no web looks exactly like another, though they may contain similar characteristics. Likewise, each student possesses their own web of motivations, which they weave by strands of their individual intellectual-mathematical and social-personal desires. In figure 20, Daniel and Andrew both have a different set of motivations. The intellectual-mathematical motivations are shaded in blue and the social-personal motivations are in orange.

Some spiders have been found to weave communal webs—where the spiders intertwine their individual threads to create a stronger, giant web (Jackson, 1979). Due to the social nature of the studied calculus classroom, this idea of a communal web will provide additional insight into student motivations for understanding mathematics. In CMT, motivations are individual in that each individual must possess the desire themselves and not all motivations are shared by others. One motivation might be more salient for one student than others. For example, Justin expressed a *desire to make connections* more than did any other student. Motivations may also be communal when a group of students choose to act on the same desires to achieve their shared purposes. In figure 20, the communal or shared motivations are represented by the circled boxes in the middle of the diagram. To further illustrate this idea of communal motivations, consider the first transcript presented in the last chapter (Figure 21):

Timecode	Speaker	Transcript	Codes (*Focused*)
(3-2) 26:44	Daniel	I still don't know the derivative of "Q" though, or what "Q" is. [Looks at p. 76 in book-recreated below] $Q'(a) = \lim_{b\to a} \frac{Q(b)-Q(a)}{b-a}$	-*Persistence* in understanding derivative and notations -*Poses problem* ***-Desire to know***
26:49	Riley	"Q" is the quantity of the function [Quietly, states fact]	***-Respond to student posed problem***

26:52	Andrew	Thank-you [chuckles]…what does that mean? [Garbled words]…we're trying to figure out what the book's talking about. 'Cuz I mean, we already know how to get the average [average rate of change], and it took us like ten minutes to figure out what he [book author] was talking about.	-***Desires meaning*** for book language and notation -***Emulate other's use of agency*** -*Question for understanding* -*Persistence* in understanding average rate of change -***Build shared meaning***
27:09	Justin	To find out we already knew what he was talking about.	-***Build shared meaning*** for average rate of change -*Statement of understanding*
27:13	Andrew	Huuh—that's horrible. [Laughs, a moment's silence.]…ok, I'll try an' figure out what he's talking about.	-*Displaying emotion* -*Persistence* in understanding derivative -***Extend scope of ideas*** -*Trying to figure things out*

Figure 21: Transcript. Shows idea of communal motivations.

In analysis, I did not always find it necessary to attribute particular motivations to individual students only. For example, in this episode, it was Daniel who initially stated the *desire to know* more about derivatives. However, as Group 1 responded to Daniel's posed problem, this *desire to know* became more of a communal motivation as students continued to work together to understand more about derivative. Thus, as Sullivan, et al. surmised, motivation is not purely individual, but can also be a "product of group factors" (p. 91). All of the other motivations reported in this thesis, such as the *desire to communicate effectively* and *precision,* were manifested in the words and actions of multiple students, but were not always communal in nature. Motivations that might have been strictly held by one individual were not identified because I was interested in the powers of motivations held by multiple students.

2. Webs are made up of a complex system of connected strands.

In a web, many strands of silk intersect each other in the web. A student's motivational web is also a complex design of intersecting motivations. In the calculus classroom, students used their agency by choosing to seek an understanding of the mathematics as opposed to blindly following mathematical procedures. When choosing to understand mathematics, students acted on one or more of their motivations. In figure 20, each box contains a motivation, and the arrows represent instances in which the pair of motivations was acted upon at the same time for one choice. For example, in Andrew's web, there is an arrow between the intellectual-mathematical motivation "desire to know" and the social-personal motivation "build shared meaning." These motivations were manifest simultaneously during at least one of Andrew's attempt to understand mathematics. Note that the motivations are tightly interconnected. In a spider web, if one were to pick up a strand somewhere in the web, many other strands would be affected. Likewise, in CMT, motivations are so interrelated that students cannot act upon one motivation without such action being affected by their other desires. One desire may be a student's main motivation for an action, but many secondary and/or related motivations are always present but sublimated.

For example, let us revisit the transcript about Group 1's exploration of derivative (3-2, 26:44-27:13). Here students' main motivation might have been a *desire to know* more about derivative and its associated notations. However, Group 1 also acted on desires of *build shared meaning, respond to a student posed problem* and *emulate the use of another's agency* in order to further their knowledge of derivative. The group's choice to understand more about the derivative in this instance was therefore activated by at least four different, yet related, motivations. At other times during the class, students had many other desires, such as a *desire to make connections* and a *desire to communicate effectively.* Although these motivations were not manifest during this particular episode, they cannot be ruled out as secondary motivations that may have also influenced student's decision making. For example, students' *desires to communicate* their understanding of the derivative *effectively* to the class may have also been a factor in the group's further exploration of the derivative.

3. Webs can have varying powers.

Spider silk is one of the strongest materials known to man. Its tensile strength can be five times greater than that of steel (Vollrath & Knight, 2001). However, some spider webs are more structurally sound than others and are therefore more effective, or powerful. Recall that power may be defined as "capacity for action or performance" (Porter, 1913, p. 1122). Consider the power of a spider web. Spider webs have one tendency or capacity for action—to catch prey. No web will look alike, yet all webs were created for this same purpose. The power of a spider web therefore depends on its ability to allure and catch prey.

Motivations to understand mathematics can also be very powerful. What is needed, Dewey (1913) said, is not an inventory of personal motives, but rather "a consideration of their *powers*, their tendencies in action, and the ways in which these can be carried forward by a given subject-matter" (p. 62).

Now, consider the powers of student motivations to understand mathematics. No motivational web will look the same as another, yet all webs are created with the same capacity for action in mind—understanding mathematics. Intellectual-mathematical and social-personal motivations to understand mathematics, when acted upon, can hold similar powers. For example, Andrew's sense of *social responsibility* and Justin's *desire to make connections* both had the capacity to urge each student to seek an understanding of mathematics. In the calculus classroom, both motivations were noted for their tendencies in action. The subject matter, mathematics, “carried forward” these powers (Dewey, 1913) because the studied calculus classroom consisted of a social problem solving environment where students were encouraged to pursue their intellectual passions. In CMT, motivations to understand mathematics differ in power according to the strength given them by the individual. Developing desires to understand mathematics is the first and most important thing a student can do. After that, students will benefit from strengthening their desires and using their agency to act more frequently on their intellectual-mathematical and social-personal motivations to understand mathematics.

Although a web presents an interesting metaphor to a student’s motivational system, the connection should not be taken too far. Unlike a web, if one strand (motivation) were removed from a student’s motivational system, the whole structure would not be destroyed. Also, not all spiders have the capacity to make webs, but all students can develop motivations to understand mathematics. Finally, most technical aspects of a web, such as details of how it is made, do not correspond to a student’s development of motivations. The power of the metaphor is in the fact that webs, like motivational systems, are very individualized, socially intertwined, complex, powerful, and beautiful.

Summary of CMT

The following sections will give a summary of Contextualized Motivation Theory by discussing: intellectual-mathematical and social-personal categorizations, unique set, communal, power, and agency. Each construct is then compared and contrasted with existing theories of motivation to show the distinctiveness of CMT.

Intellectual-Mathematical and Social-Personal

CMT serves to supplant dichotomous notions of extrinsic versus intrinsic motivation. In particular, CMT states that motivations to understand mathematics consist of two major categories: intellectual-mathematical desires and social-personal desires. When an individual strives to understand mathematics they are acting on one or more of these related desires.

A few current motivation theories have begun to recognize intellectual desires as motivations. In Maslow's theory of motivation, one of the levels of growth needs was the need to know and understand (Maslow & Lowery, 1998). Also, goal theories posit that some students have a learning goal orientation (Dweck, 1986) in which students learn solely for the sake of learning. However, these two theories do not assert that all students possess such desires nor do they attempt to explain content-specific actions such as those involved with mathematical problem solving. Social-personal desires are recognized by many other theorists, but again, these types of desires are removed from the social atmosphere of a group of students striving to understand mathematics.

Unique Set

As an individual faces the many experiences of life, they develop desires along the way. No individual has the same collection of desires as another. An individual's collection of desires is an intertwined, complex network—many desires are related to and affect each other. Each choice an individual makes is associated with one or more of these desires. Also, a single desire may be employed for more than one choice. For example, the desire to communicate effectively may be a motivation for an individual's choice to understand mathematics as well as for their choice to start a conversation with a friend.

Personal construct theories and many need and goal theories have also found motivations to be highly individualized. It may be argued that if students all had a unique set of motivations, it would be impossible for teachers to cater to each individual's desires. CMT reduces this dilemma by categorizing the motivations and attending to their powers, as explained in a subsequent paragraph.

Communal

Motivations to understand mathematics may also be communal in nature. In other words, a group of students may share a desire as they strive to achieve the same goal. A desire that was initially expressed by one individual and then adopted by others may also be considered a communal motivation.

In other theories on motivation, students can have desires similar to those of others. For example, in attribution theory, many different students believe that success is attributable to uncontrollable factors, such as innate ability (Weiner, 1972). Furthermore, desires such as the desire to work hard can be an intrinsic motivation for many students. CMT goes beyond to show how students can actually share and influence the desires of others by building communal motivations.

Power

In CMT, the power of a motivation to understand mathematics is more important than the particular motivation a student acts upon. Power may be defined as a capacity for action, or tendency in action. Webs that consist of motivations with greater power are stronger. To increase motivation, students can develop and strengthen those desires which encourage them to strive to understand mathematics.

Power is a unique construct of CMT. Dewey (1913) originated the idea of examining the powers of student motivations.

Agency

In the end, it is a student's use of agency that will determine the choice, no matter the power of the desire. For example, a student may have a strong desire to *communicate effectively*, but if he or she chooses not to act on it, the desire will not initiate action.

The majority of theories reviewed for this thesis did not explicitly mention agency as a fundamental part of motivation. Personal agency is a central part of mathematical problem solving and therefore central to CMT.

Figure 22 gives a concise summary of the constructs of CMT discussed in this chapter.

Contextualized Motivation Theory (CMT) Summary	
Intellectual-mathematical and social-personal	Motivations to understand mathematics can be categorized as intellectual-mathematical motivations or social-personal motivations.
Unique set	Individuals have their own unique set of closely intertwined motivations (desires) to understand mathematics.
Communal	A group of students can share a motivation to achieve the same goal. These motivations are communal.
Power	The power, or tendency in action, of motivations is more important than the particular motivation a student acts upon. If a certain motivation more often tends to influence a student to understand mathematics, that motivation is more powerful.
Agency	A student's use of agency, influenced but not determined by their desires, determines the choices they make.

Figure 22: Contextualized Motivation Theory (CMT) summary.

CMT's Contributions to Motivation Literature in Mathematics Education

The complexities of motivations exhibited by students in learning mathematics may be generously described as a "chaotic puzzle" (Nuttin, 1984, p. 83). CMT aims to begin to piece together that chaotic puzzle in order to illuminate the intricacies and powers of student motivations to understand mathematics in a conceptually based calculus classroom. CMT was created using a fine grain analysis—dissecting students' words and actions during collaborative problem solving—to discern student motivations. Such an analysis is rare among motivation studies, including studies in mathematics education. Using this analysis, individuals were found to possess their own collection of motivations unlike any other individual's collection. Various mathematics education researchers have also found that motivations are highly individualized (Hannula, 2002; Dowson & McInerney, 2003). However, CMT further elucidates how motivations can be social, or communal, in nature. CMT also simplifies the chaotic puzzle by showing that student motivations share similar powers, or tendencies in action.

CMT is also unique among motivational theories in mathematics education because of the nature of the motivations analyzed. Many motivation studies have examined students' motivations for *achievement* (e.g. Koaler, Baumert, & Schnabel, 2001) or motivations for *engagement* (e.g. Williams & Ivey, 2001) in mathematics. In contrast, CMT describes students' motivations for *understanding* mathematics.

CMT was not created to discount previous motivational theories or findings in mathematics education, only to contribute to the growing corpus of information related to student desires. However, few of the motivating factors coded for in this thesis could be found in the traditional motivation literature in mathematics education and popular motivation theories, especially the intellectual-mathematical motivations. CMT positions personal agency as the active power in intellectual passion, foregrounds mathematical need as the kernel of students' problem solving industry, characterizes the social nature of motivation, and encompasses conceptually driven conditions that foster student engagement in mathematics learning. These aspects of CMT carry more direct and pertinent implications for teaching mathematics with understanding than other general motivation findings. The next chapter will discuss the implications in more detail.

CHAPTER 7: FURTHER DIRECTIONS AND IMPLICATIONS

Contextualized Motivation Theory (CMT) is one of the few theories on motivation developed from students' actual lived experiences in a conceptually centered mathematics classroom. CMT can and should be refined and expanded by studying students in other mathematics classrooms where agency is valued—including high school classrooms. Also, in future studies, other forms of data could be collected and compared to CMT's claims. More self-report data could also be incorporated into future conceptualizations of CMT. As mentioned earlier, self-reported information has its limitations and should not be used as the only source of data, but it could provide valuable insights into student thinking. Hopefully, studies like this one will also encourage more domain specific explorations of motivation, which will help mathematics educators build better classrooms.

Implications of CMT for building best practices in mathematics classrooms depend on teachers, teacher educators, and researchers recognizing and facilitating the productive role that students' personal agency plays in enriching intellectual-mathematical and social-personal motivations that may be unanticipated by teachers, yet which foster student engagement in learning and that contribute to building meaningful understandings of mathematics. The question may then be asked: "We can train habits, we can impart knowledge, but how do we enhance agency?" (deCharms, 1984, p. 275). The answer to such a question is not within the scope of this thesis. The interested reader should turn to associated literature on agency. Having said this, one way to enhance the use of agency in the classroom would be to allow students to create an understanding of mathematics by making mathematical choices.

George Bernard Shaw (1921) penned the words, "Imagination is the beginning of creation. You imagine what you desire, you will what you imagine and at last you create what you will" (p. 9). Shaw pointed out that, although "imagination is the beginning of creation," an individual's imaginations stem from their desires. Therefore, it may be concluded that desire is the actual beginning of creation. If we want students to create an understanding of the mathematics, they must first have a desire to do so. It is clear that students can and do have such desires when given the opportunity to exercise their agency. However, despite the best efforts to create a classroom atmosphere which encourages the manifestation of these desires, a few students may come into the classroom with very little desire to understand mathematics. These students should not be ignored and labeled "unmotivated". Everyone has the capacity to develop such desires and possesses a right to be given the opportunity to do so.

As stated in the conclusions, intellectual-mathematical motivations were the most frequently coded motivations in this study. This may be because students acted upon intellectual-mathematical motivations more often while engaged in problem solving, or students chose not to reveal their social-personal motivations while in the classroom setting. Either way, such a finding should give hope to teachers. While teachers are fundamental in setting up a productive and respectful classroom environment where student thinking is valued and important, it is not the teacher's job to cater to other extraneous social-personal needs and desires. For example, if a student has a social-personal desire to look "cool" in the presence of their friends, such a motivation may not be able to be influenced by the teacher. However, teachers do have the ability to help educate many intellectual-mathematical desires of students in the classroom.

One way instructors can strengthen students' desires to understand mathematics is to ask probing questions that prompt explanation and reflection of mathematical work. While students were seeking to understand the book notation for derivative, one of the instructors asked: "So, tell me what you understand about this limit [points to $Q'(a) = \lim_{b \to a} \frac{Q(b)-Q(a)}{b-a}$] as B approaches A" (3-2, 31:59). This question not only encouraged students to explain their work and reflect on the meaning of the derivative, but helped facilitate students' *desire to make connections*. After the instructor asked the question, students discussed how derivates related to driving on the freeway.

One of the intellectual-mathematical motivations held by students in this study was *recognize mathematical need*. Walter and Hart (2009) noted the power of mathematical necessity. They affirmed, "Conceptually driven classroom conditions that encourage the emergence of mathematical necessity have been shown to support the growth of intellectual passion and persistence in mathematics learning" (Walter and Hart, 2009, p. 170). As has been shown, students do exhibit powers of intellectual passion and tendencies in action to choose among simultaneous intellectual-mathematical and social-personal motivations and to persist beyond obtaining correct answers to build conceptual understandings of mathematics.

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APPENDIX A: STUDENT INTRODUCTION SURVEY

Math 112H: Calculus I, Section 25
Student Introduction—Homework Assignment

Due Date: First Day of Class

Please help us to get to know you better by responding to the questions as completely as possible. One paragraph may be sufficient for some responses, while several paragraphs may be needed to provide the detail necessary to fully answer some questions. Expand the provided space as needed. Please submit your responses in the digital drop box in Blackboard.

Name:______________________________________

Please circle your current academic standing:
Freshman Sophomore Junior Senior

What is your declared (or expected) major?___________________________

Exam (if you did not take an exam, please indicate)	**Score**
ACT	
SAT	
AP Calculus AB	
AP Calculus BC	

Mathematics Background

High School Mathematics Courses	Grade Earned
University Mathematics Courses	Grade Earned

Perspectives on Mathematics Learning
1. List three necessary qualities of an excellent mathematics learner.

2. Which of the qualities you listed above, do you feel is your strongest? Please explain.
3. Which of the qualities you listed above, do you feel is your weakest? Please explain.

4. What does it mean to be a successful mathematics learner?

5. Describe an optimum classroom environment for learning mathematics. Why are these conditions optimum? What would be the practices within this environment?

Perspectives on Mathematics
6. What is mathematics?

7. What are the purposes of mathematics?

8. What do you like most about mathematics? Please explain.

9. What mathematics have you most enjoyed learning? Please be specific and explain why you find these particular topics engaging.

10. What do you find least appealing about mathematics? Why?

Perspectives on Mathematics Teaching
11. List three necessary qualities of an excellent mathematics teacher.

12. Please describe the teaching style of your best mathematics teacher

Perspectives on Technology
13. What role does technology have in learning mathematics? Please explain.

14. What technologies have helped you learn mathematics? How?

Perspectives on Responsibilities
15. What do you feel are the responsibilities of a student in this course?

16. What do you feel are the responsibilities of a teacher in this course?

APPENDIX B: CODING ORGANIZATION

Codes in bold are the focused codes. The italicized words below the focused code gives a description of that focused code. The words under each focused code represent the open codes that were clustered to form the focused codes.

Participating in Meaningful Mathematics
These open codes signify that students were working on meaningful mathematics during their extending problem solving.

Average rate of change
Plotting points
Table
Displacement
Curve
Slopes
Derivative
Average
Graph
Instantaneous velocity
Acceleration
Using equations
Speed
Average speed/velocity
Equation
Distance
Speed= d/t
Stat plot
Regression
Exponential regression
Cubic regression
r-value
Statistics
Trends
Line that goes through the middle
Tangent line
Arctangent
Negative velocity
Negative acceleration

Adapting to Mathematical Norms
Students looked to convention and outside tools to help them understand their own work

Notation
Reads book
Examples
Textbook
Calculator
Book notation
Calculate button

Displaying Emotion
Many different types of emotions were evident in student interactions. Some of these emotions also have a social purpose.

Scared
Laughing
Apologizing
Good
Joking
For fun
Surprise
Actual work?
Excitement
Stating feeling
Not boring
Good thing
Value judgment

All of us
We
Explanation of group’s method

Social Responsibility and Interaction
There are responsibilities we all tend to adopt when we are working in group settings. One is a responsibility to listen to others and try to understand them.

Relating to other group
Group answer
All of us
We
Laughing
Apologizing
Joking
Thinking about another’s idea
Listening
Kind of interesting
I don’t know because…
Don’t think
I mean
I’m guessing
Attempt to verify another’s idea
Attempt to explain another’s answer

Consensus
Coming to an agreement

Yeah
Uh huh
Consensus
Agreement
Group answer
Building consensus

Building shared meaning
This is related to consensus because usually when there is shared meaning, there is consensus. However, I chose to make it a separate category.

Same page
Building consensus
Make sense to everybody?
Reference to shared classroom knowledge
Share ideas

Teacher influence
It is hard to detect how the teacher’s presence in the classroom affects students, however, there were a few times when the teachers talked directly with the students.

Asking the teacher a question
Teacher response to student question
Teacher question
Teacher validating student
Response to teacher question

Grades
Students only talk about grades once, and it is very brief. They wonder if they get graded for presentations.

Grades
It’s school

Desire to complete task
There are times when students are so focused on a small part of the task (personally or as a group) that the time comes where there is a desire to move on.

Let’s finish
Moving on
Desire to move on
Overanalyzing
Focus
K, so…
Taking it too far

Satisfaction with answer
When students feel good about where they are in the task.

Satisfaction with answer
As good as we can
As far as we can go
We’re there

Overanalyzing

Precision
Students try to get as close as they can to a correct answer/interpretation. They realize that sometimes they cannot achieve perfection, so they try to get close.

Exactly
Pretty close, but
Exact moment
Exact
More precise
As close as possible
Wouldn't be exact, but
As close as we can
About (ish…)
Pretty close
As close to zero
Close
Approximating
Really good
Technically
Ballpark figures
Line that fits the points best
Skewed

Relating to other experiences/Prior knowledge
Students try to understand the task by using what they know about other situations to guide their thought process.

Giving a related example
It's like
Speeds in a car
Speedometer
Arctangent
Using previous knowledge
Statistics
State origin of idea
Depends
Lasers
Running
Track
Relate to other experiences
Imagining
Reference to shared classroom knowledge

Keeping context in mind
Students don't just try to do the math after they have collected the numbers they need. They keep the context of the problem in mind.

Looking at situation in context
Pauses/leaps
Running
Modeling
Look at cat's perspective
Just going
(Cat) Not paying attention

Justification
Students often asked for or gave justification for why they did what they did. This is an indication that students are striving to learn with understanding.

Justifying
'Cuz
'Cause
Any questions?
Explaining method and logic
Attempt to verify another's idea
Explaining conflict
Clarifying

Extending
Students were often observed going beyond what was asked of them by the task and the teacher.

How much further
Extending
See what it gives us
Figure that out
Could potentially
Thinking about extensions
Just to see what happens

Satisfying curiosity
This idea of curiosity is closely linked to extending. Students wanted to see what happened when they tried their ideas.

Fake it
Risk taking
Trying an idea
Leads somewhere
Just to see what happens
See what it gives us
What if
Play around with it
Try something

Trying to figure things out
Perhaps many of these codes could also be coded for desire to make sense of.

Play around with it
Try an' figure out
Figure out
Figure things out
Trying to figure out
Trying to make sense of
Trying to think
I figured (what we can do)
Wait a second!
I don't know
We figure
I think
I think you can
Thinking aloud

Poses problem/suggestion
These are occasions when a student brings up a problem or suggestion and then the group works at resolving the problem.

I don't know, but
Well
Offering a suggestion
Presenting an idea
Shows idea
What if
Suggestion
Makes a proposition
Poses problem
Want
Try something
How?
Student question
Question for information
Asking a question
Still don't know
Question for understanding
What does that mean?
Like to know

Statement of understanding
Wherein someone states or displays they already have come to know something.

Figured out
We understand
You understood
Already knew
Already know
Describing what they know
Stating what she does know
Attempt to describe r
Derivative

Responding to a student posed problem/explanations
When students are offering explanations, answering a student question, or responding to a problem posed by a student. Responding to a problem is usually manifest by students pursued course of action.

Answering question
Answering student question
Response
Response to a previous concern
Response to student question
Continuing and building on the conversation
Expounding on a previous explanation
Explaining conflict
Explaining

Clarifying
Explaining method and logic
Explanation of work
Continuing discussion
Thinking of another's idea

Explaining answers/Comparing work
Students want to make sure they are on the same page by comparing answers and asking what is going on.

Question to compare
Comparing work
Confirming answer
Stating answer/fact
Statement
Statement of answers
I got
How to find
Explanation of how to find answer
Explaining group's method
Explanation of previous work
How to get
How?
Procedure question
Asking for confirmation
Did you get?
Asking a question
Asking a clarifying question
Question for information
Question to know what's happening

Desire to know/for meaning
Students had a desire to know more about a particular mathematical topic or desired meaning. These are two very similar motivations.

Question for understanding
What does that mean?
Means (meaning)
To know
Still don't know
Attempt to make sense of
Makes more sense
Reiterating question
Repeating the question
Trying to make sense of
I don't know because…
Wanting further information
Like to know
Don't know how
Can't remember

Persistence
When students continue to work on a task or student posed problem, even when they don't get a quick answer.

Took us ten minutes
Persistence
Persistence in understanding average rate of change
Still don't know
Reiterating question
Repeat the question
Responding to a previous concern
Re-measuring distances
Continuing and building on the conversation
Much better

Self-Investment
When students are invested in the work, aside from the group.

Myself
Did them all
Doing work
Talking to self
Individual work
Seems to go by fast

Inconsistency/Cognitive or Group Conflict
There are times when student's work conflicts with their experience and knowledge. Also, there are times when students work or thoughts contradict that of another students'.

In the middle
Bugs me

Confusion
Conflict in individual’s mind
Disagreement among measurement
Don’t get that
Did same thing…
Inconsistency in answers
Isn’t right
Wait
Can’t be right
Not right
Something’s weird
Answers different than expected
Wrong
Make it weird
Negative velocity

Resolving inconsistencies and conflict
Much of the resolution of conflict in also done in students’ normal conversations.

Explanation of troubles
Resolving inconsistency
Why not
Recognizing the problem
Affirmation
Resolution of confusion
Explaining conflict
Attempt to explain another’s answer
Attempt to verify another’s idea
I mean

Enjoyment in learning
Students seem to enjoy learning.

Learning
Entertaining
Excitement
Wait a second!
Not boring
Good thing

Active learning
Students are active participants in their own learning.

Group work
Asking questions
Actually doing stuff
Not passive learners
Participation

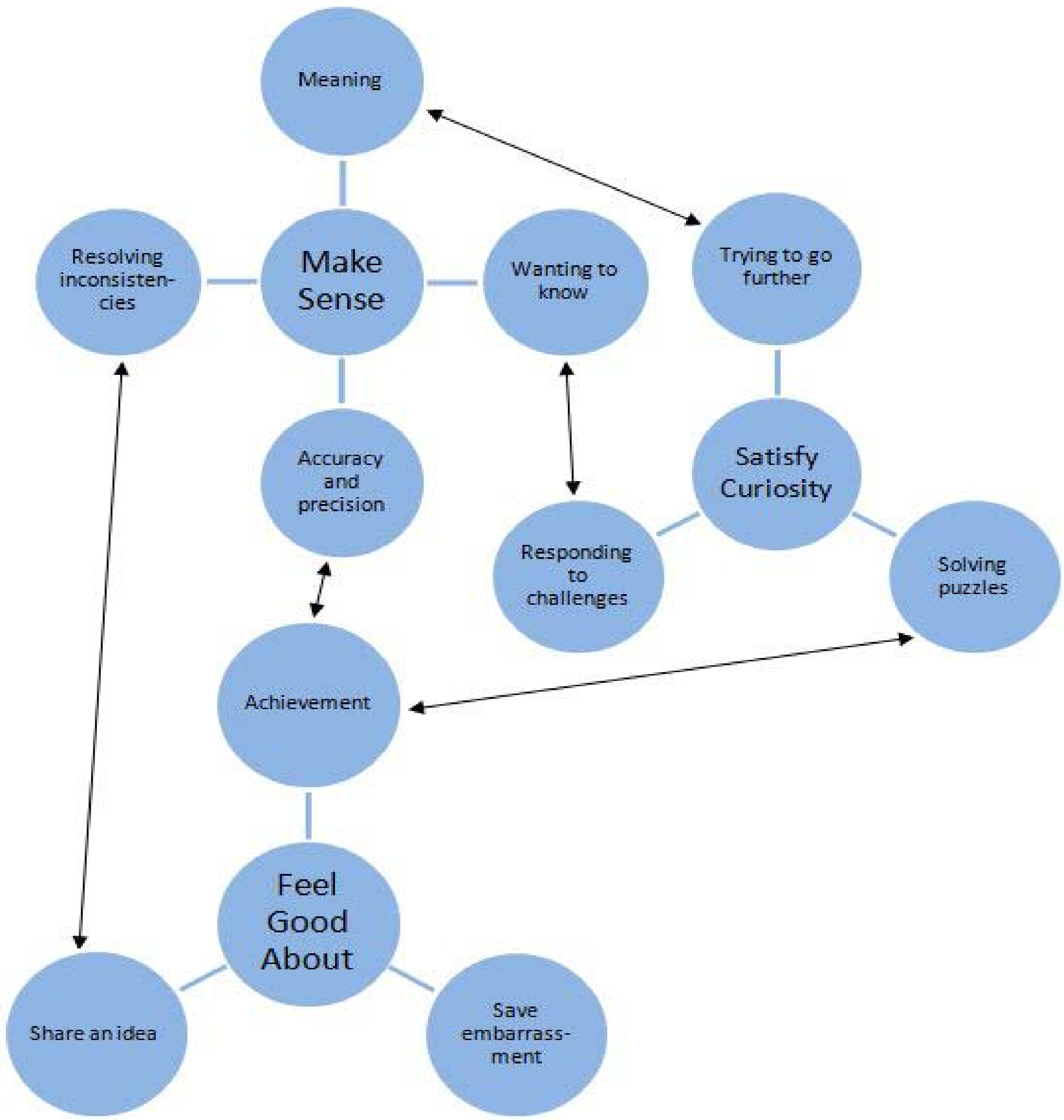
Desire to communicate effectively
Students want to be able to communicate with others and to be understood.

How to present
Present-makes sense
Preparing presentation
Share ideas
What we did
Presenting to class
Did everyone understand?
Wish to communicate ideas effectively
Reference to shared classroom knowledge
Make sense (to everybody)?
Relating to other group
Building consensus

Emulating other’s use of agency
This happens when other’s respond to a student posed problem, especially when that problem is not in a form of a question. It is when a student uses his/her agency to try to understand the mathematics and other students pick up on that decision and also strive to understand.
Thinking about another’s idea
Continuing and building on the conversation
Well
Actually do work

APPENDIX C: ANALYTIC DIAGRAM

This represents one of the early diagrams made between concepts during the analysis process. Many changes and rearrangements were made before the final diagram was constructed.



APPENDIX D: FOLLOW-UP SURVEY

This survey was distributed to students a little over a year after the calculus class ended. I authored the fifth question and some answers to that question have been analyzed in this thesis.

1. Has your major changed since taking our honors calculus course? If so, what is your current major and why did you change majors?

2. Please describe how the learning conditions in our honors calculus course helped you understand mathematics in subsequent courses.

3. How has taking our honors calculus course affected your academic studies, goals, or perspectives?

4. Reflecting on our honors calculus course, please share with us any additional compelling or central experiences that may have shaped, directed, or influenced you.

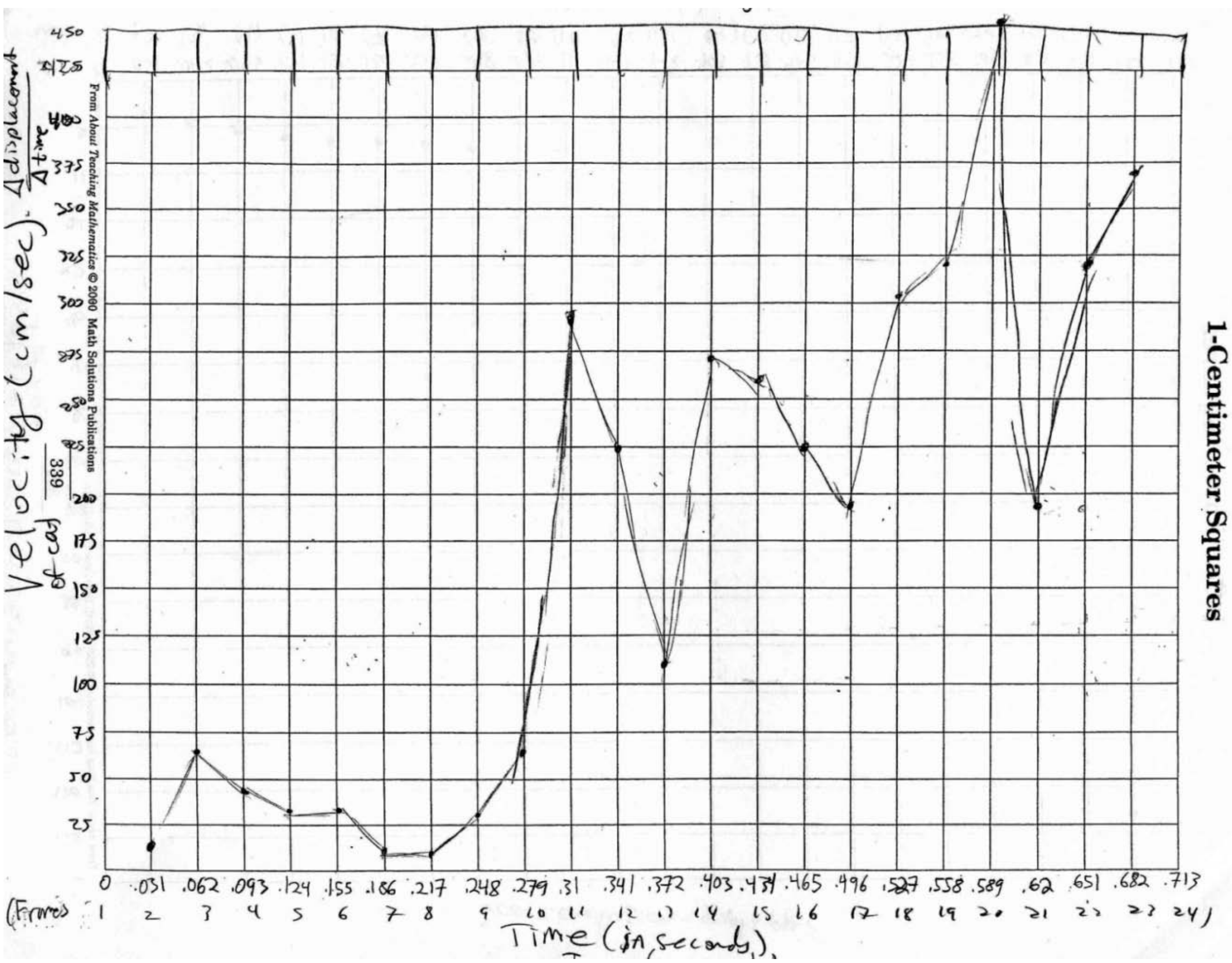
5. During our honors calculus class, we often noticed that students would work to go beyond just finding a correct answer. When you did this, why? When you did not do this, why not?

6. Please rate the following:

	Excellent	**Above Average**	**Average**	**Below Average**	**Inadequate**
Your procedural skill in and computation of honors calculus right after completing our course	☐	☐	☐	☐	☐
Your retention of procedural skill in and computation of honors calculus	☐	☐	☐	☐	☐
Your conceptual understanding of calculus right after completing our honors course	☐	☐	☐	☐	☐
Your retention of conceptual understanding of calculus	☐	☐	☐	☐	☐

APPENDIX E: VELOCITY GRAPH

This velocity graph was created by members of Group 2 by calculating the average rate of change between consecutive points on the position graph. For example, to get the velocity at frame 2, students took the change in position from frames 1 to 2 and divided it by the change in time between the frames (0.031 seconds).



APPENDIX F: CODED TRANSCRIPTS

These are some transcripts that were chosen as critical events and then coded with both open and focused codes. Focused codes are in italics. All the transcripts below aided in the development of Contextualized Motivation Theory (CMT), but not all appear in the data and analysis section in this thesis.

Timecode	Speaker	Transcript	Annotations	Codes (*Focused)*
3-2 26:44	Daniel	I still don't know the derivative of "Q" though, or what "Q" is. [Looks at p. 76 in book-recreated below] $Q'(a) = \lim_{b \to a} \frac{Q(b) - Q(a)}{b - a}$	He moved his calculator off his book and looked at the left page	-Still don't know -*Persistence* in understanding derivative and notations -*Poses problem* -*Desire to know*

3-2 26:49	Riley	"Q" is the quantity of the function [Quietly, matter-of-fact]		-Response -*Responding to student posed problem* -Reads book
26:52	Andrew	Thank-you [chuckles]…what does that mean? [Garbled words]…we're trying to figure out what the book's talking about. 'Cuz I mean, we already know how to get the average [average rate of change], and it took us like ten minutes to figure out what he [book author] was talking about.		-What does that mean? -*Desires meaning* for book language and notation -*Emulating other's use of agency* -Question for understanding -Figure out -Ten minutes -*Persistence* in understanding average rate of change -Book notation -*Building shared meaning* -Already know

3-2 27:09	Justin	To find out we already knew what he was talking about.	Emphasis on the word knew.	-We -*Building shared meaning* for average rate of change -Already knew -*Statement of understanding*
27:13	Andrew	Huuh—that's horrible. [Laughs, a moment's silence]…ok, I'll try an' figure out what he's talking about.	During the silence, each member of the group stares off into space and most have their hands on their heads	-Laughing -*Displaying emotion* -Try an' figure out -*Persistence* in understanding derivative -*Extending* scope of ideas -*Trying to figure things out*

3-2 30:25	Daniel	We understand "Q" signifies like—we figured out this could help us to understand exactly, like how to get the instantaneous--		-We understand -*Statement of understanding* -Figured out -Understand exactly -*Precision* -How to get -Instantaneous velocity -Asking teacher a question
30:29	Instructor	The instantaneous…[Daniel: Yeah] So is your question about what is "Q"?		-Teacher asking a clarifying question -*Teacher influence*
30:32	Daniel	Yeah, "Q" defined in this, I guess [pointing to the book]. I'd like to know how to find that too, but basically, I needed to know what "Q" was.	I'm unsure of what "that" is here.	-Clarifying -Like to know -*Desire to know* -Needed to know -*Recognize mathematical need* -Textbook

3-2	Justin	It's like when you're		-It's like…driving

33:07		driving in your car, and you’re looking at your speedometer, you just look at your speedometer, it’s saying 65 mph, so instantaneous velocity is at, zero, because, at that exact moment; but what we’re trying to work out here is, we’re looking at two different time frames, and say, ‘k, we’re goin’ 60 miles an hour-this is the time frame-we’re going 63 miles an hour at this time frame, what do we do if we’re somewhere right in the middle? ‘s kinda what we’re looking at.		-Speedometer *-Relating to other experiences* -Instantaneous velocity -Exact moment -Trying to work out *-Trying to figure out* -Giving a related example -Two different time frames -In the middle

3-2 35:51	Instructor	So have I sort of answered your question [garbled words]?		-Teacher question *-Teacher influence*
35:56	Andrew	I think pretty much what we’re getting to is we’re *there* [he chuckles].	Emphasis on there.	-We’re there *-Satisfaction with answer* -Answering teacher question -Satisfaction with answer
35:58	Justin	But we’re trying to think about how much further we can go [trailing off].		-Trying to think -How much further *-Extending*
36:01	Andrew	Yeah-I think that we’re taking it too far, I think we got--		-Taking it too far
36:04	Daniel	‘Bout as good as we can go.	Daniel says it in a nonchalant/ definitive manner	-Satisfaction with answer -As good as we can go *-Satisfaction with answer*
36:07	Justin	Well let’s finish the other half then. We only	He looks at the cat pictures to	-Let’s finish -Making a

		worked on number ten, let's do number twenty.	signify his getting back to work.	suggestion -Moving on -*Desire to move on*
36:13	Daniel	Wait-so-ooo…		-Wait so… -Wondering

3-2 37:01	Andrew	Ok so it was [inaudible]. Man did we overanalyze that.		-Overanalyze
37:19	Daniel	Sorry.		-Apologizing -*Displaying emotion* -*Social responsibility*
37:21	Andrew	Oh, no, it's, I think it was all of us like trying to figure out what it was talking about. That's pretty much a .0312 plus it's a zero…		-Assuring Daniel -All of us -*Social responsibility and interaction* -*Building shared meaning* -Trying to figure out -*Trying to figure out* -Continuing to work

3-2 45:15	Riley	So, we decided it [speed of cat at frame 10] was 64.5 [cm/s]?	Riley looks over at the overhead the group has created	-Speed -Clarifying question -Group answer for speed -*Consensus*
45:19	Andrew	That's for, that's the average [average rate of change] from 9 to 10. Is that what you guys got?	Riley grabs his orange paper and looks at it.	-Answering question -average rate of change -*Comparing work*
45:25	Justin and Daniel	64.5 ish		-Confirming answer -ish
3-2 45:28	Riley	64.5 from 9 to 10. The thing that bugs me on		-Bugs me -Stating answer

		that one is from 10 to 11 I got 225 [225 cm/s average rate of change].		-*Inconsistency/cognitive conflict* -Conflict in individual's mind -Major difference in numbers
45:35	Justin	That's what- yeah, I was just looking at too. But that's just, it's the cat's accelerating really quick there.	Justin says this quietly	-Agreement -Looking at too -Acceleration -Explaining conflict -*Resolving inconsistencies/conflict*
45:44	Daniel	It changes in that much from 9 to 10.		-Statement

3-2 46:04	Andrew	Well, we've got to present the information in a way that makes sense. [Daniel: Yeah, that's true] Ok, why don't we just make a table, you know, out of the rate of change from 9 to 10 and the rate of change from 19 to 20 and then that's how we display, you know the rate of change of the uh, centimeters.	Andrew is now looking directly at Daniel.	-Well -Present in a way that makes sense -*Desire to communicate effectively* -Table -*Consensus* -Agreement -Rate of change

3-2 47:19	Justin	So have we taken it [the cat task] about as far as we can go, for now? [Others: for now]	Daniel and Andrew are working on drawing tables and such for the presentation, Riley is working alone	-As far as we can go -Asking for confirmation -*Consensus* -*Satisfaction with answer*
47:24	Riley	Well, on the 9 and 10 I took from 10 to 11 it's the huge 200 whatever. From		-*Extending* -Responding to a previous concern

		9 to 10 it's 64 so I just subtracted the one from the other and got 160.5. Which makes more sen-, that's for number 10.		-*Persistence* -Makes more sense -Subtracting speeds -Offering a suggestion
47:44	Daniel	For number 10?		-Question
47:45	Riley	So that's what I'm *guessing* is instantaneous.	Emphasis on "guessing"	-Guessing -Instantaneous -Proposition

3-2 51:09	Andrew	Beyond my level. Ok, cuz I was getting scared. I'm like, everybody else seems to know what this [derivative] means. [Riley: I don't even know how to use it; Justin: I know the *word*…]	In a relieved voice.	-Derivative beyond my level -Scared -*Displaying emotion* -Derivative -To know -Meaning -Don't know how -Know word
51:19	Daniel	'Cuz, the derivative of the displacement is velocity. The derivative of the velocity is the acceleration. And that's what this little y thingy means. That means the common. The Q prime is…[Garbled words from members of the group]. So like derivative, you just find like slopes [pretends to make lines in the air with his arms] of the line.	In a mechanical voice, quickly said	-Explaining knowledge of derivative -*Statement of understanding* -Displacement -Velocity -Acceleration -Slopes
51:52	Andrew	So how about we focus on our project [the Cat Task] *now*.	Stretches out his arms, then points them forward.	-Focus now -*Desire to move on*
51:57	Riley	About 354 and 64.5.		-States answers
52:00	Daniel	Yeah, 64.5 and I don't know though, that was kind of interesting what you [Riley] did though minusing the amount. I		-I don't know -Kind of interesting -*Social responsibility and interaction*

		don't know if that gets closer.		-Subtracting velocities -Thinking about another's idea
52:11	Riley	That was the idea of at 11 o' clock you were moving 65 and at 12 o'clock you were moving 68. The difference is 3, like one subtracted from the other one.		-States origin of idea -Speeds in a car *-Relating to other experiences* -Difference

5-1 8:57	Derrick	Did you guys actually get to where you figured out the average speed, or its- [Justin: its instantaneous speed] its speed on that?		-Asking a question -Comparing work *-Comparing work* -Figured out -Average speed -Instantaneous speed
9:04	Andrew	Yeah, we did, didn't we?		-Responding to student question
9:07	Justin	Did we do instantaneous [Directed toward Andrew]?		-Unsure -Instantaneous
9:09	Andrew	We [group 2] got an instantaneous velocity, I think-is what we figured out.	Daniel is working on something—looks like measuring—while Justin and Andrew talk to Derrick and Kacy	-Response -Instantaneous -I think -Figured out instantaneous velocity
9:11	Derrick	Ok		-Listening
9:14	Kacy	What'd you get for that?		*-Comparing work*
9:16	Andrew	Um, it's pretty much you just, however you get the velocity, um 'cuz the whole deal with instantaneous velocity is you got to get as close to zero as possible, between frames [makes imaginary points in the air with his left hand] it's point zero	Andrew explains to Derrick and Kacy his understanding of why group 2's answers are close enough to instantaneous velocity to be	-Answering question -Explaining method and logic *-Explaining answers* -'Cuz *-Justification* -Velocity -Instantaneous

		three one [.031] seconds, which is pretty freakin' close to zero, so that's pretty much whatever you get for velocity is going to be instantaneous velocity.	called instantaneous	velocity -As close to zero -Pretty close -Approximating -*Precision*
9:37	Derrick	Ok [Kacy nods her head]		-Listening -Acceptance -*Social responsibility and interaction*
9:41	Daniel	We just took that from frames ten to nine, did the equation of the distance minus the distance all over the rate of change in time which always point zero three one. [Derrick: Yeah] So, in your guys' case I guess it'd be fifteen minus twelve, I think, over point zero three one?	Daniel puts aside what he was measuring as he begins to speak	-Explanation of how to find velocity -Equation -Distance -Rate of change -Relating to other group -*Desire to communicate effectively* -*Social responsibility and interaction*
10:04	Andrew	Yeah, and then that gives you the instant velocity		-Agreeing -*Consensus* -Instant velocity
10:08	Derrick	We did, we did five over point zero three one. [Kacy: mumbled words] Like five-he moves…'Cuz we did speed equals uh distance over time, and since he moved five centimeters from frame nine to frame ten, that was distance. [Andrew: Uh, huh] And then we did-we just divided it by--		-Explaining group's method -*Explaining answers* -*Justification* -Formula for speed -Speed=d/t

5-1 12:01	Daniel	Soo, we're just going to write-up [for in-class presentation]…what we did basically, right? [Justin: Yeah] Soo [pause]	When he says write-up, Daniel means for their presentation in	-Preparing for presentation -*Desire to communicate effectively*

		wait uh second-	front of the class	-Asking confirmation question -Wait a second -*Trying to figure things out*
12:15	Justin	Well if we…just that thinking out loud-so if we did the graph like this [points to the displacement graph] and you found the slope of the line *at* ten, that would be the instantaneous velocity, right? [Kacy: Right]	We see the displacement graph on the table and Justin points to it with his ruler.	-Presenting an idea -*Poses problem/suggestion* -Thinking out loud -*Trying to figure things out* -Slope of the line -Instantaneous velocity -Asking confirming question
12:23	Andrew	Well it would, be-		-Responding to question
12:25	Justin	-be more precise than what we were doing with the average between the two points [points to two points on the graph]. So we could do that while someone's writing up stuff, some people could figure that out just to see what happens.	We see an even better shot of the displacement graph.	-More precise -*Precision* -Average -Figure that out -Just to see what happens -*Satisfying curiosity* -*Extending* -Some people could -*Poses problem*

5-1 23:25	Daniel	So to make sure that I'm understanding-this [hand drawn graph on table] is the speed or velocity of the cat--	We see a shot of the velocity graph done on paper, but you wouldn't be able to see the details. It seems like they actually have a displacement graph there…	-Make sure I'm understanding -*Desire for meaning* -Question for understanding -Velocity
23:34	Justin	[Yawns] –Uh huh, for each frame.		-Agreement -*Consensus*

23:35	Daniel	-the velocity of the cat. So I could potentially draw an acceleration graph. Or a distance graph.		-Velocity -Acceleration -Distance -Potentially -Thinking about extensions -*Extending*
24:27	Justin	So are we getting grades on all these presentations, or are they just for fun?		-Question for information -Grades -*Grades* -For fun
24:31	Derrick	Probably getting grades. I would assume. It's school.		-Response -Grades -It's school -*Grades*
24:33	Daniel	I don't know…I'd guess that it's part of our working project and kind of like share ideas that we all have-to help with our write-ups.	After this short conversation, there is no more talk about grades	-Don't know -Share ideas -*Building shared meaning* -Recognizing benefit of presentations

5-2 26:54	Andrew	'Kay, so what we did is we plotted each individual, the distance the cat travelled, and then time in the frame, so that we plotted each individual frame [points to the squares indicating each ordered pair] and we got the-in the stats-plots and we got that line [cubic regression] and-	We see a camera shot of the stat plot on the calculator along with the graph of their cubic regression equation	-What we did -Presenting to class -*Desire to communicate effectively* -Distance -Stat plot
27:08	Kacy	-the line is the equation [the graph of the cubic equation]-		-Explanation of line
27:10	Andrew	-And then Kacy created the equation. Let me get to that real quick…that one [points to the equation on the calculator overhead]. This equation right here [$y = -98.4x^3 + 421.9x^2 - 69.1x + 8.7$] is the equation that we used to graph the stat-point in the line, okay?		-Equation -Calculator -Stat plot -Regression -Okay?

27:25	Instructor	Is that a cubic?		-Teacher question *-Teacher influence*
27:27	Andrew	Yeah, it's a cubic		-Response to teacher question
27:28	Unknown Male	How'd you get that equation?		-How? *-Desire to know* -Student question
27:29	Andrew	So it-we also had to use several other different uh-we tried different other graphs that didn't work quite so hot [scrolls through to show other equations on the calculator].	Andrew said this right after the question was asked, so he is not responding to the question.	-Different graphs -Explanation of previous work
27:40	Unknown Male	Where'd you get them from?		-Reiterated question
27:41	Kacy	So the equation basically like, what we did on Wednesday when we did the, what was it, the exponential regressions? Instead of doing that one, you'd-we used the cubic…regression	Now we see a whole bunch of different lines on the calculator, representing all the different regressions the group tried.	-Response to student question -Exponential regression -Reference to shared classroom knowledge *-Desire to communicate effectively* *-Building shared knowledge* *-Relating to prior knowledge* -Cubic regression
27:55	Andrew	Yeah, so the cubic matched the line. Like…It was closest to "r", it wa-what was it, it was ninety..?		-Agreement -Continuing explanation -"R" -Question for information

28:01	Justin	Point nine nine [actually .99614]?		-Response
28:02	Andrew	Point nine nine nine, so then as you get closer to one, the better the graph matches the-y'know- the points on the plot.		-Explanation of r-value
28:10	Kacy	Then with that, we were just able to take the derivative by doing the Calculate button on the tenth and twentieth frame to get one hundred and forty-three point three six two centimeters per second, and three hundred and twenty-five point five three one centimeters per second. Then, someone did a handy little conversion to do it miles per hour. So it's just easier to look at that way, to see the difference. 'cuz it's three point two [miles per hour] and seven point two eight [miles per hour]. Any questions?	The group puts the overhead back up that shows the answers they got when using the calculator.	-Derivative -Calculate button -Statement of answers -Conversion -Easier -Any questions? -Ready to answer questions *-Desire to communicate ideas effectively*
28:48	Justin	Yeah, any questions? Did everybody understand that?		-Did everybody understand? *-Desire to communicate ideas effectively*

5-2 31:21	Andrew	Make sure we're all on the same page.	Note: Derrick does not talk during this whole conversation	-Same page -Building consensus *-Consensus* *-Building shared meaning*
31:26	Daniel	How do you do the, like "r"…		-How? -Procedure question
31:29	Andrew	That's something like, I learned in my statistics class, [Daniel: 'k] 'cuz we use a lot of stat plots and all that stuff, and you have to find a line that uh, fits all the statistic plots when		-Something I learned -Statistics *-Desire to make connections* -Drawing on other classes -Stat plots

		you're trying to find like y'know like define like trends or whatever [Daniel: Yeah] And so like "r" is just, can't remember how to find "r", I can't remember.		-trends -"r" -Can't remember -How to find
31:47	Daniel	Wait-how did you find "r"?		-How? -*Student posed problem* -Procedure question
31:49	Andrew	Um, when you actually define the line, it'll give you like-do that's-		-Responds to question -Attempt to explain
31:52	Kacy	-'cuz I never really…I just knew that closer to one [Daniel and Justin: Yeah] is better and I never, well I knew there was…		-Never really -Stating what she does know -*Statement of understanding*
31:59	Andrew	'Cuz if you take Stats [Justin: Yeah], you'll – they'll talk about it.		-Stats -Attributing possession of information to others
32:01	Justin	'Cuz when you're in Stats, you'll have points that are like all over the place, you know, and so it just makes the line that goes through the middle, judging on how close those are.	A pretty good camera shot of Justin leaning over explaining something to Daniel while everyone else is looking on	-Stats -Points all over -Line that goes through the middle
32:10	Andrew	'Cuz they try and define trends, not each individual point, and so they try to get a line that'll define the trend best. And I can't remember if like- it's the distance between the points or something. I can't remember how to find "r", but- [Daniel: 'k] and that's how they figure out that the line fits the points best.		-They -Trends -Can't remember -Attempt to describe r -Line fits the points best -*Desire to know*

5-2 39:07	Andrew	But you can still define the one forty-five as instantaneous 'cuz you have to get it as close to zero as possible. But you can technically describe it as instantaneous even though it's point zero three one seconds difference.	Note: Derrick still does not make any noise. The camera is not focused on him, so we don't know what he is doing.	-Instantaneous -As close as possible -Technically -*Precision*
39:19	Justin	And then it just all depends on what you're talking about too. I mean-with the cat, you know, that's pretty dang close to instantaneous, if you're talking like *lasers* or something like that [Kacy: Giggles], where you're measuring in point, point tenth decimal stuff, you know, then it's like-[Andrew: Microns] er Microns, and stuff like that. So it just all depends on what you're doing.	Justin talks with his hand	-Continuing and building on the conversation -*Persistence* -*Emulating other's use of agency* -Depends -Pretty close -*Precision* -Instantaneous -Lasers -*Desire to make connections* -Looking at situation in context -*Keeping context in mind*
39:36	Daniel	So, the instantaneous derivative, we just used our graph to find out that at point ten.		-Instantaneous -Derivative -Graph -Explanation of what they did
39:42	Andrew	So yeah, we, we found out that it [velocity at frame 10] was one forty-three from the graph would be the slope of the line.		-*Consensus* -Slope -State answer -Relating slope to velocity
39:47	Justin	So at that point [makes an imaginary point with his right hand] when time equals zero, that's the instantaneous velocity when time equals zero. 'Cuz like with these, time	Andrew and Daniel don't just cut off Justin's explanation. They just get on a slight	-Continuing explanation -*Explanations* -Instantaneous velocity

		is equal to-	tangent.	
40:25	Justin	But does that all make sense, like our results make sense to everybody? [All: yeah]	Speaks with concern	-Make sense (to everybody)? *-Building shared meaning* *-Desire to communicate effectively* -Agreement *-Consensus*

6-2 44:24	Derrick	Wait, to find number 24, don't you just do 24 minus 23? Err, like the frame 24 minus 23?	Andrew stops talking to Daniel and they listen to Derrick's question	-Wait -Procedural question *-Comparing answers*
44:33	Daniel	Yeah.		*-Consensus*
44:33	Kacy	That would find 23.	Looks up into the air as if she is thinking	-Disagreement - *Inconsistency/Group conflict*
44:34	Daniel	Wait.		-Wait
44:35	Kacy	Wait.		-Wait -Confusion
44:36	Derrick	No, to find number 2, to find frame number 2 you do 2 minus 1, so the only one that's going to be zero is number 1.		-Disagreement -Explanation of how to find velocity *-Resolving inconsistency*
44:43	Kacy	Oh yeah.		-Agreement -Resolution of confusion
44:43	Derrick	You can find out number 24, but, cuz I went through and did them all.		-Did them all *-Self-investment*
44:48	Andrew	Oh, you already did everything?		-Surprise *-Displaying emotion* -Already
44:49	Derrick	Yeah.		
44:50	Andrew	While I was sitting here like yacking on, like you got actual work done?	Andrew was not completely off task. He was talking to Daniel about	-Sitting here -Actual work

			getting a USB port to transfer information between calculators	
44:53	Derrick	I was just, letting you figure things out, I guess.		-*Figure things out* -Giving others opportunity
44:57	Andrew	Ok. So I guess it's my turn to do some work. [Laughs] K, so wait, I wanna actually, you know try something myself, so. Let's go distance.	Andrew begins to write on his paper. Derrick kind of sits doing nothing for a while.	-Want to try something -Laughing -*Displaying emotion* -Try something myself -*Self-investment* -Distance -Doing work
45:21	Daniel	*Woah*, what did you [Derrick] get for frame 11? Err, for velocity.		-Surprise at own answer -Question to compare -*Comparing work* -Velocity
45:33	Derrick	10.		-Answer
45:34	Daniel	You got 10?		-Surprise -*Inconsistency*
45:36	Derrick	Yeah.		
45:36	Kacy	What were you dividing it by?		-Question to compare
45:38	Daniel	I got 64.		-State answer -*Inconsistency in answers*
45:38	Kacy	Well you sh, you have to divide it by .031 which are the seconds.		-Explanation of how to find velocity -*Resolving inconsistency*
45:43	Derrick	Oh, you divided all these numbers by .031?		-Question to clarify procedure
45:45	Kacy	Yeah.		-Response
45:46	Derrick	So when you, so you're like figuring out the change in y and then you're dividing them by .031.	Derrick seems to be sincerely wondering.	-Repeat the question
45:50	Daniel	That can't be right [referring to his own		-Can't be right -*Cognitive conflict*

		work].		
45:54	Derrick	It's not right, I'm not right at least.		-Not right *-Inconsistencies*
45:56	Daniel	I did something really weird 'cuz in my velocity in frame 10 he's at 64 [cm/s] and [frame] 11 he's 322 [cm/s] and in 12th [frame] he's 32 [cm/s].	He looks up and smiles after he finishes talking	-Did something weird -Velocity -Answers different than expected *-Cognitive conflict*
46:07	Andrew	You did something wrong [nodding his head, but still focusing on his own work].		-Wrong
46:09	Derrick	That sounds right, because like when you, when you calculate the distance, like in frame 10 he only moves 2 [cm] and in frame 11 he moves 10 [cm].		-Sounds right -Affirmation of another student's work *-Social responsibility* -Distance *-Resolving inconsistencies*
46:16	Daniel	Oh. Well, I don't know.		-Don't know
46:17	Derrick	So, it's just the way, like how the cat's not running at a constant speed, he's like, his body's like…		-Speed -Explanation of answers *-Relating to context* of cat
46:24	Daniel	Has rest periods [rising intonation]?		-Question *-Relating to other experiences*
46:26	Derrick	Yeah, 'cuz like…		-Response -Agreement
46:28	Andrew	Well, 'cuz like when he, when he takes a step, he like, his body contracts and he pauses [puts his arms out in front of him and pretends to pause], and then he leaps again [lifts his hands up as if to jump like a cat].		-'Cuz like -Further explanation -Pauses/leaps -Talking about *context* -Imagining
46:34	Derrick	Yeah, that's [the cat		-Make it weird

		pausing then leaping] gonna make it weird.		
46:36	Daniel	Oh, I got a negative.		-Got a negative
46:37	Derrick	So why are you [asking Kacy] dividing them by .3, 'cuz the change in y is gonna be…		-Asking why -Reiterating question -*Question for understanding*
46:41	Daniel	No, that's right, velocity can be negative [talking to himself].		-Velocity -Negative

6-2 50:02	Derrick	This class seems to go by pretty fast.	As they talk, everyone continues to work on what they were doing.	-Seems to go by fast -*Self-investment in focused effort* -General statement -Not boring
50:05	Daniel	Yeah.		-*Consensus*
50:06	Derrick	That's a good thing.		-Good thing -*Value judgment* -Stating feeling
50:07	Kacy	It is.		-*Consensus*
50:08	Daniel	Cuz we're like learning and like, it's…entertaining I guess?	Intonation rises at the end of the sentence.	-Learning -Entertaining -*Enjoyment in learning*
50:15	Derrick	Yeah, we're not just sitting here taking notes; we're actually like doing stuff, [mumbled: sometimes]		-Not passive learners -Actually doing stuff -*Active learning*

8-1	Kacy	Did any of you guys run the thing [run down the hallway to model the movement of the cat] on Wednesday? [Others: No, I did] 'Cause if you think about that it makes acceleration seem…er makes acceleration make	Camera is still at another table when we first hear Kacy say this.	-Running down hall -Modeling -Acceleration -Comparing experience in model to task -*Relating to other experiences* -Make more sense

		more sense. [Andrew: Yeah, that's true] 'Cause you're going and then you have to like pull yourself [pulls her head back] back kind of. [Andrew: Negative g-force]	Now the camera is on our table	-'Cause, -*Justification*
45:06	Daniel	It's weird though because the cat wouldn't be worried about that, though. You know, he'd just be worried about running fast.		-Weird -Look at cat's perspective -*Context*
45:12	Andrew	He wasn't even worried about running fast, he was just going.		-Just going -Continuing discussion
45:15	Daniel	Oh yeah, but I mean like he wouldn't be like, "okay, now I'm going to stop and pull back, 'cause like…"		-I mean -*Relating to other experiences* -'cause like
45:18	Andrew	Well, I mean when you're doing a general walk yourself you're not really even paying attention yourself about your velocity or your [garbled] and acceleration [Daniel: True] You're just going.		-I mean -Not paying attention, just going -*Relating to other experiences* -Velocity -Acceleration
45:28	Derrick	I think about it.		-Joking
45:32	Kacy	[Laughing] Just doing little walking experiments.		-*Social interaction* -Laughing -Experiments
45:33	Andrew	When you're running track. You have a negative acceleration every time you take a hurdle. [Students start working on their own again]		-Running track -*Relate to other experiences* -Imagining -Negative acceleration

9-2 01:44	Derrick	I'm trying to think of how you figure it out [finding the slope of the tangent line without a calculator].		-Trying to think -How to figure it out -Slope -Tangent line -Without calculator -*Poses problem*

01:54	Daniel	Well what if we just took two lines, what if we just took from frame nine to ten and from frame ten to eleven and found the linear slope with that. [Andrew: The two different lines?] Yeah. That might…		-Well what if -Linear slope -Brainstorming -That might… -*Responding to student posed problem*
02:11	Andrew	Can you compare, like take the slope, like do the rate of change from one slope to another slope to find the slope for ten?		-Can you…? -*Presents idea* -Rate of change -Slope -Comparing two slopes
02:17	Daniel	Wait a second! If we have those two lines then we could take the arctangent thingamabobber or something like that and find the angle.	Spoken quickly	-Wait a second -*Excitement* -Arctangent -*Relating to previous knowledge*
02:24	Andrew	Do you know how to do that [take arctan]?		-Do you know how -*Desire to know*
2:25	Daniel	I don't but I can draw it and maybe fake it. I can fake it and maybe it will lead somewhere.		-Fake it -Risk taking -Trying an idea -Lead somewhere
02:48	Daniel	So do you know how to take that? [Kacy: Do what?] Okay, this is my idea. So we have this point, this point, and this point [draws three points]. Okay, so we can draw a linear line and we could figure this out by hand. Two linear lines [draws two lines intersecting at an obtuse angle]. And then we go from here. If we took these two lines…I don't know. Take the tangent of here. The arctangent or something? That angle…[Draws an arc between to the two lines]		-Do you know how? -This is my idea -Shows idea -Tangent -Arctangent -Arctangent or something
03:29	Andrew	So would the arctangent be		-Clarifying question

		like a line from there?		
03:33	Daniel	It would be like this. I don't know what I'm doing. There's something like-		-Explanation -Don't know what I'm doing -Something like
03:41	Kacy	-Oh! You're right! There's something [Daniel: there's something like that] Like a triangle thingy.		-*Excitement* -Triangle thingy