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Achieving Complex Motion with Fundamental Components for Lamina Emergent Mechanisms

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ACHIEVING COMPLEX MOTION WITH FUNDAMENTAL COMPONENTS FOR LAMINA EMERGENT MECHANISMS

by

Brian G. Winder

A thesis submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Science

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BRIGHAM YOUNG UNIVERSITY

GRADUATE COMMITTEE APPROVAL

of a thesis submitted by

Brian G. Winder

This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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Date	Dr. Larry L. Howell
Date	Dr. Brian D. Jensen

BRIGHAM YOUNG UNIVERSITY

As chair of the candidate's graduate committee, I have read the thesis of Brian G. Winder in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

ACHIEVING COMPLEX MOTION WITH FUNDAMENTAL COMPONENTS FOR LAMINA EMERGENT MECHANISMS

Brian G. Winder

Department of Mechanical Engineering

Master of Science

Designing mechanical products in a competitive environment can present unique challenges, and designers constantly search for innovative ways to increase efficiency. One way to save space and reduce cost is to use ortho-planar compliant mechanisms which can be made from sheets of material, or lamina emergent mechanisms (LEMs). This thesis presents principles which can be used for designing LEMs.

Pop-up paper mechanisms use topologies similar to LEMs, so it is advantageous to study their kinematics. This thesis outlines the use of planar and spherical kinematics to model commonly used pop-up paper mechanisms. A survey of common joint types is given, as well as an overview of common monolithic and layered mechanisms. In addition, it is shown that more complex mechanisms may be created by combining simple mechanisms in various ways. The principles presented are applied to the creation of new pop-up joints and mechanisms, which also may be used for lamina emergent mechanisms. Models of the paper mechanisms presented in Chapter 2 of the thesis are found in the appendix, and the reader is encouraged to print, cut out and assemble them.

One challenge associated with spherical and spatial LEM design is creating joints with the desired motion characteristics, especially where complex spatial mechanism topologies are required. Hence, in addition to a study of paper mechanisms, some important considerations for designing joints for LEMs are presented. A technique commonly used in robotics, using serial chains of revolute and prismatic joints to approximate the motion of complex joints, is presented for use in LEMs. Important considerations such as linkage configuration and mechanism prototyping are also discussed.

Another challenge in designing LEMs is creating multi-stable mechanisms with the ability to have coplanar links. A method is presented for offsetting the joint axes of a spatial compliant mechanism to introduce multi-stability. A new bistable spatial compliant linkage that uses that technique is introduced.

In the interest of facilitating LEM design, the final chapter of this thesis presents a preliminary design method. While similar to traditional methods, this method includes considerations for translating the mechanism topology into a suitable configuration for use with planar layers of material.

ACKNOWLEDGMENTS

I would like to thank my graduate advisor, Dr. Spencer Magleby, for his guidance, advice and support at every stage of this research. I also thank Dr. Larry Howell and Dr. Brian Jensen for their direction, support and encouragement.

I owe many thanks to the students in the Compliant Mechanisms Research lab. All have helped tremendously in various ways, and I would especially like to thank Joey Jacobsen for his insight throughout the course of this project. Also, many thanks to Tyson Triplett for his work on the illustrations in Chapter 2.

This work would not have been possible without the support of my family, and especially my wife, Kate. I thank her for her loving patience.

I would be neglectful if I did not thank the source of my inspiration and comfort throughout the course of my schooling. For that, and for life itself, I thank my Heavenly Father.

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Chapter 1

Introduction

1.1 PROBLEM STATEMENT

Cost and space reduction are ever-present concerns for engineers designing mechanical products in a competitive environment, especially with large production runs where economies of scale are a significant factor. As the number of produced parts increases, small savings per part make bigger differences in the total production cost. A proven way to dramatically reduce production costs is to simplify parts and reduce or eliminate assembly steps for components and products.

Using Lamina Emergent Mechanisms (LEMs) is one way to simplify both fabrication and assembly. LEMs are compliant mechanisms that can be fabricated from planar materials (e.g. sheet goods), with motion out of the fabrication plane. They potentially employ less expensive manufacturing techniques, use low cost sheet materials, require little or no assembly, and are compact in the initial state. In addition to exhibiting manufacturing simplicity, LEMs also have the potential to be used in applications requiring complex motions.

One challenge, however, is that LEM components and design procedures are not currently well-defined. Recent work has presented some basic components for single-layer LEMs [1]. While it is possible to create more complex motions through the use of multiple layers, components and methods for doing so are not well-developed, making it difficult to use LEMs in many practical applications. Thus, in the areas where they have the potential for the most impact, they are sometimes not employed because fundamental principles and components for LEM design are not well-defined.

The main objective of this thesis is to identify fundamental components for lamina emergent mechanisms, specifically joint types that may be used for complex mechanism motion and how they may be created and combined using planar layers of material. This thesis simplifies the LEMs design process by examining the elemental components of complex systems.

1.2 THESIS APPROACH

The main content of this thesis is broken into four chapters. Chapter 2 outlines the use of kinematics to model paper pop-up mechanisms, a well-developed field that is closely related to LEMs. Complex paper pop-up mechanisms are studied by modeling their elemental parts. Chapter 2 also presents new joint types and mechanisms not previously used for pop-up designs, and shows ways that simple mechanisms may be combined for complex motion. Because Chapter 2 examines elemental components for planar paper layers, the principles discussed can readily be applied to the simplification of the LEM design process.

Chapter 3 builds on the work in Chapter 2 by extending the discussion on joints suitable for LEMs. It is shown that many types of complex joints may be approximated with serial joint chains in planar layers of material. Chapter 4 outlines a technique for introducing bistability in spatial compliant mechanisms made from planar layers, which allows LEM designs with energy storage properties. The example linkage shown is a new bistable spatial compliant linkage that can be made from planar layers of material. The discussion in Chapter 5 centers on a preliminary framework for designing LEMs.

The research behind this thesis has been enhanced by preparing papers for peer-reviewed publications. Chapters 2 and 3 of the thesis reflect publications in the *Design Engineering Technical Conferences (DETC)* [2, 3]. Also, Chapter 4 is written with the intent for publication when a suitable venue is found. Chapter 5 is intended as a preliminary foundation for further investigation of a LEM design process for publication. Because the chapters of this thesis were written as standalone works, each chapter outlines relevant academic literature as it becomes necessary.

The appendices of this thesis contain information of interest to the reader. Appendix A outlines two brainstorming sessions that were held in the Compliant Mechanisms Research lab, in order to define possible uses for LEM technologies. Many different people in the group contributed to the list shown. Appendix B shows pictures and cutouts used to demonstrate the ideas outlined in Chapter 2 at the conference at which that paper was presented. The reader is encouraged to copy or print those pages, and cut out and assemble the pop-up mechanisms contained therein.

Chapter 2

Kinematic Representations of Pop-up Paper Mechanisms

2.1 ABSTRACT

Pop-up paper mechanisms use techniques similar to the well-studied paper folding techniques of origami. However, pop-ups differ in both the manner of construction and the target uses, warranting further study. This chapter outlines the use of planar and spherical kinematics to model commonly used pop-up paper mechanisms. A survey of common joint types is given, including folds, interlocking slots, bends, pivots, sliders and rotating sliders. Also included is an overview of common one-piece and layered mechanisms, including single-slit, double-slit, V-fold, tent, tube strap and arch mechanisms. Each mechanism or joint is described using a kinematic or compliant mechanism representation, and cutouts for all of the objects are found in Appendix B. In addition, this chapter shows that more complex mechanisms may be created by combining simple mechanisms in various ways. The principles presented are applied to the creation of new pop-up joints and mechanisms. The new mechanisms employ both spherical and spatial kinematic chains. Studying pop-up mechanisms allows various engineering applications to benefit from the unique perspective provided by pop-up artists. Possible applications include deployable structures, packaging and instruments for minimally invasive surgery.

2.2 INTRODUCTION

Pop-up book design (sometimes called paper engineering [4]) has long been considered an art, with its practice limited to a few specialized designers. However, there have been attempts at disseminating knowledge of design principles more widely. This is advantageous because, as Song and Amato [5] and Balkcom [6] show, certain princi-

Figure 2.1: A pop-up mechanism from Sabuda and Reinhart [10].

ples that apply in the design of paper folding have useful application in other areas (e.g. airbag folding, sheet metal forming, protein folding, etc.). However, academic literature on the subject of pop-up mechanisms applies mostly to computer simulation and graphical display. The mechanisms used in pop-ups (Fig. 2.1 shows a complex example) have been examined insofar as to produce workable equations for computer modeling of simple mechanisms [7–9]. Very little is discussed on underlying principles. There are some books written on the design and fabrication of pop-up mechanisms by well-known pop-up designers, but those focus on mechanism embodiments and offer little explanation in terms of the kinematic principles behind the mechanisms.

Origami literature includes lengthy discussions on kinematic principles and demonstrates in-depth understanding of folding theory (see, for example, [6]). However, pop-ups differ from traditional origami in that many are made from more than one sheet of paper and most require cutting. Also, mechanism motion between open and closed positions for a pop-up is often just as important to the designer as the folded positions themselves. Because of this, different rules apply to pop-up books than to origami, even though similar kinematic principles can be used throughout.

Not only are the kinematics of pop-up mechanisms unclear, but traditional methods of examining paper mechanisms ignore the compliance of paper links [11, 12], because many paper mechanisms can be fully simulated without taking deflection into account.

Attempts in the academic community to model the motion of paper mechanisms in a non-idealistic sense (with flexible links) quickly become complex discussions of developable surfaces and continuum mechanics [6, 13, 14]. There is a need for a simpler, but still sufficiently accurate framework in which to look at paper mechanisms, if we are to use their principles to design new mechanisms.

The *Pseudo-Rigid Body Model (PRBM)* [15] shows that many compliant mechanisms may be represented accurately by modeling them as rigid-link mechanisms with torsional springs at their revolute joints. The joint locations and spring constants depend on the mechanism being modeled. Using the PRBM greatly simplifies compliant mechanism analysis because traditional kinematic approaches may be used.

The objective of this chapter is to formalize the linkage between pop-up design, and both kinematic principles and compliant mechanism principles. This provides a basis for using common paper mechanisms in other applications, and also for applying kinematic principles to create new paper mechanisms. This chapter will classify selected mechanisms in an effort to generalize the approach to paper mechanism design.

The motivation in solidifying the link between kinematic principles and paper mechanism design is to allow designers to better understand and predict complex mechanism behavior. There is value in examining pop-ups because the artists have created many interesting mechanisms with complex motion by approaching the subject from a different point of view. Engineers can learn from this unique perspective and apply the principles to applications beyond paper engineering, especially in areas requiring complex motion from compact mechanisms, such as deployable structures and instruments for minimally invasive surgery.

2.3 MECHANISMS IN PAPER ENGINEERING

Figure 2.2 categorizes many types of movable, collapsible and pop-up products. The first documented use of a movable book is in the 13th century A.D., but it is only in recent times that movable books have gained popularity in children's literature. Pop-up books can be considered to employ techniques from movable, collapsible and pop-up products.

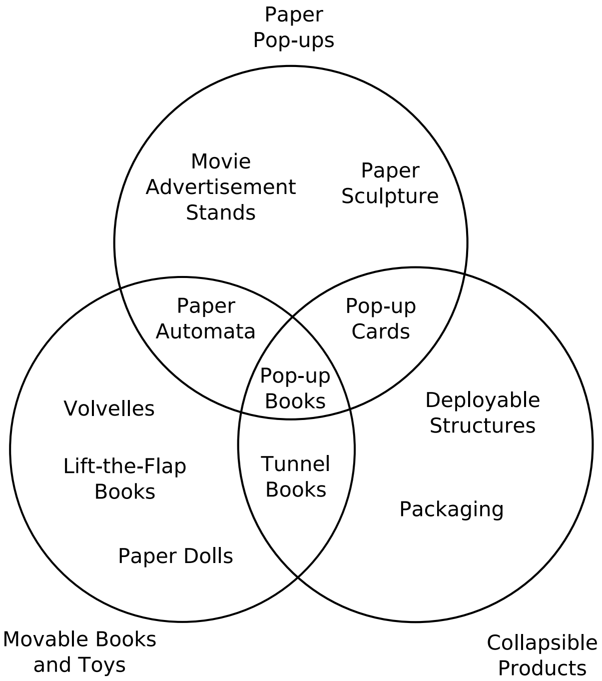

Figure 2.2: The relationship between pop-up books, collapsible products and movable products (adapted from [16]).

Jackson [17] defines a pop-up as "a self-erecting, three-dimensional structure, formed by the action of opening a crease," and thus excludes mechanisms such as "lift-up flaps, pull tabs and other two-dimensional paper-engineered devices." Pop-up books are used to create unique, exciting motions, and to better illustrate complex ideas where they can be more accurately represented and understood in three dimensions.

The key element in many pop-up mechanisms is the ability to fold flat in one or more positions. In pop-up books, the mechanism is well-hidden inside the book when it is closed, but when the book is opened, the mechanism often expands to fill space above the page or even extend beyond the page limits. The initially hidden pop-up adds an element of surprise to delight the reader.

However, designing mechanisms to accomplish the task of expansion into three dimensions can be a laborious process. The following sections present a framework in which to view pop-up design, so that designers can have more tools at their disposal.

2.4 KINEMATICS PRINCIPLES

To put our presentation of pop-up mechanisms into context, this section introduces linkage topologies as commonly represented in kinematics.

In their discussions on modeling for pop-up mechanisms, Lee et al. [7] and Glassner [8] show that for a common pop-up mechanism called a V-fold, the axes of the joints all intersect at the same point. This means that the individual links rotate about this fixed point. Lusk [18], in defining spherical mechanisms, uses identical conditions. Thus, spherical mechanism theory may be used to describe many pop-up mechanisms. As Balkcom [6] outlines, spherical kinematics may also be used to model vertex folds in origami (vertex folds are a more general case of V-folds, with open chains and large numbers of folds possible).

Spherical kinematics is also considered a general form of kinematic principles. Planar kinematics is a special case, with the joint axes intersecting at infinity (meaning the joint axes are parallel) [18]. Hence, spherical mechanism theory encompasses planar mechanism theory. Consequently, most pop-up mechanism motion, whether planar or spherical in nature, can be described by spherical kinematics. For simplicity, however, this chapter will use planar kinematics in the treatment of planar mechanisms.

Figure 2.3 shows common spherical and planar kinematic chains; both a slider-crank and a four-bar linkage are shown for each type. The links for each mechanism are represented as shown here for simplicity and broader application, but in reality any two mechanisms with the same relative position of the joint axes (assuming adequate link stiffness) are equivalent. Many pop-up designers use this principle, combined with effects that do not affect the kinematics, to conceal the mechanism topologies used in their designs.

Planar and spherical mechanisms are constrained to move within planar and spherical surfaces, respectively. Mechanisms which can move through space and are not constrained to a surface are called spatial mechanisms. Spatial mechanisms are classified by the joints used to make them because the greater number of available joints makes it possible to have many different single degree-of-freedom mechanisms for a given number of links. For example, two different spatial four-bar mechanisms with one degree of freedom are RCCC and RSRC mechanisms. Moving in one direction around the linkage, RCCC

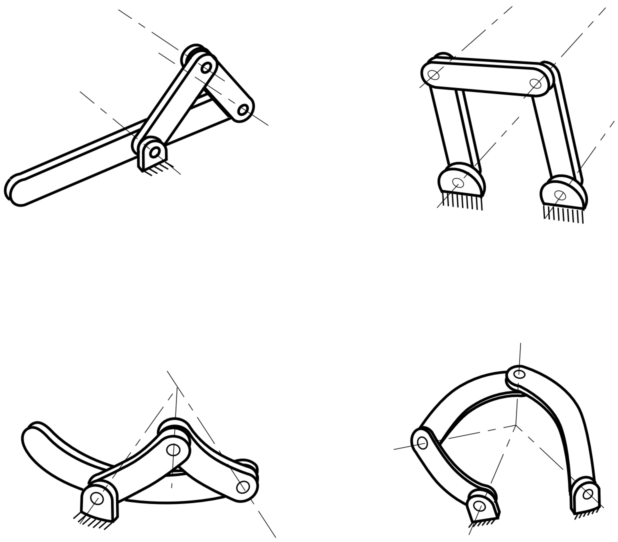

Figure 2.3: Planar topology (above) vs. spherical topology. The left images show slider-cranks and the right images show four-bar mechanisms.

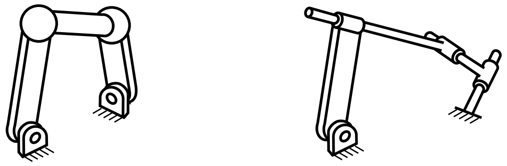

Figure 2.4: Spatial topology. The left image shows an RSSR and the right image shows an RCCC. The spheres shown represent spherical joints.

mechanisms have one revolute (R) and three cylindric (C) joints, while RSRC mechanisms have one revolute, one spherical (S), another revolute, and one cylindric joint. Figure 2.4 shows two common spatial four-link mechanisms: the RSSR and the RCCC mechanism.

Gruebler's equation (and Kutzbach's modification of it) applies to the above planar, spherical and spatial mechanisms for determining degrees of freedom. For each mechanism, the mobility (number of degrees of freedom), M, is determined by the number of links, L, and the number of degrees of freedom removed by the joints. For planar and spherical mechanisms, the equation is

$$M = 3(L-1) - 2J_1 - J_2 \tag{2.1}$$

and for spatial mechanisms, Kutzbach's equation becomes

$$M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5 \tag{2.2}$$

[19], where J_i is the number of joints with i degrees of freedom.

Note that there are certain mechanisms for which Kutzbach's equation does not apply because of unique topology [19]. For example, a parallelogram mechanism becomes one of these mechanisms if it has five or more links. This mechanism has one degree of freedom, not zero as the equation predicts.

2.5 KINEMATICS PRINCIPLES APPLIED TO COMMON PAPER MECHANISMS

Birmingham [20] states that "true pop-ups" can be classified as extensions of the *V-fold*, the *Parallelogram* and/or the *45° fold*. Carter and Diaz [4] categorize mechanisms as *parallel folds*, *angle folds*, *wheels* and *pull tabs*. This chapter classifies pop-up mechanisms according to the kinematics best used to describe them (e.g. spherical or planar mechanisms). We discuss pop-up mechanisms in terms of kinematic linkages. We generally employ the the four-link chain, because it is the simplest one degree-of-freedom closed-loop mechanism with one degree-of-freedom joints. Most pop-ups can be categorized as combinations of spherical or planar four-link mechanisms.

Figure 2.5 shows a planar four-link pop-up mechanism. We made this mechanism by cutting two slits in a piece of cardstock, then folding according to the diagram in Fig. 2.6. A mountain fold is a convex fold, and a valley fold is a concave fold [21].

For simplicity in explaining the kinematics of paper mechanisms, we use the conventions shown in Fig. 2.7. For this chapter, the right base page is ground (link 1), and the left base page contains link 2. Link 3 is located between links 2 and 4, and link 4 is located between links 3 and 1. Joint 1 is the *spine* or *gutter* (the central fold between base pages), joint 2 is between links 2 and 3, joint 3 is between links 3 and 4, and joint 4 is between

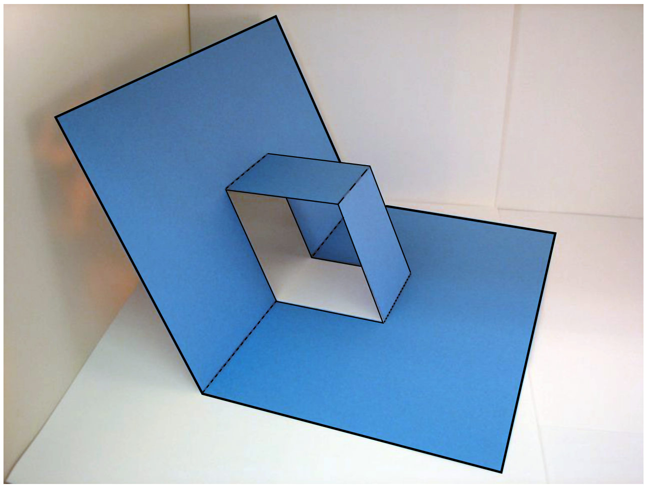

Figure 2.5: A simple paper pop-up mechanism.

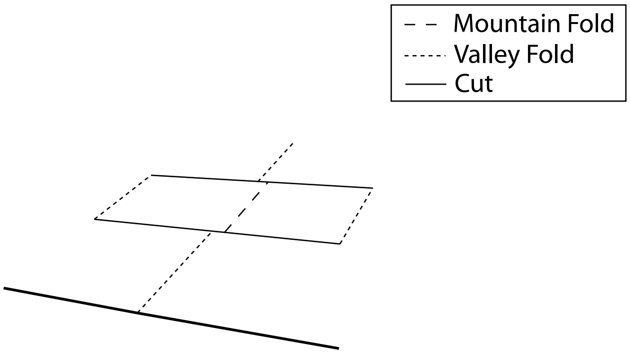

Figure 2.6: The required cuts and folds for making a simple paper pop-up mechanism.

links 4 and 1. For a four-bar, joint 4 is a revolute joint, while it is a prismatic joint for a slider-crank.

The 0° position for these mechanisms is with the book closed, 90° is with the pages perpendicular, and the 180° position is with the book fully open. The positions refer to the angle between the base pages (links 1 and 2). Some pop-up mechanisms do not allow the book to open all the way to the 180° position. Four-bar mechanisms will not open to the 180° position when

$$L_1 + L_2 > L_3 + L_4 \tag{2.3}$$

where L_i is the length of link i.

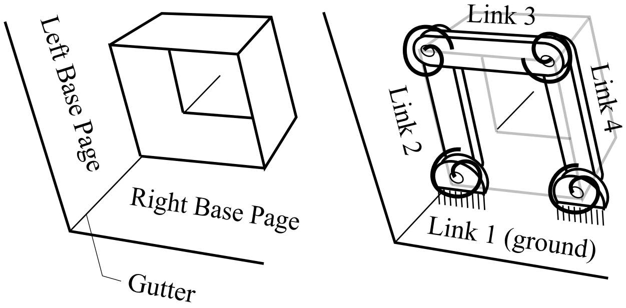

Figure 2.7: Linkage topology conventions used in this paper.

Many pop-up mechanisms are *flat folding*, which is a term taken from origami signifying the ability to "be pressed flat without crumpling [21]." This refers to the completed (folded) position of an origami structure. Origami literature does not consider unfolded structures in this definition because in most origami, the unfolded position is a flat piece of paper and origami structures do not move between folded and unfolded positions like pop-up mechanisms do. For pop-up mechanisms, however, we must consider both the folded (0°) and unfolded (180°) positions in the definition of flat folding, because the unfolded position is not necessarily flat.

For a four-bar mechanism to fold flat in the 0° position, links 2 and 3 must add to be the same length as links 1 and 4 added together (see [7]), or

$$L_1 + L_4 = L_2 + L_3 \tag{2.4}$$

For a four-bar mechanism to fold flat in the 180° position, it must follow

$$L_1 + L_2 = L_3 + L_4 \tag{2.5}$$

Deltoid four-bar mechanisms require that

$$L_1 = L_2 \tag{2.6}$$

and

$$L_3 = L_4 \tag{2.7}$$

Parallelogram four-bar mechanisms require that

$$L_1 = L_3 \tag{2.8}$$

and

$$L_2 = L_4 \tag{2.9}$$

Note that solving for L_2 in Eqn. (2.4) and substituting into Eqn. (2.5) gives Eqn. (2.8), and solving for L_1 in Eqn. (2.4) and substituting into Eqn. (2.5) gives Eqn. (2.9). Thus, a parallelogram four-bar mechanism will always fold flat at both 0° and 180°.

Simple paper mechanisms can be classified as either one-piece or layered. The following sections define these classifications.

2.5.1 One-piece Mechanisms

For this chapter, *one-piece* mechanisms are defined as those which do not require assembly. Carroll et al. [22] described mechanisms that can be made from a single layer but still require assembly. These follow more complex rules, so they are grouped with layered mechanisms. Because one-piece mechanisms do not require glueing or other assembly, complex one-piece mechanisms often are much easier to manufacture than layered mechanisms of similar complexity.

One-piece mechanisms are by definition "orthoplanar mechanisms," which means that they can be manufactured in a plane with out-of-plane motion. In addition, they are changepoint mechanisms, with more than one possible movement in their planar position. However, the nature of paper folds and mechanism layering usually negates the possible changepoint effect.

One-piece mechanisms fold flat at 180°, but may not fold flat at 0°. Designs that fold flat in two positions follow Eqns. (2.4) and (2.5). For these devices, the most "poppedup" position is at 90°, so these are often called 90° structures (e.g. in [16]). They are popular in creating pop-up greeting cards.

It is impossible to create a prismatic joint with our definition of one-piece mechanisms, so cuts, folds and bends (revolute joints) are used exclusively. While other joints may be used in mechanisms made from a single sheet, those require assembly, thus violating the requirement for a one-piece mechanism. Consequently, one-piece slider-crank mechanisms are not possible.

2.5.2 Layered Mechanisms

Layered mechanisms are not restricted to a single sheet of paper, but require assembly. Layered mechanisms consist of one or more paper mechanisms anchored to the base pages. Single-sheet mechanisms that require assembly are also members of this category. Multiple layers open the possibilities of many new joints and mechanism types, and thus more complex motions. For example, while one-piece mechanisms fold flat at 180°, with a layered technique it becomes possible for mechanisms to pop-up at 180°. For a four-bar mechanism to pop-up at 180°, it must follow

$$L_1 + L_2 < L_3 + L_4 \tag{2.10}$$

While there are many elaborate examples of layered mechanisms, the majority are combinations of V-folds, tents, tube straps and arches, which are discussed later in the chapter.

2.5.3 Joints

The nature of paper allows freedom to choose more than one joint type for a given motion. Many of the common joints used in pop-up books can be classed into one of a few types. Single degree-of-freedom joints include *folds*, *interlocking slots*, *bends*, *pivots* and *sliders*. These joints limit motion to either translation or rotation. There are considerably fewer two degree-of-freedom joints in use, but one example discussed here is the *rotating slider*.

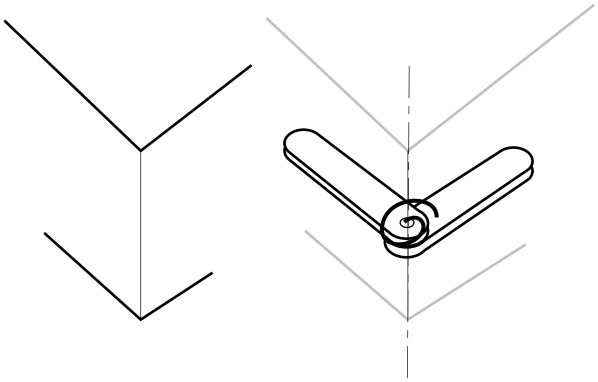

Figure 2.8: A fold (crease) and its PRBM representation.

Pop-up designers often can create “joints” in continuous sheets. In reality, these “joints” are areas of compliance (*flexures*) which allow relative motion between stiffer sections. The compliant joints in this section are represented with the Pseudo-Rigid Body Model (PRBM).

Folds

The *fold* is the most common joint used in making pop-ups. A fold is created by scoring, indenting or creasing (thus mechanically yielding) a piece of paper along a joint axis [17]. The paper on either side of the joint exhibits much greater stiffness than the joint itself and rotates about the axis of the crease. Thus, the crease acts as an equivalent revolute joint between two links. It is modeled with a revolute joint and torsional spring at the center of the fold, as shown in Fig. 2.8. In many cases, the moment produced by the joint is so small that the torsional spring can be ignored.

Interlocking Slots

Interlocking slots are revolute joints that are constructed of separate pieces of paper. Slots are cut in each piece, and these nest with each other to constrain relative motion. Interlocking slots are rigid-link joints. The main advantages of interlocking slots stem mostly from the fact that they do not require one continuous piece of paper: the paper does not yield (and will last longer) and more complex mechanisms are possible. Figure 2.9 shows an interlocking slot.

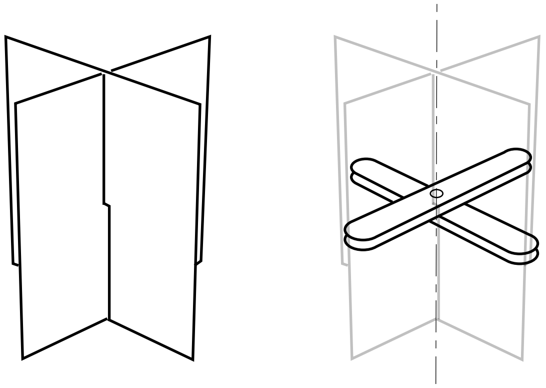

Figure 2.9: An interlocking slot and its kinematic representation.

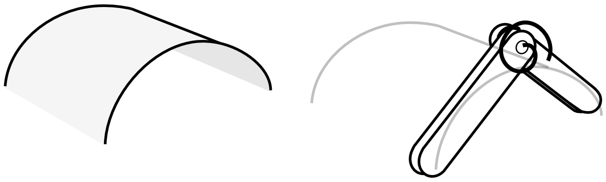

Figure 2.10: A bend and its PRBM representation.

Bends

A *bend* is defined here as an uncreased section of paper that does not stay planar (exhibits curvature) when the mechanism moves. Bends can be modeled with revolute joints and torsional springs. A bend is shown in Fig. 2.10 with its PRBM representation. For the mechanism shown, the PRBM places the revolute joint and spring in the center of the deflected piece. The bends in this chapter are modeled this way.

Pivots

We define a *pivot* as a revolute joint with the ability to rotate more than 360 degrees, with its axis of rotation perpendicular to its attachment plane. Pivots were among the first joint types used in movable books, and still see widespread use (their most common use is for *volvelles*, or rotating discs). However, they limit motion to rotation within a plane and

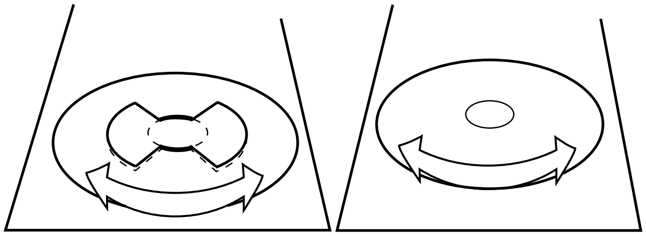

Figure 2.11: A pivot and its kinematic representation.

thus the mechanisms for which they are used often do not fit Jackson's definition of pop-ups. The possibility exists, however, for combining them in mechanisms exhibiting out-of-plane motion. Pivots may be made either with creative combination of multiple paper layers or with non-paper fasteners. They cannot be made fully compliant, so the PRBM is not necessary to model them. Figure 2.11 shows a pivot and its kinematic representation.

Sliders

Sliders (prismatic joints) are common in pop-up mechanisms because they allow a larger range of motion and present many unique motion opportunities. The layered nature of pop-up books lends itself quite well to the creation of sliding joints, because little extra is needed to completely constrain a joint for one degree-of-freedom translation. While the motion of a sliding joint is often within the plane of the page, it can be used to create out-of-plane motion by linking it to a pop-up mechanism. Many pull-tab mechanisms employ sliders.

It is not necessary to employ the PRBM for these mechanisms. A representation of a slider is shown in Fig. 2.12.

Rotating Sliders

Rotating sliders (half-joints) allow one rotational and one translational degree of freedom. They are commonly used, and can be easily mistaken for sliders. In fact, most literature does not differentiate between the two joints. However, these joints do not restrict

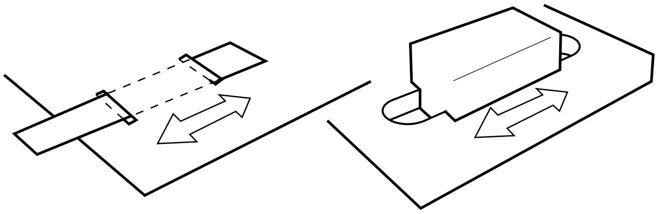

Figure 2.12: A slider and its kinematic representation.

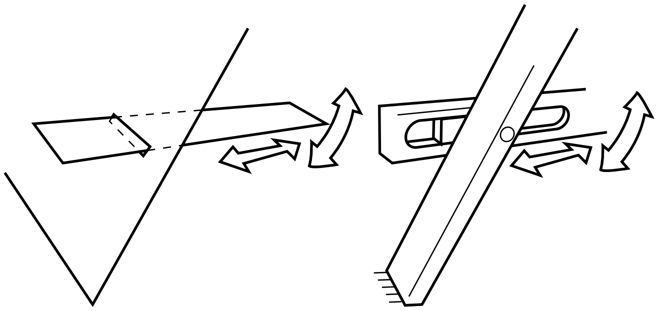

Figure 2.13: A rotating slider and its kinematic representation.

motion to the plane of the page, and their out-of-plane motion is often purposely hidden to create special effects. A rotating slider is created with a slot through which a paper link moves. The slot allows two degrees of freedom. Figure 2.13 shows one such joint and its kinematic representation.

2.5.4 Planar Mechanisms

As previously discussed, planar mechanisms have parallel joint axes. For simple pop-up designs, the joint axes are also parallel to the gutter axis. For these reasons, Carter and Diaz call these mechanisms *parallel folds* [4]. The most common planar pop-up mechanisms are *planar double-slit* mechanisms, *tents*, *tube straps* and *arches*. Planar double-slit mechanisms are one-piece mechanisms, while tents, tube straps and arches require layered techniques. The following sections explain each of these examples.

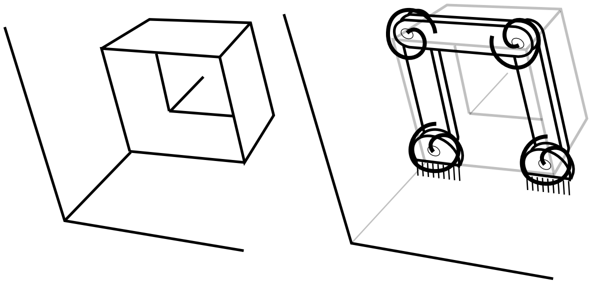

Figure 2.14: A double-slit device and its PRBM representation.

Planar Double-slit Mechanisms

Planar double-slit mechanisms are produced when two cuts are made across the gutter, and three folds are made with their axes parallel to the gutter. Double-slit devices can be spherical or planar four-bar mechanisms. Spherical double-slit mechanisms are discussed in a later section. The cuts across the gutter do not have to be straight for either of these mechanisms, thus allowing interesting possibilities to come from a simple mechanism. A planar double-slit device is shown in Fig. 2.14.

Tents

A *tent* is a planar four-bar mechanism created with a layered technique. The links can be of any arbitrary length as long as they follow Eqn. (2.4) so that they can fold flat. Figure 2.15 shows an example of a tent mechanism.

The deltoid tent is a special case which follows Eqns. (2.6) and (2.7). The parallelogram mechanism is a special case tent that follows Eqns. (2.8) and (2.9).

Tube Straps

Tube straps [4] are mechanisms that allow one base page to drive a mechanism on the other base page. They are called tube straps because the paper is doubled up (for rigidity), forming a flattened tube. The kinematic equivalent to a tube strap is the driver

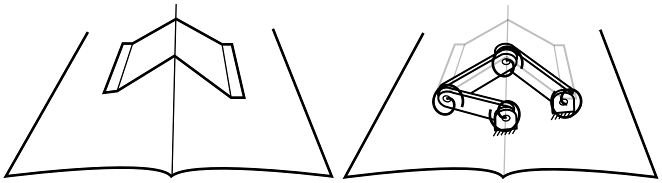

Figure 2.15: A tent and its PRBM representation.

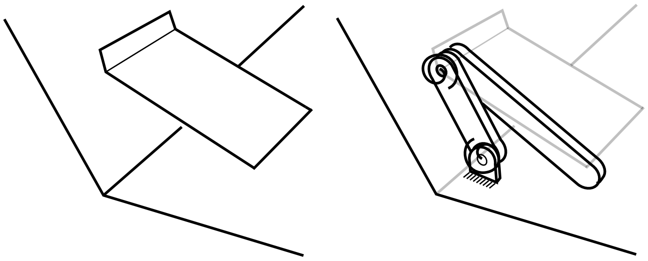

Figure 2.16: A tube strap and its PRBM representation.

dyad, which by itself is an open chain mechanism. Thus, tube straps appear in pop-ups in combination with other mechanisms. Figure 2.16 shows an example of a tube strap.

Arches (Knee Mechanisms)

Arches [20], which are kinematically equivalent to *knee mechanisms* [4], can be modeled as planar slider-crank mechanisms. The only difference between the two mechanisms is that one uses a fold between links 2 and 3 (knee mechanism), while the other uses a bend (arch). This means that their PRBM representations are equivalent except for the torsional spring constant between those links.

These mechanisms may be represented in two ways: with three links and a half joint between links 3 and 1, or with four links, where link 3 is connected to link 4 by a revolute joint, and link 4 is connected to link 1 by a prismatic joint. We choose to represent

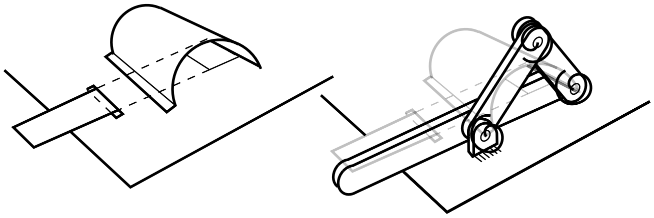

Figure 2.17: An arch and its PRBM representation.

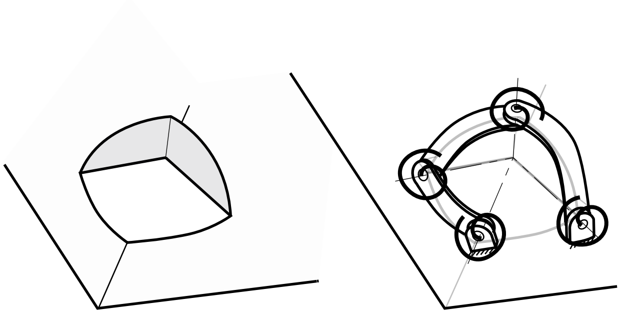

Figure 2.18: A single-slit device and its PRBM representation.

arches in the second way because it more accurately reflects the actual paper geometry. Figure 2.17 shows an example of an arch mechanism. Arches can be actuated either with a user-actuated slider (pull tab) or with a tube strap connected to link 4.

2.5.5 Spherical Mechanisms

Simple pop-ups which employ spherical mechanisms have joint axes which all intersect at a point on the gutter. Carter and Diaz call these mechanisms *angle folds* [4]. The following sections explain the most common spherical pop-up mechanisms: *single-slit* mechanisms and *V-folds*.

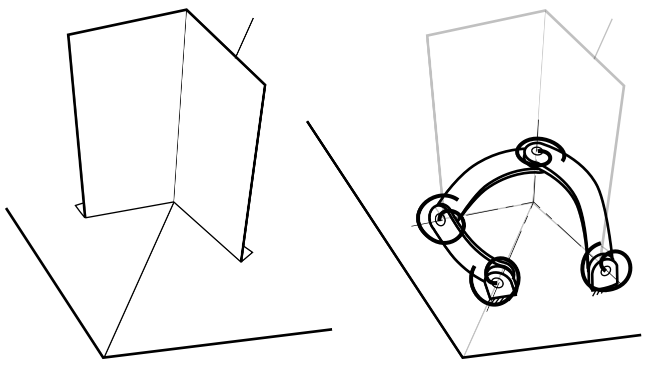

Figure 2.19: A V-fold and its PRBM representation.

Single-slit and Spherical Double-slit

Single-slit and *spherical double-slit* mechanisms are spherical four-bar devices made with one-piece techniques. Single-slit devices require one continuous slit to be made across the gutter, along with three folds intersecting the gutter at a single point. As with double-slit devices, the cut for a single-slit mechanism does not have to be straight. A single-slit device is shown in Fig. 2.18. Spherical double-slit devices have two cuts across the gutter and three folds whose axes intersect the gutter at a point.

More complex one-piece mechanisms can be made (i.e. with more than two slits), but they combine single-slit and double-slit topologies to arrive at their final shapes.

V-folds

The *V-fold* is a spherical four-bar mechanism created with a layered technique. Although similar to a single-slit mechanism, a V-fold is connected to the base pages with folded tabs. It features a fold whose axis intersects the gutter axis. This mechanism has many uses and variations in pop up design. It can pop-up toward the front or back of the book and to varying heights. Figure 2.19 shows an example of a V-fold.

2.6 COMPLEX MECHANISMS

Birmingham [20] states that simple pop-up mechanisms can be combined to make more complex devices. There are many ways to combine mechanisms, but we will group them into two categories: *series* and *parallel* combinations. We refer to the mechanism

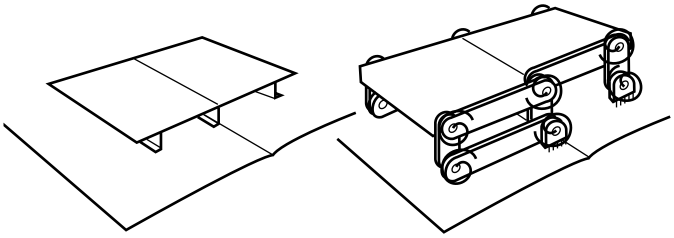

Figure 2.20: A floating layer and its PRBM representation.

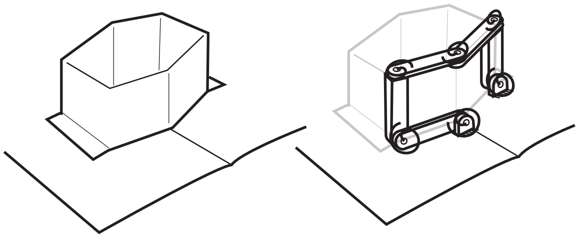

Figure 2.21: A solid shape and its PRBM representation.

most directly attached to the main mode of actuation (the mechanism that spans the gutter, the mechanism connected to the pull tab, etc.) as the *primary mechanism*, and the *secondary mechanisms* are any others that exist on the page.

Mechanisms combined in parallel both have a link attached to ground. Thus, if one or more joints of a secondary mechanism are attached to a base page, the primary and secondary mechanisms are in parallel. This principle is commonly employed in pop-up books where the fold between a link and ground becomes the gutter for another mechanism. This allows the secondary mechanism to be actuated with the primary mechanism. An example of a parallel mechanism combination, called a floating layer, is shown in Fig. 2.20.

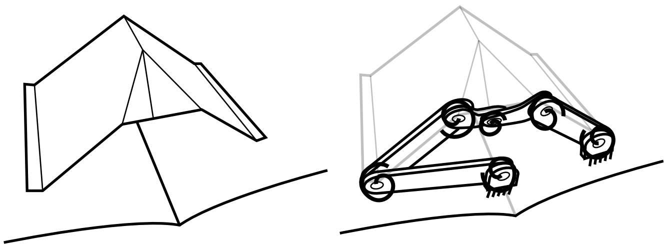

Figure 2.22: A 45° fold and its PRBM representation.

Mechanisms combined in series do not share any attachments to ground, and in general, the secondary mechanism does not have any stationary links. Examples of series mechanisms include solid shapes and 45° folds. These are shown in Figs. 2.21 and 2.22.

In many cases, combining simple mechanisms to make more complex topologies produces spatial mechanisms. Some ways to create a spatial mechanism are by combining a spherical and a planar mechanism, by combining two planar mechanisms with movement in different planes, and by combining two spherical mechanisms whose joint axes intersect at different points. Note that the mechanisms in Figs. 2.21 and 2.22 are spatial mechanisms.

Many complex mechanisms combine one-piece and layered techniques. One-piece techniques are commonly used for secondary mechanisms where layered mechanisms are the primary mechanisms because when the base pages are fully open, the layered mechanisms can create the 90° positions so favorable for one-piece designs. The technique used (one-piece or layered) has no bearing on the mechanism classification, as it is the mechanism behavior that is important.

In addition to the many ways of combining mechanisms, it is possible to make mechanisms that look more complex by changing link shapes. Thus, links may be curved, partially cut away, extended beyond their joints, etc., in order to fit the form of the item being represented by the pop-up. This is among the most common ways of creating complexity in pop-ups and representing objects more realistically. An example of changing link shapes is given in Fig. 2.23.

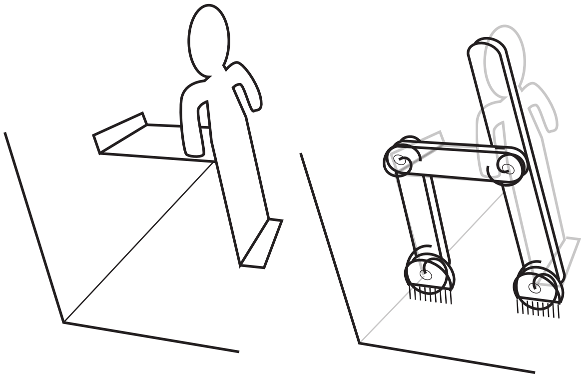

Figure 2.23: A mechanism with a complex link shape and its PRBM representation.

2.7 KINEMATICS APPLIED TO NEW POP-UP MECHANISMS

Pop-up designers often employ a top-down approach, using a long list of existing mechanisms to perform the ideal motion for a given design. However, with an understanding of kinematics, a bottom-up approach becomes feasible, whereby designers can create new mechanisms to move in a certain way. This leads to new and innovative designs for pop-up mechanisms.

2.7.1 New Paper Joints

We mentioned previously that pop-up designers use few joints that employ two or more degrees of freedom. We are not aware of the current use of any joints with three or more degrees of freedom, probably because even the most complex pop-ups are often just combinations of planar or spherical devices, so more complex joints are not required.

However, thinking in terms of spatial devices makes it obvious that there is more room for exploration in the area of paper pop-ups. The use of kinematics principles allows us to create new joints for use in pop-ups. Two common spatial joints we will discuss are the *cylindric joint* and the *spherical joint*. Pieper [23], in discussing robotic manipulators, presents ways to create equivalent spherical and cylindric joints with combinations of revolute and/or prismatic joints. His discussion forms the basis for creating these joints out of

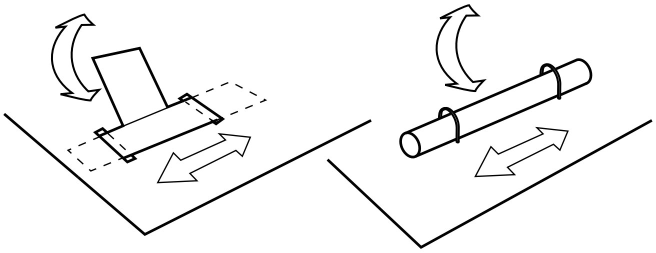

Figure 2.24: A paper cylindric joint and its kinematic representation.

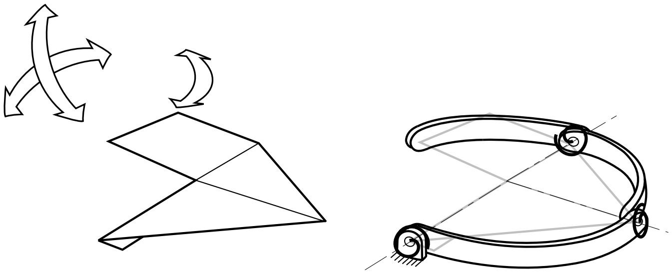

Figure 2.25: A paper spherical joint and its PRBM representation.

flat sheets of paper. There are other joint types that are used for spatial mechanisms that could be made with paper, but we limit our discussion to these.

The Paper Cylindric Joint

The *paper cylindric joint* has one rotational and one translational degree of freedom, much like the rotational slider mentioned in this chapter. However, for the cylindric joint, the axes of rotation and translation are collinear. A paper cylindric joint is made in much the same way as a slider, with two slots through which a paper link moves. The paper cylindric joint, however, also requires a fold (or a bend) to align with the direction of translation. Figure 2.24 shows a paper cylindric joint.

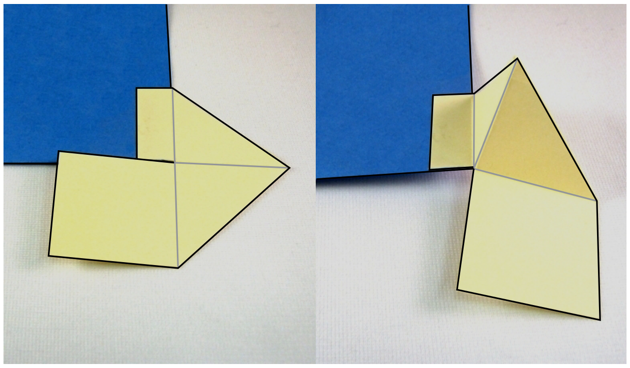

Figure 2.26: Photographs of a paper spherical joint in two positions.

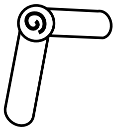

Figure 2.27: A simplified representation of the spherical joint PRBM.

The Paper Spherical Joint

Robot wrists often use "...three-jointed spherical open chain[s]" to produce motion equivalent to that of a spherical joint [24]. We apply this same technique to make a new spherical joint purely out of paper. The *paper spherical joint* consists of a single paper folded three times at right angles. This joint allows three rotational degrees of freedom about orthogonal axes, but no translational degrees of freedom. The paper folds may only move ± 180°, so the paper spherical joint is also limited to this range of motion about all its axes of rotation. In many cases its movement is limited even more by the page to which it is attached. Figure 2.25 shows a paper spherical joint and its PRBM, and Fig. 2.26 shows photographs of a paper spherical joint. Because of the visual complexity of representing the PRBM graphically for this joint, we use the representation shown in Fig. 2.27 when this joint is used in mechanisms.

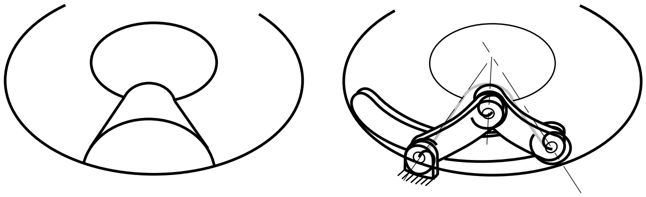

Figure 2.28: A circular arch and its PRBM representation.

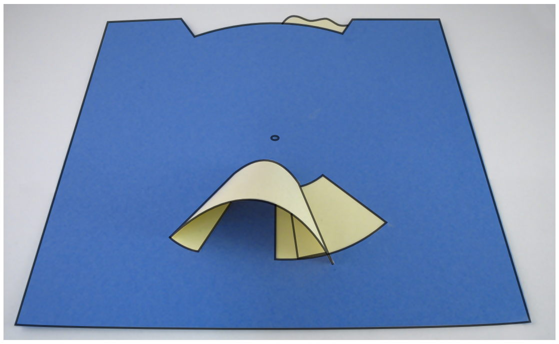

Figure 2.29: A photograph of a circular arch.

2.7.2 New Paper Mechanisms

Exploring spherical and spatial kinematics allows new paper pop-up mechanisms to be considered. This chapter has presented common joints and some new joints that may be used in creating pop-up paper mechanisms. We now present new mechanisms that utilize the joints presented thus far. The spherical slider-crank mechanism, for example, is a common mechanism that has not (to our knowledge) been used in the creation of pop-ups, so we present here its use for pop-ups.

In addition, the new paper joints discussed allow many new possibilities for spatial pop-up mechanisms. One such possibility is the RSSR mechanism, which is presented here as well.

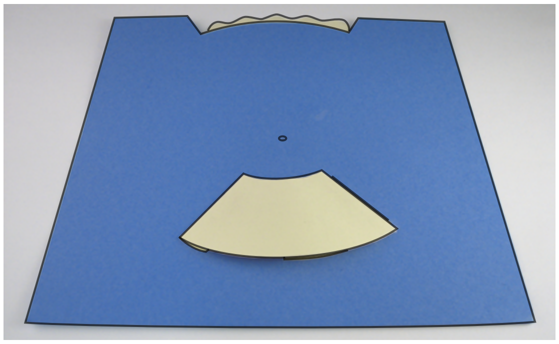

Figure 2.30: A photograph of a circular arch in the flat-folded position.

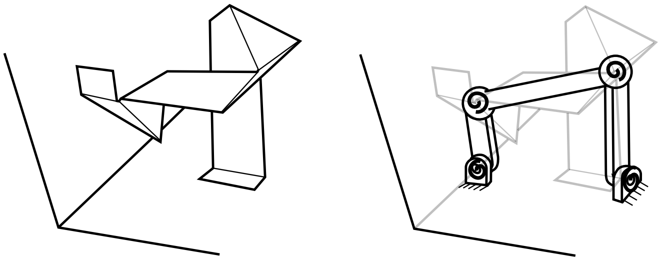

Figure 2.31: A figure-8 (paper RSSR mechanism) and its PRBM representation.

The Circular Arch

The *circular arch* is a new pop-up mechanism that employs the spherical slider-crank, a common spherical four-link mechanism. This mechanism is shown in Fig. 2.28 with its PRBM representation. It requires a layered technique and consists of a slider attached to the base page by a pivot, and a strip of paper attached to the slider on one side and the base page on the other. It is called a circular arch because attaching the slider with a pivot constrains its motion to a circle contained on the base page. The four joints required to make a circular arch are a pivot, a bend and two folds.

Figures 2.29 and 2.30 show photographs of a circular arch. As the light-colored, wavy tab at the top of each picture rotates by user input, the slider moves and the strip of paper raises off the base page.

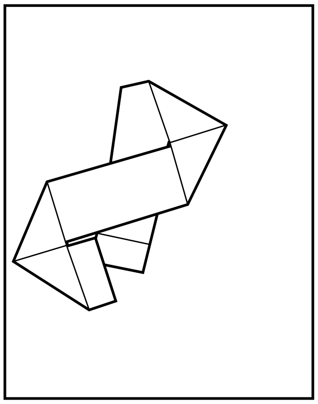

Figure 2.32: A figure-8 folded flat.

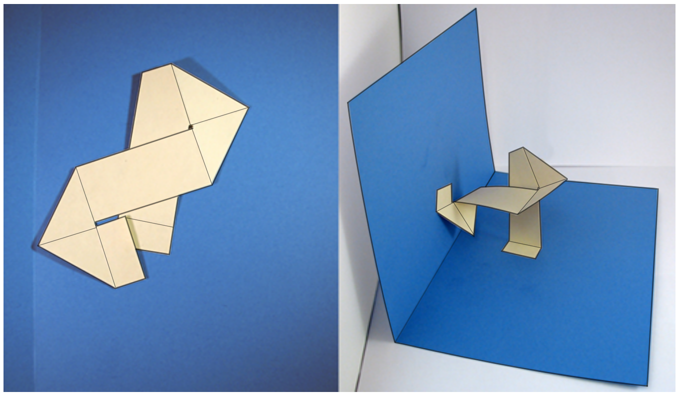

Figure 2.33: Photographs of the figure-8 paper mechanism. Both the flat position (left) and the raised position are shown.

The Figure-8 (Paper RSSR)

The RSSR mechanism is a spatial mechanism with two degrees of freedom. However, it only requires one input because the extra freedom allows the link between the two S joints to rotate about its axis. The RSSR mechanism may be created using the new paper spherical joints presented earlier.

Figure 2.31 shows a paper RSSR mechanism. This mechanism folds flat at 0°, but not at 180°. This particular paper RSSR will not move past 90° without flexing links. It consists of two paper spherical joints between two folds. We call it a *figure-8* because when the mechanism is folded flat, links 3 and 4 (the links attached to the base pages) closely resemble that shape, as shown in Fig. 2.32. Figure 2.33 shows photographs of a figure-8 in its flat and raised positions.

2.8 CONCLUSION

This chapter has shown how kinematic principles may be applied to pop-up mechanisms. Models have been demonstrated for common pop-up joints. The kinematic representation was used to develop new paper joints not previously used in pop-ups, and the joints in this chapter were demonstrated in two new mechanisms.

There is value for engineers to consider inspiration for solutions to problems (such as obtaining complex motion from compact mechanisms) from people with diverse intellectual backgrounds (such as pop-up artists). This can result in innovative solutions to problems in seemingly unrelated applications. The principles discussed here form a basis for additional work in research for applications such as deployable structures, instruments for minimally invasive surgery, and many other applications where weight and space savings are important. The ongoing research in these areas could benefit from a better understanding of collapsible mechanisms made from thin sheets.

Chapter 3

A Study of Joints Suitable for Lamina Emergent Mechanisms

3.1 ABSTRACT

One way to save space and reduce cost in a competitive environment is to use ortho-planar compliant mechanisms which can be made from sheets of material, or lamina emergent mechanisms (LEMs). One major challenge associated with LEM design, however, is creating joints with the desired motion characteristics, especially where complex spatial mechanism topologies are required. This chapter presents some important considerations for designing joints for LEMs. A technique commonly used in robotics, using serial chains of revolute and prismatic joints to approximate the motion of complex joints, is presented for use in lamina emergent mechanisms. Important considerations such as linkage configuration and simple prototyping are also discussed.

3.2 INTRODUCTION

Using Lamina Emergent Mechanisms (LEMs) is one way to simplify both fabrication and assembly in a competitive design environment. LEMs are compliant mechanisms that can be fabricated from planar materials (e.g. sheet goods), with motion out of the fabrication plane. They potentially employ less expensive manufacturing techniques, use low cost sheet materials, require little or no assembly, and are compact in the initial state. Figure 3.1 shows an example of a LEM. In addition to exhibiting manufacturing simplicity, LEMs also have the potential to be used in applications requiring complex motions.

One challenge in achieving complex motions, however, is creating joints out of planar layers of material. Jacobsen et al. [1] have presented methods for tailoring flexibility in layered materials, and this work builds on principles discussed there.

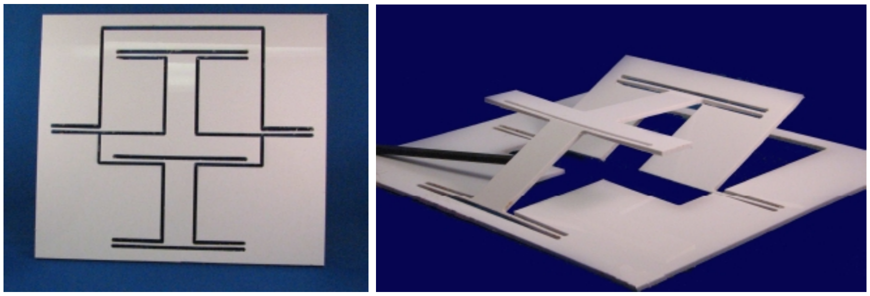

Figure 3.1: A lamina emergent mechanism, shown in both the planar (left) and non-planar states.

The objective of this chapter is to facilitate the design of LEMs by developing techniques and defining fundamental principles for designing complex joints from planar layers of material.

3.3 BACKGROUND

The mechanism characteristics that define LEMs are shown in Fig. 3.2. LEMs utilize components and techniques found in the existing mechanism classes of compliant mechanisms and ortho-planar mechanisms. This section discusses these two topics in order to establish a basis for studying joints for LEMs. Related research in paper pop-up mechanisms and robotics is also discussed.

3.3.1 Compliant Mechanisms

Mechanisms which gain some or all of their motion through the deflection of flexible members are called compliant mechanisms. Compliant mechanisms offer many distinct advantages over traditional mechanisms, in that they can be lighter, cheaper, simpler, more precise, more reliable, and use fewer parts. Designers often use compliant mechanisms when the goal is simplified designs. Fully-compliant mechanisms have no rigid-link joints, consisting entirely of flexible members [15].

The pseudo-rigid-body model (PRBM) models flexible members in compliant mechanisms as rigid links connected by pin joints and torsional springs, as shown in Fig. 3.3. The PRBM is useful because it allows simplified models suitable for use with traditional kinematics analysis techniques [15].

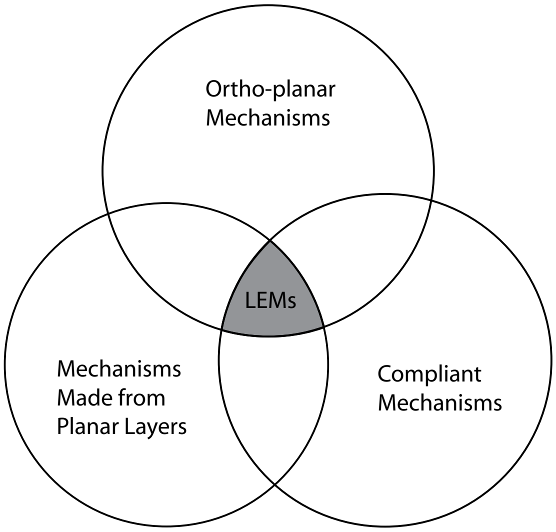

Figure 3.2: A Venn diagram showing the mechanism characteristics that define LEMs.

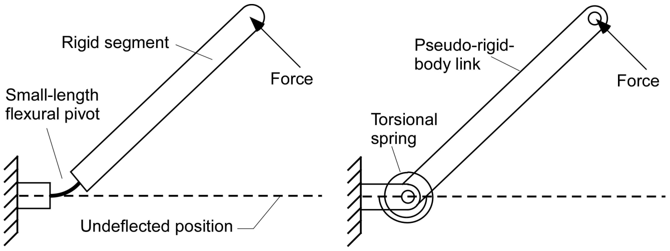

Figure 3.3: The pseudo-rigid-body model (PRBM) representation (right) for a beam with a short flexure.

3.3.2 Ortho-planar Mechanisms

Ortho-planar mechanisms are mechanisms for which all the links can be located in a plane, with mechanism motion out of that plane [25]. A main advantage of ortho-planar mechanisms is that they are compact in their planar state but can expand out-of-plane to perform a given function. Figure 3.4 shows an ortho-planar mechanism in its planar and non-planar states. Ortho-planar mechanisms can be compliant or rigid-link mechanisms. The focus of this chapter, however, is compliant ortho-planar mechanisms, for which the pseudo-rigid-body model is used.

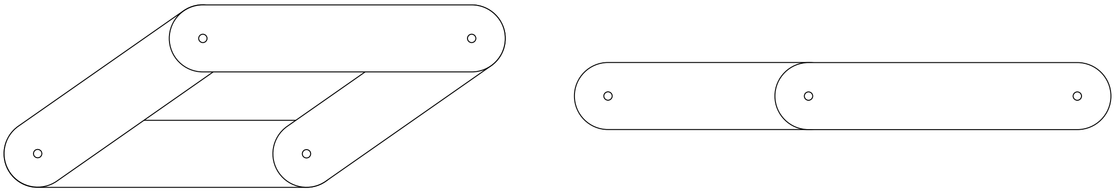

Figure 3.4: An example of an ortho-planar linkage. A parallelogram four-bar is shown in its out-of-plane (left) and in-plane states.

Methods have been proposed for creating flexible joints in the plane of a lamina so that an ortho-planar mechanism may be easily produced [1]. The flexibility of compliant segments in LEMs is governed by the geometry, boundary conditions and material properties. Large-deflection compliant joints for LEMs are created by modifying these properties for flexible segments.

3.3.3 Paper Crafts

Paper crafts such as origami and pop-up paper mechanisms can form a basis for thinking about lamina emergent mechanisms. Paper may be readily employed in LEMs because it is manufactured in sheets, and many pop-up and origami devices are in fact LEMs. The advantage of using paper to study aspects of LEMs is that it is easy to create joints in a paper lamina by creasing it.

Recent work shows how to model paper mechanisms using the PRBM [2]. Typical joints and fundamental mechanisms commonly used in paper pop-up mechanisms were outlined, and new joints based on kinematic principles were introduced. Of note is the idea that complex pop-up mechanisms are most often produced by combining simpler mechanisms in various ways. This chapter expands on concepts discussed in that work, principally the idea of creating multi-degree-of-freedom joints from flat sheets.

3.3.4 Open-chain Mechanisms

Spherical 3R open chain mechanisms are often used to create robotic spherical wrists [26, 27], because it can be easier to actuate three separate revolute joints than a single spherical joint. Duffy [28] showed ways to approximate spatial joints with serial joint chains, and McCarthy [29] captured this idea in a table presented on modeling spatial serial chains for robotics. The table lists examples of serial open chain linkages that use revolute and prismatic joints and can be constructed so as to approximate the motion of T, C and S joints, among others. This work serves as a key foundation for the serial chain complex joints presented in this chapter.

3.4 ELEMENTAL JOINTS

Designing LEMs can be challenging, especially with the scarcity of information currently available on the subject. Dividing LEMs into smaller elemental parts which can be easily modeled makes it easier to see how those parts fit and interact in a more complex embodiment. Thus, the presentation here discusses joints as basic elements of LEMs. These joints may be combined in different ways to create complex mechanisms.

Joints with up to three degrees of freedom may be used for planar and spherical mechanisms, but pure rotational or translational joints are often used because in the cases where multi-degree-of-freedom joints might be used, equivalent mechanisms may be created by combining single-degree-of-freedom joints. Thus, the revolute joint and the prismatic joint are the fundamental components for planar or spherical mechanisms. This section discusses those two joint types as they can be used for LEMs.

For simplicity, the joints are represented as they would appear with creased paper, where the creases represent the joint axes. Ways to make compliant joints from other sheet materials are addressed in [1].

3.4.1 Revolute (R) Joint

The revolute joint provides a single rotational degree of freedom between connected links in a linkage. Figure 3.5 shows an example (links A and B are two adjacent links). For LEMs, it is relatively straightforward to create a revolute joint within the plane of a material or between layers. In these cases, the joints are either contained in the plane of fabrication or perpendicular to it, as shown in Fig. 3.6; multiple sheets are needed in the latter case (shown at the far right in Fig. 3.6).

It is a much harder task to create a revolute joint with its axis out of the plane of fabrication, but not perpendicular to it. In cases where this is necessary, the planar layer

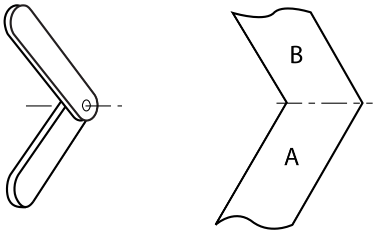

Figure 3.5: A revolute joint.

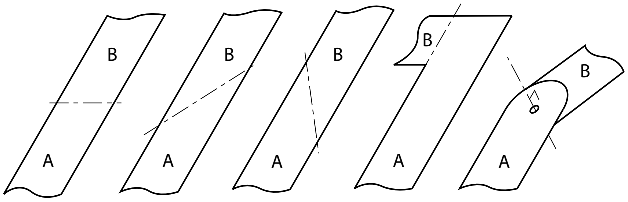

Figure 3.6: Some possible axis directions for a revolute joint between two links. For LEMs, it is easiest to create revolute joints with their axes contained in or perpendicular to the plane of fabrication.

may be bent as part of the manufacturing or assembly process so the joint axis may lie in a different orientation with respect to the original plane of fabrication, as shown in Fig. 3.7. It is important that these bends be constrained either by geometrical constraints or by their own stiffness, so that the joints stay in the proper orientations. This allows joints fabricated in the original layer of material to have any angle relative to the original fabrication plane (not just normal as a pin joint between layers would be). An example of this principle implemented in a linkage is shown in Section 3.6.3 (Fig. 3.18).

3.4.2 Prismatic (P) Joint

The prismatic joint, or sliding joint, provides a single translational degree of freedom between connected links in a mechanism. Lamina emergent mechanisms are well-suited to creating prismatic joints, given the nature of the raw material. For LEMs, pure translational motion from a single joint can be easier with an assembly process. Figure 3.8 shows an example of a prismatic joint.

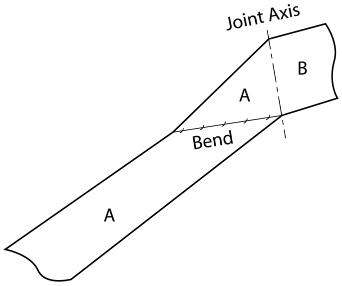

Figure 3.7: One possible axis direction created by bending the lamina.

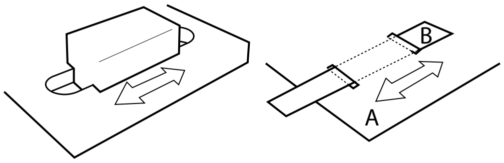

Figure 3.8: An example of a prismatic joint.

3.5 SERIAL CHAINS OF ELEMENTAL JOINTS

While planar and spherical mechanisms use primarily the single degree-of-freedom joints discussed in the previous section, spatial mechanism topologies have a much wider range of joints available. Technically, a joint with up to six degrees of freedom may be used in a spatial mechanism, though the usefulness of such a joint in a mechanism is questionable (not to mention difficult to construct) for a joint that is not actuated. Joints of more than three degrees of freedom are seldom used in practice.

One major challenge associated with using spatial mechanisms for LEMs is that it is difficult to make joints of more than one degree of freedom from planar layers of material. However, as mentioned previously, it is possible to create combinations of single-degree-of-freedom joints to mimic the motion of more complex joints. Table 3.1 (adapted from [29]) shows some joint types and the way that similar motions can be created with

Table 3.1: Serial joint chain approximations of spatial joints for LEMs (adapted from [29]).

		Degrees of Freedom		
Joint Name	**Serial Chain Approximation***	**Trans.**	**Rot.**	**Total**
Revolute (R)	R	0	1	1
Prismatic (P)	P	1	0	1
Universal (T)	RR (Perpendicular axes)	0	2	2
–	PP	2	0	2
Cylindric (C)	PR (Parallel axes)	1	1	2
Half	PR (Perpendicular axes)	1	1	2
Spherical (S)	RRR (Perpendicular axes)	0	3	3
–	PPP	3	0	3
–	PRR	1	2	3
Planar (E)	PPR	2	1	3
–	PPRR	2	2	4
–	PRRR	1	3	4
–	PPPR	3	1	4
–	PPRRR	2	3	5
–	PPPRR	3	2	5

*Note: Here we assume that all joint permutations are equivalent (e.g. RPP = PRP = PPR) for simplicity, though separate permutations will have different motion characteristics, especially for chains with higher degrees of freedom.

joints from planar layers of material. Some of the most common joints in the table are named, and those joints are discussed in this section.

Decomposing these high-order joints into simpler parts when moving to a LEM topology effectively adds links to the mechanism (however small they may be) while removing higher order kinematic pairs. Thus, the total mobility of the mechanism remains the same.

The concept of serial chain joint approximations becomes extremely useful when designing LEMs because it allows the traditional bulky versions of joints to be replaced with joints that can be created from planar materials. This section discusses ways to create cylindric, half, spherical, universal and planar joints from planar layers of material, by combining the elemental joints in serial chains.

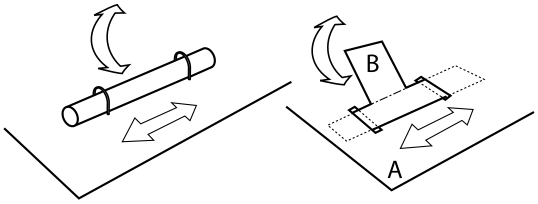

Figure 3.9: A cylindric joint serial chain approximation.

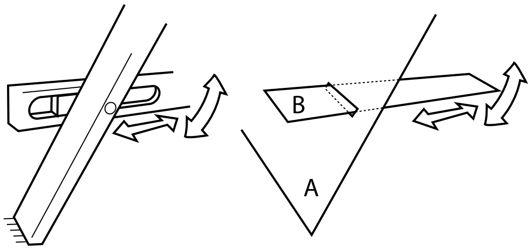

Figure 3.10: A serial chain representation of a half joint (left) and its representation as a LEM.

3.5.1 Cylindric (C), Half and Spherical (S) Joints

Cylindric, half and spherical joints were presented in a previous paper [2], so they are only briefly mentioned here for completeness.

The cylindric joint has one rotational and one translational degree of freedom, with the axis of rotation and the direction of translation collinear [23, 30], so an approximation is made by combining a prismatic joint and a revolute joint in a serial chain. Figure 3.9 shows a cylindric joint.

The half joint is a higher pair with one rotational and one translational degree of freedom. It was presented previously as a "rotating slider [2]" because of the way it is used in pop-up paper mechanisms. Figure 3.10 shows a half joint.

The three rotational degrees of freedom of the spherical joint may be approximated by making a serial chain of orthogonal revolute joints with intersecting axes [23, 30]. Figure 3.11 shows a spherical joint.

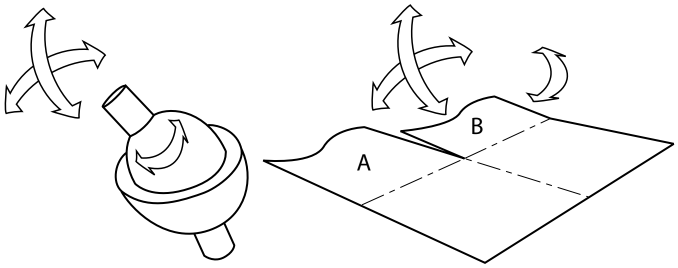

Figure 3.11: A spherical joint serial chain approximation.

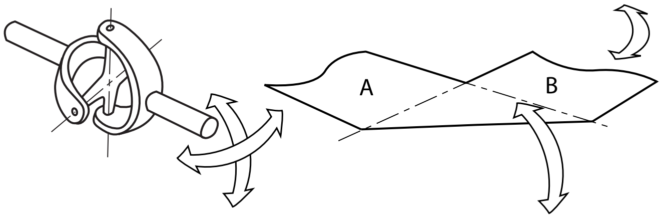

Figure 3.12: A serial chain universal joint in the in-plane position.

3.5.2 Universal (T) Joint

The universal joint has two rotational degrees of freedom with perpendicular joint axes. Like the spherical joint, the universal joint can be approximated by a serial chain of revolute joints with intersecting axes. The challenge with this joint is that neither joint axis is parallel to a link axis, so for the planar layer representation, one of the joints is contained in, and one normal to the original plane of fabrication. Figure 3.12 shows a universal joint in the in-plane position, and Fig. 3.13 shows the out-of-plane position which aligns the joint axes with those of the traditional joint.

3.5.3 Planar (E) Joint

The planar joint (or E joint, for the German word for a plane, *Ebene* [31]) has two translational degrees of freedom and one rotational degree of freedom. The directions of translation are normal to each other, and the axis of rotation is orthogonal to both. Thus, this joint is easily decomposed into a serial chain of two prismatic joints and a revolute

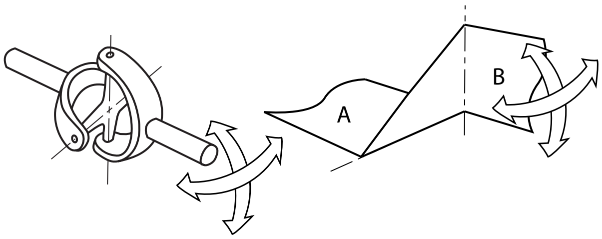

Figure 3.13: A serial chain universal joint in the out-of-plane position.

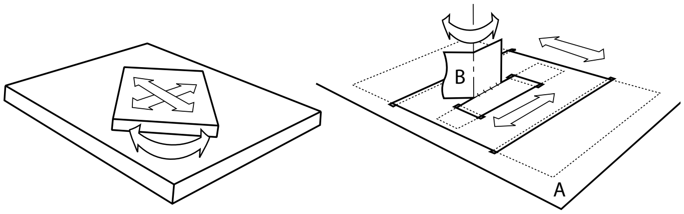

Figure 3.14: A planar joint constructed with a serial chain of elemental joints.

joint, as shown in Fig. 3.14. However, in this case the decomposition is not necessary, as it is also simple to create the original joint with planar layers of material, as Fig. 3.15 shows.

Duffy [28] presented ways for creating an equivalent planar joint without using PPR chains. Among them are the RRP chain and the RRR chain. The RRR chain approximation of a planar joint is shown in Fig. 3.16. It is useful for LEMs because it does not require any prismatic joints and all of its joint axes are parallel.

3.5.4 Some Notes on Spatial Joints

One important characteristic of many of the joints that have been shown is that they have positions with all of their links in a single plane, but they still retain their degrees of freedom. Not all of the serial chain joint approximations have an in-plane position (consider, for example, a PPP chain with mutually orthogonal axes), so those joints that do not have an in-plane position may not be as well-suited for use in LEMs as the others.

While it is useful to have these special joints available for use in LEMs, it is recognized that they are approximations and may not function as intended in all situations. The designer will have to judge whether their use is appropriate based on the given application.

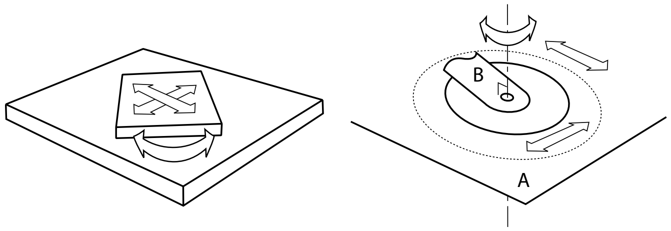

Figure 3.15: A planar joint, with a simpler topology.

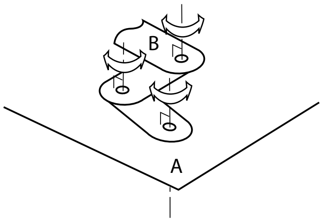

Figure 3.16: A planar joint constructed with a serial chain of three revolute joints.

It is also important to note that while the serial chain joint approximations provide designers with a wide range of possibilities, it may be that not all of the possible joints can be created with planar layers of material. For example, as of this writing, there has not been discovered a way to couple translation and rotation in such a way as to make a helical (H) joint from laminae. Luckily, this is the only lower pair for which that is the case, and approaches for the most common joints–R, P, C, T and S–have been identified (the lower pairs are the R, P, C, H, S and E joints [32]).

3.6 DESIGNING MECHANISMS

Combining serial joint chains with links allows the creation of complex lamina emergent mechanisms. This section discusses some basic considerations for combining joints into mechanisms and gives three examples of spatial linkages that can be made into LEMs.

3.6.1 Links: Connecting the Joints

The configuration of the links in a mechanism is important in determining mechanism characteristics. Often in mechanism design, the assumption of rigid links between joints is made. This is necessary because flexible links can allow the joints to misalign, often turning the mechanism into a structure. However, with compliant mechanisms, flexible links are often used for energy storage, given that the mechanism can be designed in such a way as to keep the joints in the correct alignment.

Many ways to create link flexibility are shown in [1] and [15]. For a mechanical linkage, the boundary conditions are set by the joint positions, so material and geometry changes are the main factors in creating link flexibility.

3.6.2 Multi-layered LEM Configurations

Sometimes a mechanism's joint configuration makes it necessary to have overlapping links during some part of the motion. Lamina emergent mechanisms can be designed with multiple layers to achieve this motion.

Multi-layered configurations are useful in LEMs because they allow certain mechanism characteristics not otherwise possible. For example, the joint motion in some mechanisms will be limited if the mechanism is made from a single continuous lamina, thus limiting the range of motion of the mechanism. Such is the case for the Goldberg 6R linkage discussed in Sections 3.6.3 and 3.7.3.

Another example of when it may be useful to use multiple layers is for mechanism configurations which are not ortho-planar. Many of these require metamorphic topologies as well. A metamorphic mechanism is one in which the mechanism mobility changes as the links move from one configuration to another [33]. Refer to [22] for a process used to create metamorphic topologies.

3.6.3 Examples

Thus far, the discussion has centered on designing joints for LEMs, so it is useful to give some examples of linkages with complex motion that can be created with LEMs. Here we present the Bricard 6R, Altmann 6R and Goldberg 6R spatial linkages to showcase LEM joint possibilities.

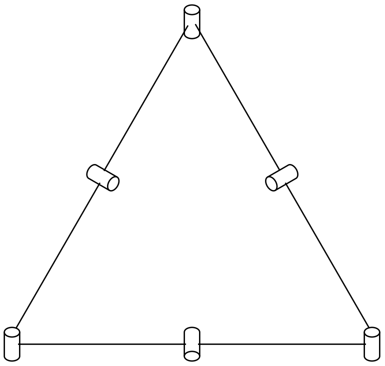

Figure 3.17: The Bricard 6R linkage.

The Bricard 6R Linkage

A Bricard 6R linkage [34] is shown in Fig. 3.17. It has been extensively studied (e.g. in [35–37]), so it is not explained in detail here. This linkage has special characteristics that make it useful for a lamina emergent mechanism implementation, and it will be shown in different forms in the next section.

The trihedral Bricard 6R linkage shown in Fig. 3.18 (often called a kaleidocycle in this form [38]) is a good illustration of creating different joint axis directions by bending the lamina, as outlined in Section 3.4.1.

The Altmann RTRT Linkage

An Altmann RTRT linkage [39] is shown in Fig. 3.19. Baker [40] and Phillips [32], among others, have studied this linkage. If the T joints are represented as a serial chain of R joints, the mechanism can be viewed as a special case of the Bricard 6R linkage [40]. Thus, it serves well to illustrate the idea of combining chains of basic elements in order to create approximations of more complex joints. As noted by Phillips [32], all of the links of this linkage can become coplanar, and thus it is also well-suited for implementation as a LEM.

A paper representation of Altmann's linkage is shown in Fig. 3.20.

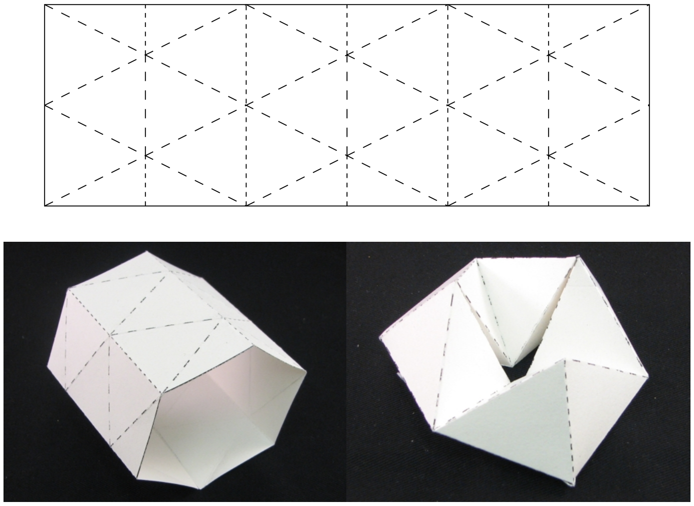

Figure 3.18: An orthogonal Bricard 6R linkage made with paper. The paper cutout, an intermediate assembly step and the final linkage are shown (counterclockwise, from top). Note that the intermediate step can collapse to an in-plane configuration, and that the metamorphic process in moving to the final step consists of a sequence of folds.

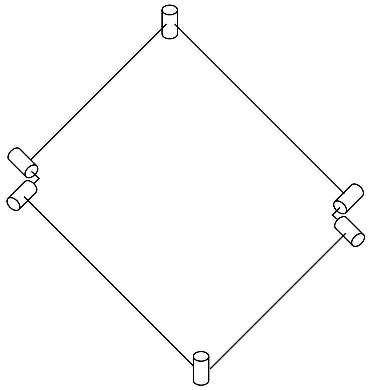

Figure 3.19: Altmann's linkage.

The Goldberg 6R Linkage

Another linkage with useful characteristics for a LEM implementation is the Goldberg 6R linkage [41]. Fig. 3.21 shows a Goldberg 6R linkage.

3.7 PRACTICAL ISSUES IN CREATING JOINTS AND LINKAGES

Previous sections discussed making approximations of complex joints out of planar layers. This section discusses some of the practical issues encountered when designing

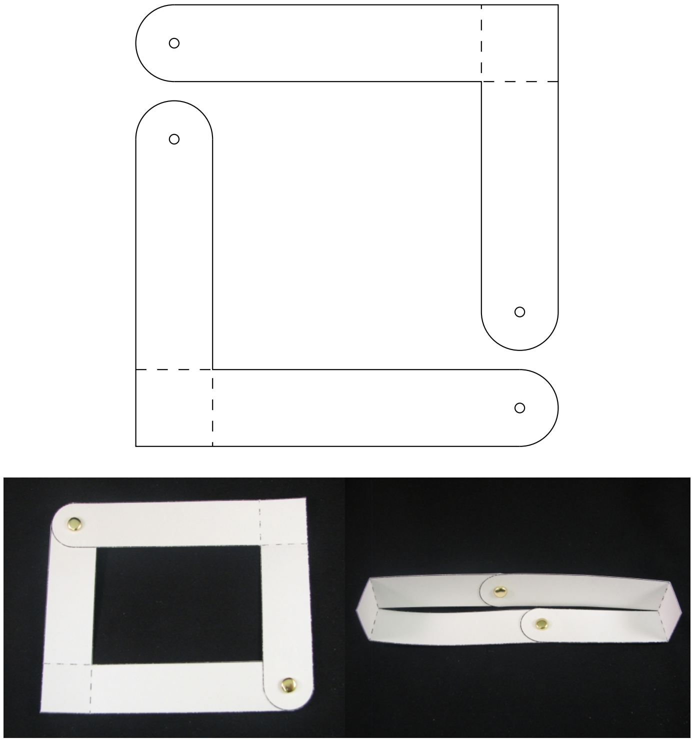

Figure 3.20: The paper cutout (top) and an assembled Altmann linkage made with paper. The two sets of creased joints are serial chain approximations of universal joints.

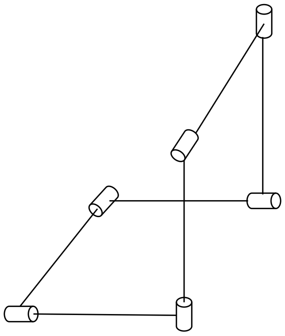

Figure 3.21: The Goldberg 6R linkage.

joints to take advantage of the principles discussed previously. Specifically, this section presents a discussion on linkage considerations, quick prototyping approaches, creating practical designs and some limitations imposed by lamina emergent mechanisms.

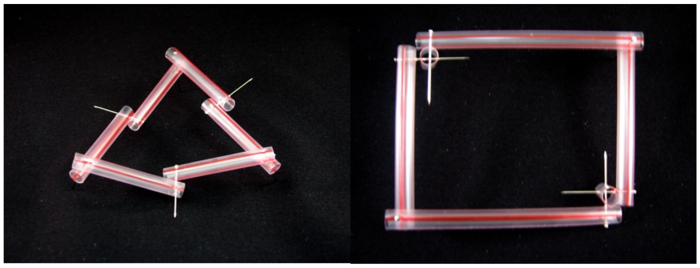

Figure 3.22: Using drinking straws and push pins is a convenient way to prototype many different types of linkages. The examples shown are a trihedral Bricard 6R linkage (left) and an Altmann linkage.

3.7.1 Prototyping

It is advantageous to have a fast method for prototyping linkage configurations. One good way to do this is to create an equivalent linkage from paper, using either creases or metal fasteners for the joints, depending on joint orientation. This is an especially good medium for the rapid creation of prototypes for lamina emergent mechanisms. Another good way to create quick prototypes is with push pins and hollow tubes (such as drinking straws), as shown in Fig. 3.22.

One advantage of these quick prototyping methods is that they allow the designer to evaluate joint alignment. Linkages with revolute joint axes that are parallel, intersecting, or offset from each other by 90° can usually be made into LEMs because the joint axes can be contained in or perpendicular to the lamina. The special case Bricard and Goldberg 6R linkages shown in this chapter are used because their axes are all offset by 90°.

Not only do these simple prototypes allow the designer to quickly visualize joint alignment, they also allow visualization of useful linkage characteristics. For example, in creating the Bricard 6R linkage shown in Fig. 3.23, it was noted that the positions where all of the links are in-plane alternate between a flat and a raised configuration. While the prototype in Fig. 3.22 shows some important linkage characteristics, the high aspect ratio of the LEM (paper) links in Fig. 3.23 accentuates this particular characteristic, which could be useful for volume-critical applications.

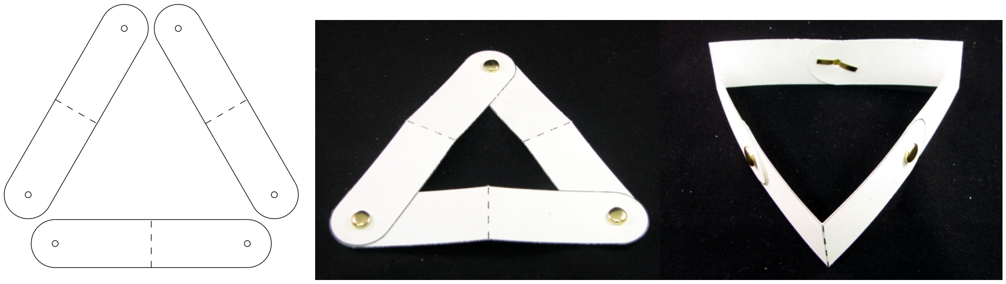

Figure 3.23: A convenient way to prototype LEMs. This is a Bricard 6R linkage, with the paper cutouts (top) and two of its positions shown.

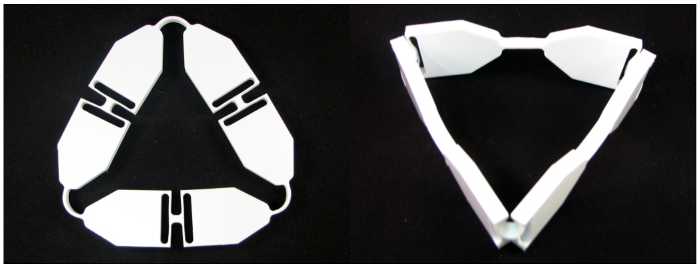

Figure 3.24: A fully-compliant device made from a thin layer of polypropylene, with motion similar to a Bricard 6R linkage. The left position is the manufactured position.

3.7.2 From Paper to Other Materials

Paper is a convenient medium for prototyping linkages, allows quick proofing of joint and link locations, and it can be used for some LEMs applications. Some related materials, such as cardboard and cardstock, have similar properties and show potential for many high volume applications, particularly in advanced packaging [42]. However, there are many other possible LEMs applications that will require the use of other materials.

Sheet metal and silicon (for microelectromechanical systems) are examples of materials in which LEMs could be fabricated using layers. Creasing such materials is not feasible for large deflection joints, thus, the low stiffness joints in other materials, represented by creases in paper prototypes, must be achieved in other ways. A number of different types of flexures exist for just that purpose [43–45] and it is anticipated that more will be developed as research in the LEMs area continues. Often, joint creation is a matter of choosing a design and sizing the joint geometry for the correct force-deflection characteristics. Figure 3.24 shows a LEM with motion similar to a Bricard 6R linkage (such as shown in Fig. 3.23), but here it is made from a thin polypropylene sheet. The creases in the

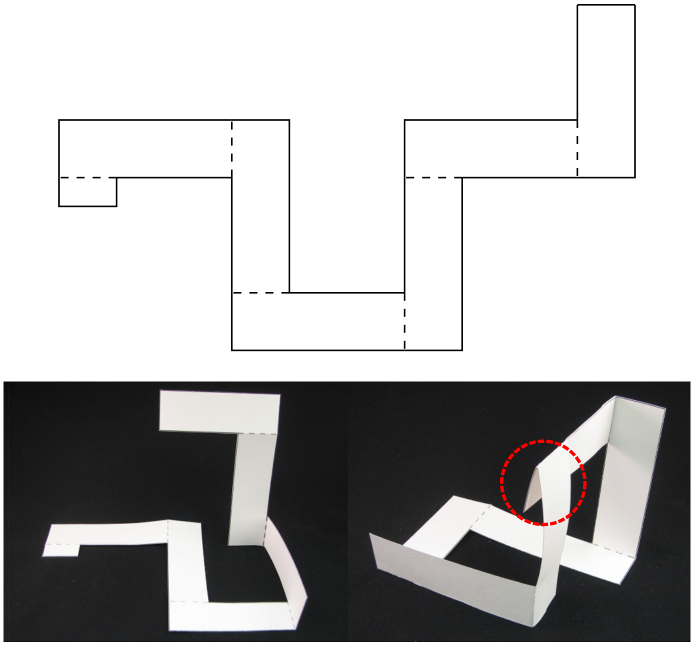

Figure 3.25: A Goldberg 6R linkage made as a LEM with a single layer. The initial cutout, an assembly step and the final linkage are shown (counterclockwise, from top). The linkage has a joint (circled, at right) which limits the overall motion of the linkage.

paper linkage in Fig. 3.23 have been replaced by compliant joints, called Lamina Emergent Torsion (LET) joints [46], that use torsion to achieve large deflection.

3.7.3 Some Limitations

In many cases where revolute joints are used in LEMs, the relative motion of links is limited to ±180°. However, if the required motion is less than 360°, it is sometimes possible to design the linkage so that the required motion falls within the range of movement and the joint is not self-limiting. The Goldberg 6R linkage of Fig. 3.25 is an example of this. While none of the joints in the linkage are required to move more than 360°, the linkage in Fig. 3.25 has two joints that are oriented so that their motion is limited to between 90° and 180°. Because the linkage requires the joints to move more than 90°, its motion is severely limited. The linkage in Fig. 3.26 fixes the problem so that all of the joints can move ±90°, so the full range of linkage motion is possible.

This represents a trade off between simplicity and functionality. The linkage with the full range of motion requires multiple layers of material, while the linkage with limited

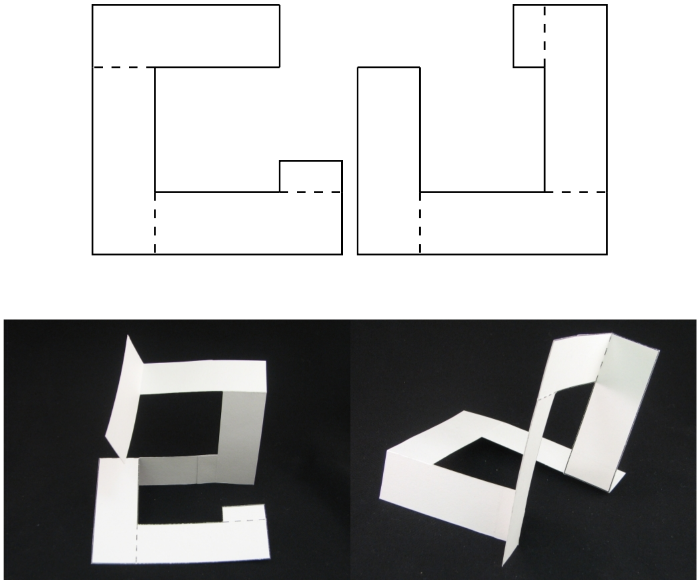

Figure 3.26: A Goldberg 6R designed to eliminate the limiting joint found in the previous figure. The initial cutouts, an assembly step and the final linkage are shown (counterclockwise, from top). The final configuration requires multiple layers, but does not have any self-limiting joints.

motion is simpler to make because it can be made with a single layer. In situations such as this, the designer is to weigh the options and choose the most suitable device for the application.

3.8 CONCLUSIONS

The use of lamina emergent mechanisms in design can yield significant advantages. However, it can be difficult to implement certain mechanism topologies because of the difficulty of creating complex joints from planar layers of material.

This chapter has shown a way to simplify the creation of complex joints. Utilizing serial chain approximations of complex joints can allow designers to have more tools at their disposal. This, along with the practical design considerations discussed in the chapter, allows easier application of lamina emergent designs in areas where they can be of use.

Chapter 4

Using LEM Joints to Create Bistable Compliant Spatial Linkages

4.1 INTRODUCTION

Chapter 3 showed ways to create linkages with complex spatial motion using serial joint chains. This allows complex motion to be accomplished with relatively simple elements. However, this still does not make it possible to turn every type of rigid-link spatial mechanism into a LEM topology.

This chapter outlines a design principle and an example linkage that were discovered while exploring the possibilities of LEMs. The principle of angular offsets in joint axes allows the creation of stable equilibrium points in compliant mechanisms, and the example linkage employs that principle.

4.2 BACKGROUND

The linkage presented in this chapter is a bistable compliant linkage that is derived by offsetting the Bennett mechanism joints. Previous chapters have discussed compliant mechanisms and the pseudo-rigid-body model (PRBM); this section outlines multi-stable mechanisms and the Bennett mechanism as the background for the rest of the chapter.

4.2.1 Multi-stable Mechanisms

Studies have shown that in traditional compliant mechanisms, a number of ways exist to create multi-stability [47,48]. Because lamina emergent mechanisms are compliant mechanisms, they are amenable to creating multi-stable mechanisms as well.

Multi-stable mechanisms can be useful because they do not require energy input to stay in their equilibrium states. They use energy only to move between stable positions,

Figure 4.1: A bistable compliant hair clip.

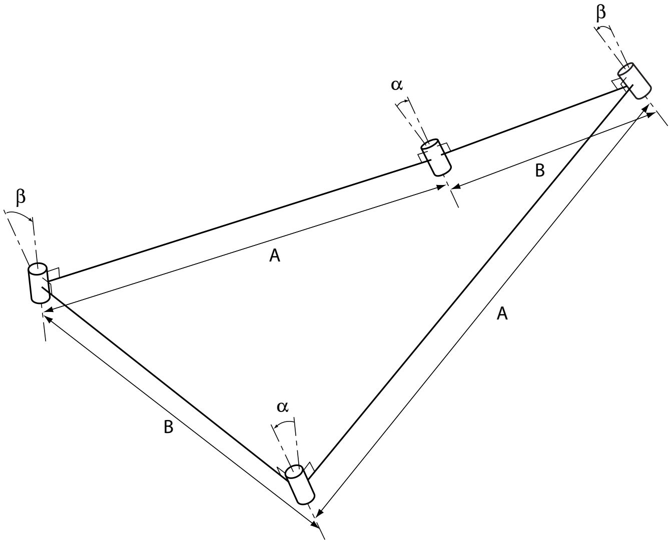

Figure 4.2: The Bennett linkage topology. The specific orientations of the joint axes allow the rigid-link mechanism to move.

so a multi-stable mechanism has the potential to use less energy to maintain a position than mechanisms which don't employ multi-stability. Figure 4.1 shows an example of a multi-stable mechanism.

4.2.2 The Bennett Mechanism

The Bennett mechanism [49] is a mobile 4R mechanism without parallel or concurrent joint axes (i.e. it is not a planar or spherical mechanism). It is a well-studied overconstrained spatial mechanism that requires specific geometry in order to be mobile [50]. The topology for a Bennett mechanism is shown in Fig. 4.2, and a model made with drinking straws and push pins is shown in Fig. 4.3.

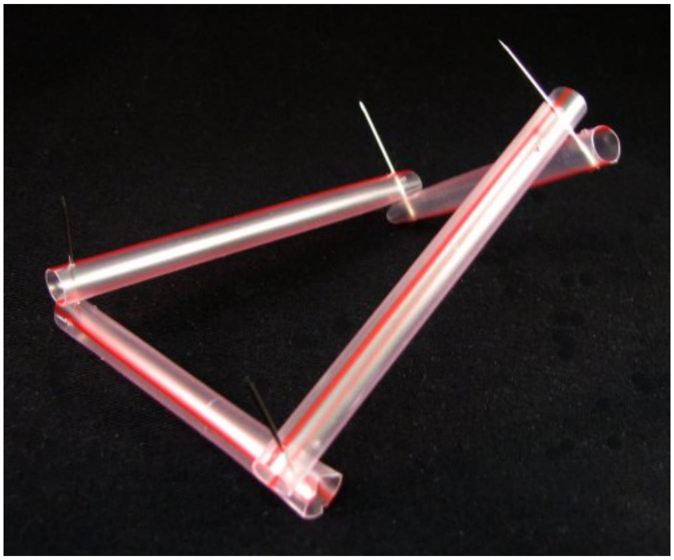

Figure 4.3: A Bennett linkage created with drinking straws and push pins.

For a Bennett mechanism, adjacent joint axes have common normals, so the orientation of a joint axis is determined by a "twist" from the previous joint axis about the common normal (the link axis).

4.3 CREATING MULTI-STABILITY BY ANGULAR OFFSET OF JOINT AXES

In a traditional mechanism sense, introducing angular offset of joints in a linkage is not usually useful because it can easily turn a mechanism into a structure–especially for spatial overconstrained mechanisms like the Bennett mechanism where it is special geometry that allows them to move in the first place. However, compliant mechanisms with offset joint axes can retain freedom of movement because of their flexibility.

Angular joint offset can produce special benefits in compliant mechanisms: depending on the type and amount of offset, it can produce stability in certain mechanism positions. When two adjacent links become coplanar, the axis of a revolute joint connecting those two links can be in any orientation within the plane of the links, or perpendicular to that plane, as in Fig. 4.4. As long as the two links maintain their orientation, the revolute joint is unstressed (the joint is aligned). But when the two links move relative to each other, if the mechanism motion requires the two links to move about an axis other than the joint axis between those two links, a rigid-link mechanism will not move because of the angular

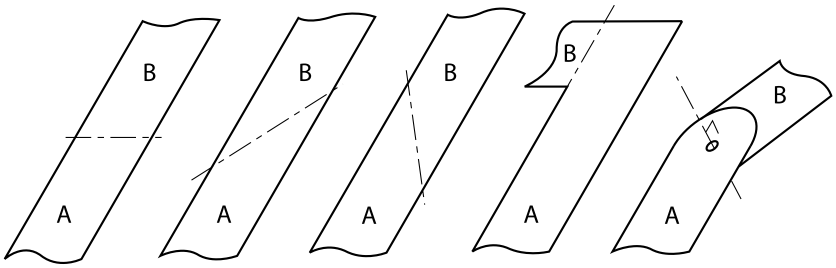

Figure 4.4: Some possible axis directions for revolute joints between coplanar links.

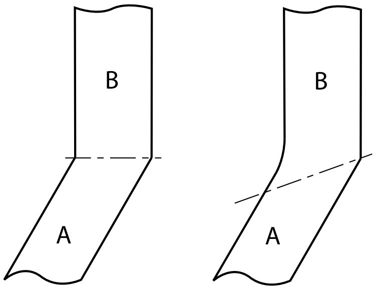

Figure 4.5: A joint axis between two links (left) and a joint axis offset by an angle, which requires a link to bend in order for the relative motion between the two links to remain the same.

offset of the joint axis (i.e. the mechanism has become a structure). However, if the joint or the links are flexible, the mechanism will move, storing energy in the process, and the mechanism will tend to move back toward the unstressed state if left alone. Thus, the position where the two links are coplanar is a stable equilibrium position. Figure 4.5 shows an example of links that are deformed as a result of an angular offset of the joint axis between them.

By the same reasoning, spatial compliant mechanisms with adjacent links that are coplanar at any point in their motion have the potential to become stable in those positions if the joint axes are offset by an angle from their rigid-link mechanism orientation. Mechanisms with more than one coplanar link position have the potential to be made multi-stable.

Note that planar mechanisms have coplanar links by definition, so if the joints are offset by arbitrary angles, a planar compliant mechanism will usually turn into a spatial compliant mechanism, except in the special case where the joint axes intersect at a point.

4.3.1 Creating LEMs from Mechanisms with Offset Joint Axes

It was noted in Chapter 3 that mechanisms with joints offset from one another by 90° make good candidates for becoming lamina emergent mechanisms. Mechanisms with joint axes offset as described in this chapter also can be good LEM candidates because the joints are contained in the plane of the links in at least one position.

This principle sometimes allows a rigid-link mechanism that would otherwise not make a good LEM candidate to become one by offsetting the joint axes, because the angular offset can allow the joints to lie in the plane of fabrication. As outlined previously, the offset often turns the rigid-link mechanism into a structure, but it gives the compliant-link mechanism at least one stable equilibrium point.

4.4 A LEM-BASED BISTABLE SPATIAL LINKAGE

This section outlines a new linkage that was created by offsetting the joint axes for a rigid-link Bennett linkage. The joint axes are offset in such a way that two of them lie in the plane of the links, and two of them are perpendicular to that plane. This allows the linkage to be created with planar layers of material. It seems that even if this configuration had been considered by others, it would not have been considered useful because the angular joint offset produces a structure in both of its in-plane positions. Allowing link compliance is the key to making this configuration work.

4.4.1 Offsetting the Bennett Linkage Joint Axes

The Bennett linkage is usually not a good candidate for a LEM, because of limitations on its configuration. It can have configurations where the link axes all are coplanar [37], but in these configurations, the joint axes are neither contained in nor perpendicular to the plane containing the links, so creating the mechanism with planar layers of material is difficult. However, offsetting some of the joint axes from their Bennett linkage orientations, as shown in Fig. 4.6, allows two of the joints to be contained in the plane of the links and the other two to be perpendicular to it, as shown in Figs. 4.7 and 4.9. This creates a new linkage which is no longer a Bennett linkage, but is a good LEM candidate, because

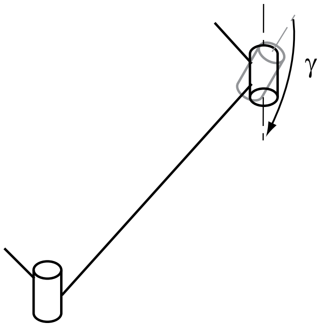

Figure 4.6: One way to create an angular offset of the joints in a Bennett linkage. Two joints in the linkage are shown. The first joint is not offset from its Bennett linkage orientation, but the second joint (gray) is offset by an angle γ to create the new joint (black).

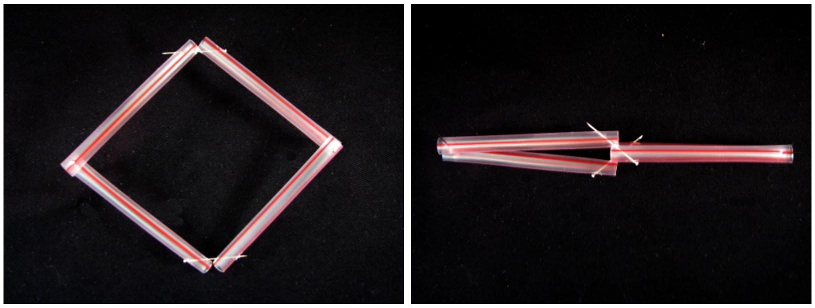

Figure 4.7: An offset Bennett linkage in its two stable positions. The position at left shows coplanar links, and the right position shows collinear links.

the links must be compliant for the linkage to move, and it can be made with joints perpendicular to the plane of the links with the other joints in that plane. A LEM configuration is shown in Fig. 4.8.

Offsetting the Bennett linkage joint axes in this way creates a structure, but compliance in the links allows the linkage to move, and gives it stability in the two positions where all of the links become coplanar.

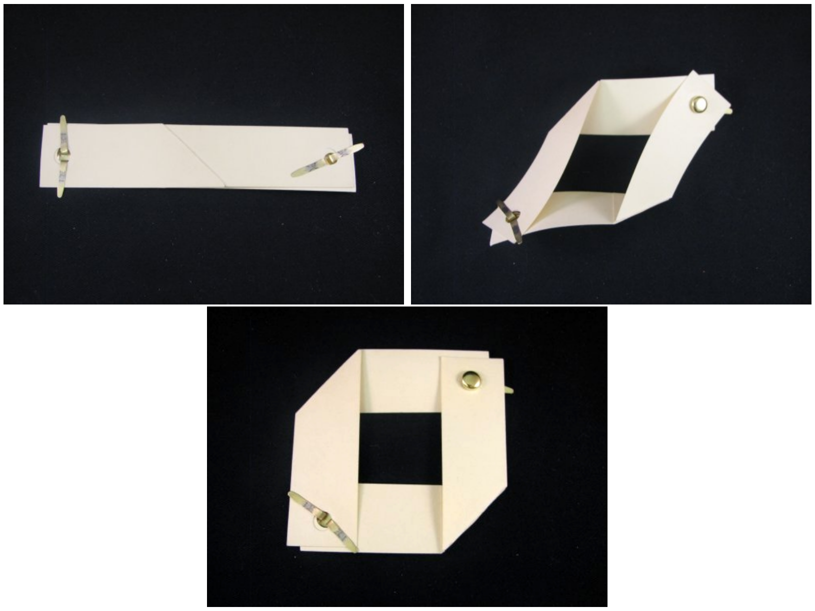

Figure 4.8: An offset Bennett linkage made from planar layers of material. The collinear stable position, a non-equilibrium position and the coplanar stable position are shown (clockwise, from top left).

4.4.2 Linkage Models

Figure 4.9 shows the topology of the new linkage created by offsetting the Bennett linkage joint axes. It has one position where all of the links are collinear and one where they are all coplanar.

As shown in Fig. 4.9, in both the coplanar position and the collinear position, joints 2 and 4 are contained in the plane containing the links, and joints 1 and 3 are perpendicular to it. The in-plane joint axes are offset at the same angle but in opposite directions when all of the links are collinear.

The linkage shown in Fig. 4.9 is a structure in both of its positions. In order for it to move, two of the links must flex. Thus, it is useful to represent the linkage with the PRBM, much like the paper linkages shown in Chapter 2. Figure 4.10 shows a three-dimensional conceptual representation of the PRBM, where the creases and curved links of the paper model are replaced by pin joints and torsional springs. The PRBM has six joints altogether. The two additional joints (5 and 6) that allow the links to bend must be stiffer than the other two flexible joints (2 and 4) for the linkage to be bistable.

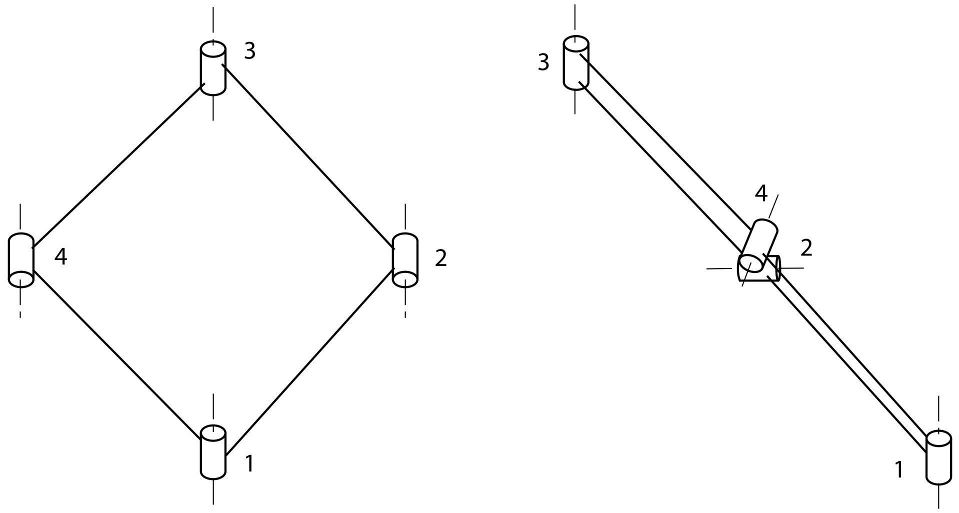

Figure 4.9: Rigid-link models of the offset Bennett linkage where the linkage is a structure. These are the stable positions in the compliant linkage. The left image is the coplanar link position and the right is the collinear position.

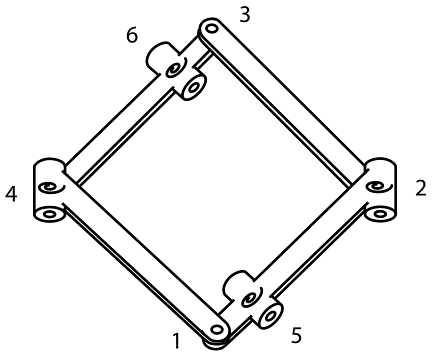

Figure 4.10: A conceptual PRBM for the offset Bennett linkage.

4.4.3 Linkage Properties

Because the linkage has one position where the links are collinear, the sum of the lengths of two of the links has to equal the sum of the other two. In addition, the linkage can either be a deltoid or a parallelogram, as shown in Figs. 4.11 and 4.12. The linkage must have one or both of these configurations in order for the links to reach both coplanar positions. The configurations in Figs. 4.8 and 4.13 meet the criteria for both deltoid and parallelogram mechanisms because their links are all the same length.

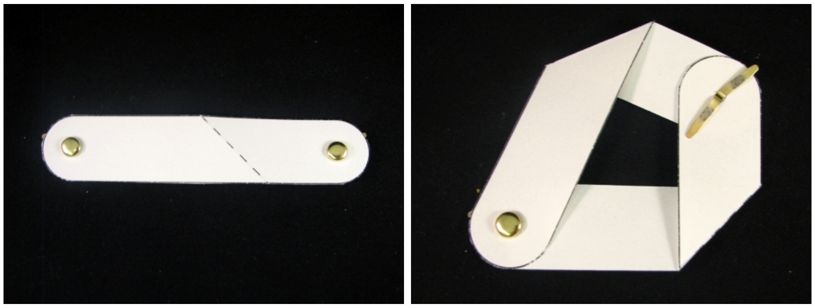

Figure 4.11: The deltoid configuration.

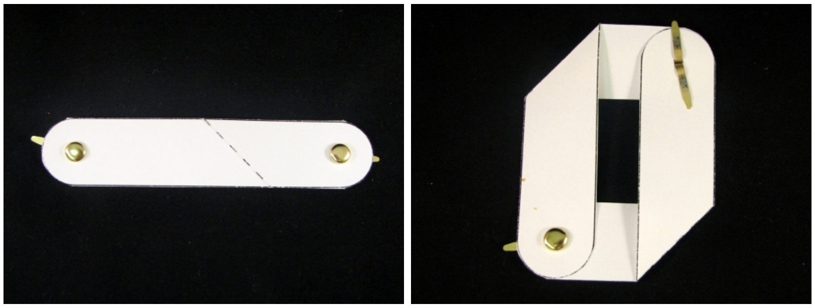

Figure 4.12: The parallelogram configuration.

The angles of the offset joint axes have to be equal for the linkage to reach a second coplanar link position. Also, the angle of the misaligned joint axes determines the "spread" of the linkage when the links are coplanar but not collinear. This can be seen by comparing Figs. 4.8 (bottom) and 4.13 (right). The two linkages have the same link lengths, but one opens wider than the other when the links are coplanar.

Perhaps the most interesting part of this linkage is that it is an ortho-planar bistable compliant spatial linkage. Ortho-planar spatial linkages are somewhat rare, as are ortho-planar bistable linkages, and this linkage is both spatial and bistable.

4.5 CONCLUSION

This chapter discusses the creation of multi-stability in mechanisms by using an angular offset of joint axes and gives an example of a linkage that implements it. The principle of angular joint offset creates opportunities for LEM-based mechanisms to be multi-stable. The approach presented in this chapter could be applied not only to the Bennett mechanism, but also to a number of different mechanisms, especially those with links which become

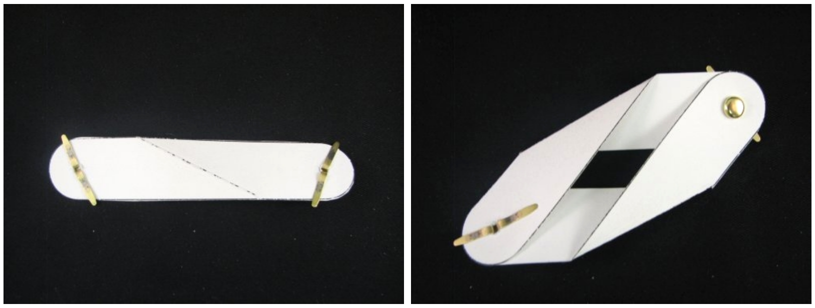

Figure 4.13: A configuration with a larger angle of offset that makes the links “spread” less. Compare the right image with the bottom image of Fig. 4.8

coplanar in one or more positions. Application of this approach allows multi-stability to be incorporated more easily in LEMs.

Chapter 5

A Preliminary Framework for Designing LEMs

5.1 INTRODUCTION

The ultimate goal of researching lamina emergent mechanisms is to facilitate their design. One way of aiding designers is to define a design method that takes advantage of basic LEM elements. The methods used to create the LEMs presented in this thesis are summarized here as one possible design technique. It is hoped that this will serve as a guideline for a future, more rigorously tested synthesis method.

The steps presented here should be viewed as basic guidelines, as the designer may need to modify the approach to suit a particular application. For example, some of the steps may need to be done out of order or in iterative loops.

It may be noted that the method here is not very different from a generic mechanism design method, except for a few specialized steps. This is important because it is often easiest to design LEMs as rigid-body mechanisms, then use various techniques to translate the mechanisms into their final LEM configurations. The design steps note important deviations from the traditional design process.

5.2 STEP 1: DEFINE NEEDS

This first step is crucial: ill-defined needs often make the synthesis process hard because there is not a set target. In this step, strength, weight, dynamic, energy use and spatial constraints should be defined if possible.

It is at this point that the decision is made as to whether it is advantageous to use LEMs or not. Some considerations for determining this include whether the mechanism needs to fold to an in-plane configuration, whether mechanism arrays are required and the importance of cost and ease of manufacturing.

Other considerations: Is it a mechanism or a structure in its final state? How many degrees of freedom in the initial and final states?

5.3 STEP 2: GENERATE TOPOLOGIES

The topologies in this step should be able to fulfill the requirements from step 1. However, it is not yet necessary to consider mechanisms in terms of their lamina-emergent topologies. It is generally better to find a mechanism with the right motion characteristics in this step in order to separate the process of implementing the LEM from the process of kinematic analysis. The more these steps are separated, the easier it is to move through the process. Here, energy storage elements can be modeled separately from mechanism links (it is a good idea to use the PRBM), and layered topologies do not need to be used. The specific implementation of the design in a lamina-emergent format will be defined in a later step. Note that while the resulting mechanisms in this step do not need to be compliant mechanisms made from planar layers, all the mechanisms resulting from this step should still be ortho-planar if the final configuration needs to be ortho-planar.

Since LEMs can incorporate any of the three mechanism classes (planar, spherical, spatial), many different designs will be possible for similar motions. For simplicity, the mechanism should have planar or spherical topology unless it requires a more complex three-dimensional movement not possible with planar or spherical mechanisms. Where spatial mechanisms are required, it is important to remember that not all of the spatial joints are possible in a LEM configuration, and that any approximated spatial joints may have some limitations depending on the configuration (see Sections 3.5.4 and 3.7.3 of Chapter 3).

5.4 STEP 3: SELECT MECHANISM CONFIGURATION AND JOINT ORDER

The purpose of this step is to narrow the field of possible mechanism candidates and begin the process of moving to a LEM topology. The most suitable configuration from the previous step should be selected so that it may be transformed. The link and joint configuration and order should be defined for translation in the next step. Joints and links should be kept to the simplest possible for easier translation.

It is advantageous here to have a fast method for prototyping mechanism configurations to determine suitability for translation. One good way to do this is with push pins and drinking straws, or with paper, as shown in Section 3.7.1 of Chapter 3.

5.5 STEP 4: TRANSLATE THE CONFIGURATION INTO A LEM

This step is where the LEM elements defined in this thesis are put to use to make a topology that has the required characteristics. Joint types and joint order should be defined, as well as the configuration of serial chain approximated joints. There are many permutations within mechanism topologies and configurations. Energy storage considerations are not as important in this step as defining the basic layout of the mechanism.

It is important to note that in many cases, using LEMs will limit the relative motion of links to $\pm 180^\circ$. However, if the required motion is less than 360°, it is often possible to design the mechanism so that the required motion falls within the range of movement and the joint is not self-limiting (see, for example, Section 3.7.3 of Chapter 3). In some cases, rearrangement of the joints may require creative arrangement of multiple layers.

In addition, metamorphic topologies should be defined in this step. For mechanism configurations which are not ortho-planar, but need to be LEMs, it is most likely necessary to use a metamorphic topology. In many cases, there will be a number of possible configurations; it may be possible to use different links to achieve the same change in the mechanism degrees of freedom. The designer is left to decide which configuration is most advantageous. Refer to [22] for a process used to create metamorphic topologies.

5.6 STEP 5: SIZE THE LINKS AND JOINTS

This is the final mechanism design step, where all of the considerations are finally taken into account. Important considerations are the material type and thickness, and link dimensions, which should be sized for the application. These parameters can be changed to match the required energy storage properties of the mechanism (this is most important where stable mechanism positions are required). Compliant joints need to be sized for the right amount of strength and stiffness.

It is also important to remember in this step that some links made from planar layers of material may not perform well under compression (i.e. they may buckle) and may need to be resized.

5.7 STEP 6: ANALYZE

The type and amount of analysis required in this step depends upon the application for which the mechanism is being designed. Important considerations may be degrees of freedom in the initial and final states, stiffness, stability, strength, forces on the links and joints, etc.

This step may need to be performed in iterative loops in conjunction with Step 5, depending on the results of the analysis.

5.8 STEP 7: BUILD

The build process is left to the designer. In many cases it is necessary to combine steps 5 through 8 into a cycle in order to make progressively improved prototypes leading up to a final design. Starting with simple prototypes and moving on to increasing complexity is one way to make this process easier. For example, with a mechanism array, it is advantageous to build a single mechanism and iterate on that, then combine all the mechanisms into an array.

5.9 STEP 8: TEST

The degree to which the final mechanism is tested depends on the desired application. The testing process may require fatigue testing, ultimate strength testing, user testing, etc.

5.10 CONCLUSION

This chapter presents a preliminary method for designing mechanisms that can be made from layered materials. It is anticipated that this will serve as a foundation for future work in methods for LEM design. Defining a design method simplifies the design process and gives the ability to use LEMs in a wider variety of applications.

Chapter 6

Conclusions and Recommendations

6.1 CONCLUSIONS

Mechanisms made from planar layers of material, or lamina emergent mechanisms (LEMs), are useful for specialized applications, such as those where space, weight and cost reduction are important. However, due to the relatively little information currently available for designing LEMs, it is difficult to use them readily in the applications where they can be of the most benefit.

This thesis simplifies the design of lamina emergent mechanisms by identifying fundamental components that can be combined to create mechanisms with complex motion. It demonstrates ways to combine these elements into mechanisms made from planar layers of material. Combining the principles presented in this thesis can facilitate the design of lamina emergent mechanisms by allowing designers to create LEMs with complex motion, thus expanding the number of applications in which LEMs may be employed.

Chapter 2 discusses an approach to modeling pop-up paper mechanisms and presents new joints and mechanisms that are useful for LEMs. The principle of combining simple mechanisms to achieve complex motion is also presented. This principle allows designers to accomplish a wide variety of tasks with various combinations of the same elemental components. Chapter 3 also uses combinations of simple elements to create mechanisms with complex motion. It is shown that complex joints can be created with serial chains of elemental joints. This is important because it allows LEMs the freedom to use complex joints and spatial motion, thus expanding the possible applications in which they may be used. Chapter 4 presents a method for introducing multi-stability in compliant spatial mechanisms, using the elemental components of LEMs discussed in the preceding chapters. This is advantageous because it allows energy storage mechanisms to be made with simple manufacturing techniques. The discussion in Chapter 5 introduces a preliminary method for designing LEMs. This combination of design steps is one way to employ the fundamental LEMs components of the previous chapters. The types of simple joints available for lamina emergent mechanisms can be combined to create complex mechanisms and motion.

6.2 RECOMMENDATIONS

Lamina emergent mechanisms show great potential for many applications, so researching LEMs in order to simplify the design process can make a great impact. The work presented in the main body of this thesis represents a small part of the work to be done in the fledgling area of LEMs research. This section gives recommendations for future work that would advance the state of the art.

6.2.1 Exploring Paper Mechanism Possibilities

The information presented in Chapter 2 only scratches the surface of possible topics for research in paper mechanisms. Additional work would build upon the principles discussed in that chapter. It would be useful, for example, to have a complete catalog of paper joints and mechanisms with their kinematic representations (similar to [4] or [20], but from a kinematics standpoint), so that these could be more readily employed in other research where space and weight savings are important. Moreover, there likely are many undiscovered possibilities for new pop-up mechanisms; applying kinematics and compliant mechanism principles could lead to their discovery and widespread use in pop-ups and LEMs.

There are many other principles which can be used in the creation of new pop-up mechanisms. For example, bistability could be employed in more pop-up designs. There are a number of pop-up mechanisms already in use that employ stability, but almost all of the known types are planar slider-crank mechanisms in various forms. It is possible to create bistability with other mechanisms, but difficult when the mechanism is required to fold flat. Further research would identify fundamental principles for creating flat-folding multi-stable mechanisms. The results would be useful to designers wishing to use the same principles for LEMs.

In addition, it may be useful to more fully explore ways to combine simple mechanisms to create complex topologies. Many of the topologies created when combining paper mechanisms in parallel or series are spatial mechanisms. Exploring these mechanisms could result in many new possibilities for spatial LEMs.

Machine Assembly of Paper Mechanisms

In general, paper mechanisms with multiple layers, folds and joints make for the most exciting user experience, however, these pop-up mechanisms are more costly because of the increased material and labor cost. While most pop-up mechanisms are printed, cut and scored by machine, many of these more exciting complex mechanisms must currently be assembled by hand. Single sheet and simple multiple layer mechanisms may be assembled by machine, but more complex mechanisms usually are not.

Any pop-up mechanism that cannot be assembled by machine requires hand assembly and more time for production. While machine assembly may not mean cheaper manufacturing at first (due to equipment cost, setup and maintenance), examining principles that allow complex paper mechanisms to be assembled by machine could be useful for reducing volume production costs for both paper mechanism and LEM manufacturing. Some exploration should occur into joint and linkage types and their effect on cost and manufacturing methods.

6.2.2 Exploring LEM Possibilities

Because lamina emergent mechanisms are ortho-planar compliant mechanisms made from planar layers of material, there are many advantages to using LEMs in mechanism design. Some desirable characteristics are that they provide a convenient way to build arrays and can be made multi-stable in order to save energy. This section discusses these topics in suggesting further research for lamina emergent mechanisms.

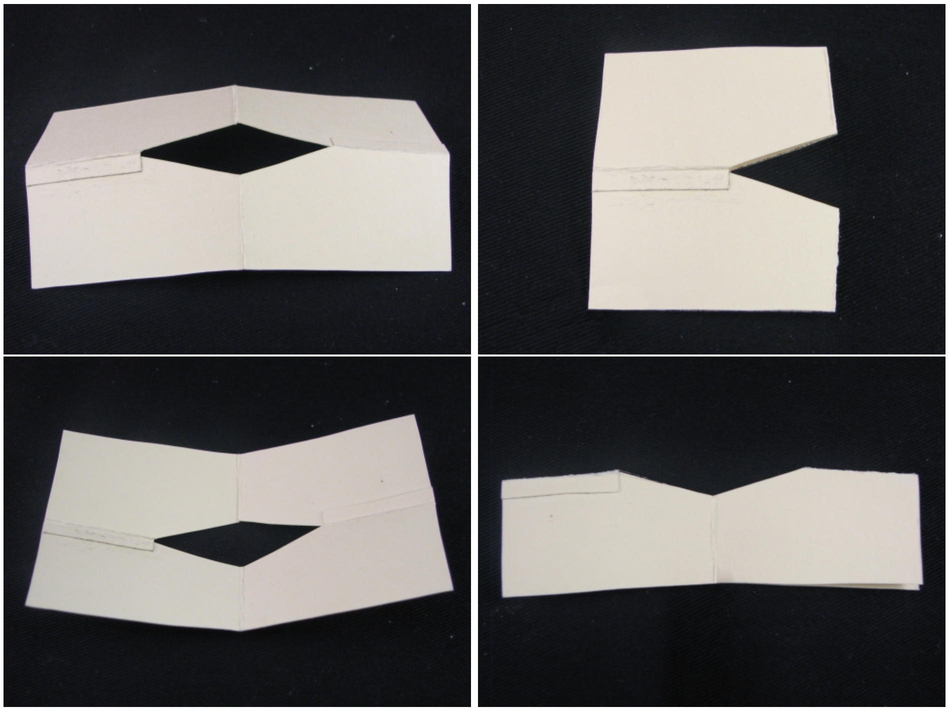

Figure 6.1: A spherical four-bar mechanism which snaps between its two configurations (left) and has two flat-folding positions (right).

Arrays

The simplified manufacturing processes and the use of planar layers of material mean that if a single mechanism can be made, it is generally easy to make many of the same mechanism with little added cost or complexity. Thus, LEMs lend themselves well to the creation of mechanism arrays. This is true whether the arrays are accomplished with multiple layers or single layers.

As demonstrated by pop-up paper mechanisms, complex arrays can be created with the combination of simple mechanisms. Mechanism arrays come in a number of different forms, the most common types being those in which the array is actuated as a group and those in which elements of the array are actuated individually (pop-up books generally employ the first type for greatest effect). Further research in this area would include exploring the use of lamina emergent mechanisms to create both types of arrays in single- and multi-layer topologies. This research would be useful in devices for surface texturing, for example, where large mechanism arrays are needed in compact spaces.

Creating Multi-stable LEMs

Manufacturing mechanisms from layered materials means that often, the links of a given mechanism are wider than they are thick, and thus are more flexible in one direction than another. Often when stable equilibrium positions are desired, it is important that the mechanism retain its flexibility in certain directions while still remaining rigid to movement in unwanted directions. Thus, LEMs are well-suited to be adapted as multi-stable mechanisms. Researching many ways to create multi-stability in LEMs would allow them to be more useful to designers.

Chapter 4 illustrated one way to create multi-stability by angular joint offset. Further work, including development of more examples and synthesis methods, is warranted in this area.

There are a variety of other ways to make multi-stable LEMs. It would be useful to explore these areas and define principles for creating mechanisms with these properties. Some ways to create multi-stable LEMs are:

1. Adding a separate multi-stable layer to the primary mechanism layer. In order for the mechanism to be stable, the total potential energy curve for all combined mechanisms must still exhibit multi-stability.

2. Adding a stiff spring to the input link of a mechanism that has two or more different configurations for one input link position. This principle is used often with bistable slider-crank mechanisms. However, there are many mechanisms (including numerous spatial mechanisms) with this property that have yet to be examined. Any mechanism with more than one configuration for a given input link position has the potential to be multi-stable.

3. Some mechanisms will not readily move between the open and the crossed configuration without disassembly. If forced to do so by bending the links, the mechanism has a higher energy state between its two configurations. Limiting the movement of these mechanisms in each configuration is yet another way to make a multi-stable mechanism (in this case, bistable). One example of this is the spherical mechanism shown in Fig. 6.1. Not only is this mechanism bistable, but it can be made with sheet materials and has more than one position where all of the links are in-plane.

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Appendix A

LEM Brainstorming Session (Thursday, 12 Jul 2007 and Wednesday, 31 Oct 2007)

This appendix contains information from brainstorming sessions which took place on 12 Jul and 31 Oct 2007. The main purpose of the sessions was to identify areas that may benefit from the application of lamina emergent mechanisms.

A.1 FLAT-FOLDING DEPLOYABLE PRODUCTS OVERVIEW

- Can be single- or multi-layer
- Can be structures or mechanisms in the initial and final states
- Suggestions:
  - The act of opening accomplishes a task
  - Low-cost, disposable sheet goods
  - “What is an application that would benefit by becoming disposable?”

A.2 IDEAS

- Entertainment Products
  - DVD case, CD case, Christmas tree
- Toys
  - Small playsets (GI Joe, Barbie, dollhouses, castles, toy car parking garages), large play structures, cool toys (e.g. Hobermann sphere), pop-up books, clothes (pockets on demand)
- Recreational Products
  - Surf board, camping equipment, compact folding snowshoes, backpacking stove, campfire ring, umbrella
- Collapsible Products
  - Clothes hangers, ice cube trays

- Packaging
  - Boxes, product display, cushioning (suspension)
  - Pallets, crates, shipping containers
- Storage/shelter
  - Shelving, garbage storage, one step metering mechanism: the act of opening accomplishes a task (for stuff that's destroyed by the elements: brown sugar, tang, etc.), shelves in refrigerator, kitchen cupboards that present their content, freezer drawers that do the same
  - Car parking lift for home use
  - Geodesic dome for house, tent, playground equipment, etc.
- Sheet Goods
- Surface Texturing
  - Buttons, shoes, air circulation surfaces, tires, mechanical displays (billboards, etc.), braille, changing coefficient of friction, locomotion (caterpillar, esophagus)
- Space Structures
  - Antennas, solar panels, rovers, human shelters (on land, in space, in water)
- Medical Devices
  - Minimally invasive surgery, contact lens storage (reusable case), wheel chair, swivel handicap chair for cars, wheelchair access ramp coupled to the car door
- User Interfaces
  - Keypad, pop-up security feature, keyboard that folds flat for cleaning, keyboard that emerges from a desk or other surface (for emergency relief efforts, airplanes, one-time factory settings on electronics)
  - One dollar cardboard keyboard: single day/one time use, secure, sanitary (for internet cafe, hospital)
- Packaging/Presentation/Dispensing
  - Cereal box lid made bistable to get rid of the nesting slot on the top
  - Replace cereal box bag with sealed cardboard
  - Locking box flaps (either with bistability or locking when put into tension)
  - Deployable structure for securing items (constant force mesh, etc)
  - Mechanism to get rid of some environmental issue by replacing styrofoam/plastic with paper.

  - Wet wipe dispensing
  - “Pop-up book” product presentation
  - Plastic containers (Tupperware, box for Christmas decorations, etc)
  - Cup holders in cars
  - Contact lenses

- Military Applications
  - Backpack items, communications, tents, hangars, weapons, antennas, disposable repeater stations, vehicles, deployable bridge, aircraft landing gear

- Utility
  - Car parts coupled to door opening (things you maintain regularly become more accessible when the car door/hood/trunk is open)
  - Trunk contents present themselves when the trunk is opened.
  - Hiding car doors (sportscar doors or sliding doors)
  - Doors on a building that either collapse flat when opened or slide like an aircraft door
  - Cupboard/freezer doors
  - Shower that emerges from the bathroom wall (for use on a plane, train, motorhome, etc)
  - Bed
  - Ironing board
  - Micro-actuated Velcro

- Small Appliances (for travel)
  - Hair dryer, curling iron, blender, ice cube trays.

Appendix B

Paper Mechanisms

This appendix contains scale drawings for creating the pop-up mechanisms showcased in Chapter 2. The reader is encouraged to copy or print the mechanisms and pages onto 8.5” x 11” cardstock (between 65 lb. and 80 lb. paper usually works well), cut them out and glue them together as shown in the photographs on the even-numbered pages. Figure B.1 shows a legend detailing the meaning of each of the line types shown in the cutouts.

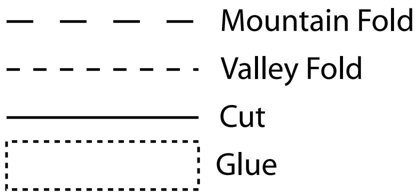


Figure B.1: A legend for the paper mechanism templates in this appendix.

Fold lines should be indented with a blunt tool (a dried out ballpoint pen works well) before they are folded to ensure crisp creases. Use the edge of a ruler to guide the tool and keep the lines straight. Be careful to just compress the paper fibers, as cutting or tearing them will decrease the life of the mechanism.

Note that the dotted boxes that are intended for gluing have letters and numbers inside. In some cases, mechanisms have boxes with both numbers and letters. The numbers should first be glued in ascending order, then the letters, finishing with the letter that is farthest from the beginning of the alphabet. The sides with printing on them should be glued facing each other so that the numbers and letters are hidden in the final mechanism.

It is important to make sure that the mechanisms will fold flat, and the final gluing step will ensure that if done correctly. The easiest way is to apply glue to the appropriate tab, then fold the base pages flat and apply pressure where the glued tab is. This may mean that the last glued tab will be slightly offset from its corresponding dotted box on the base page, depending on the position of the other glued tabs in the mechanism.

B.1 PAPER JOINTS

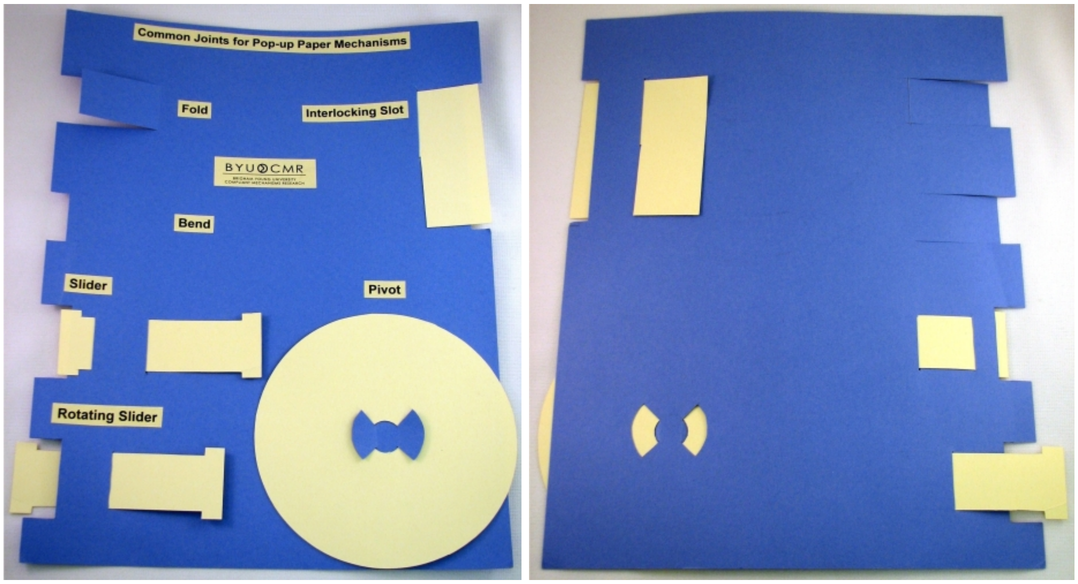


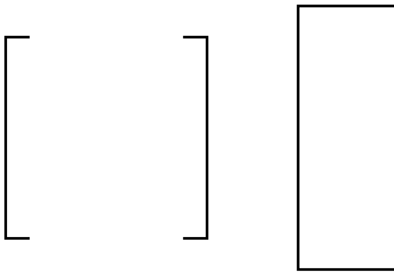
Figure B.2: Common joints used in pop-up paper mechanisms.

The paper cutouts that attach to this page are found on page 106. Note that the base page shown on page 85 is slightly modified from the photograph shown here to accommodate the binding of this thesis.

The joints on this page have tabs (called wing tabs) that require folding to fit through their corresponding holes. Care should be taken to avoid sharply creasing the tabs, so that they can unfold and keep the paper cutouts attached to the page.

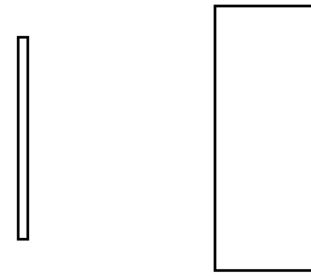
Bend

Slider

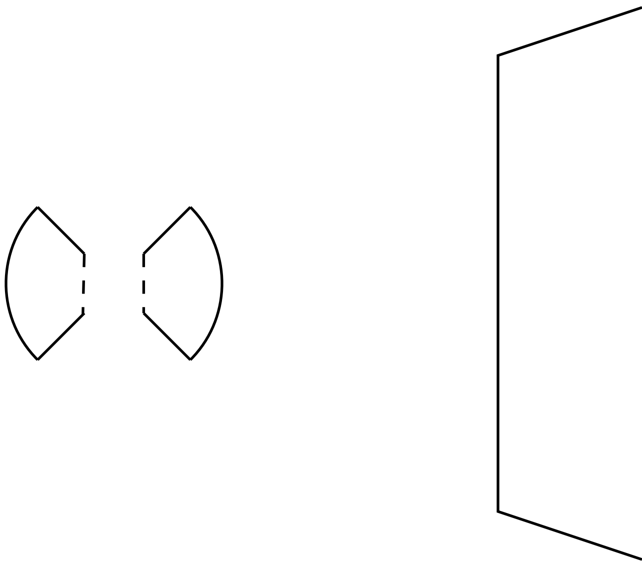


Interlocking Slot

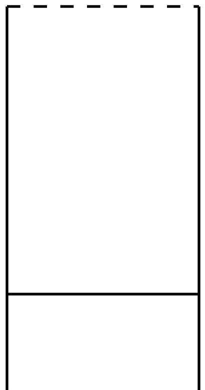
Rotating Slider



Pivot



Fold



B.2 PARALLEL FOLD (PLANAR) MECHANISMS

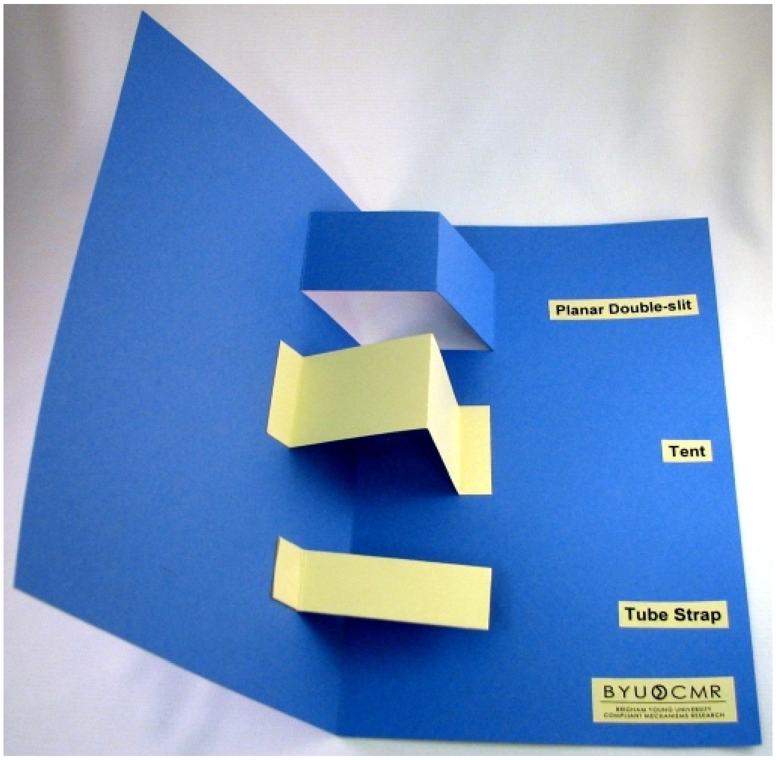
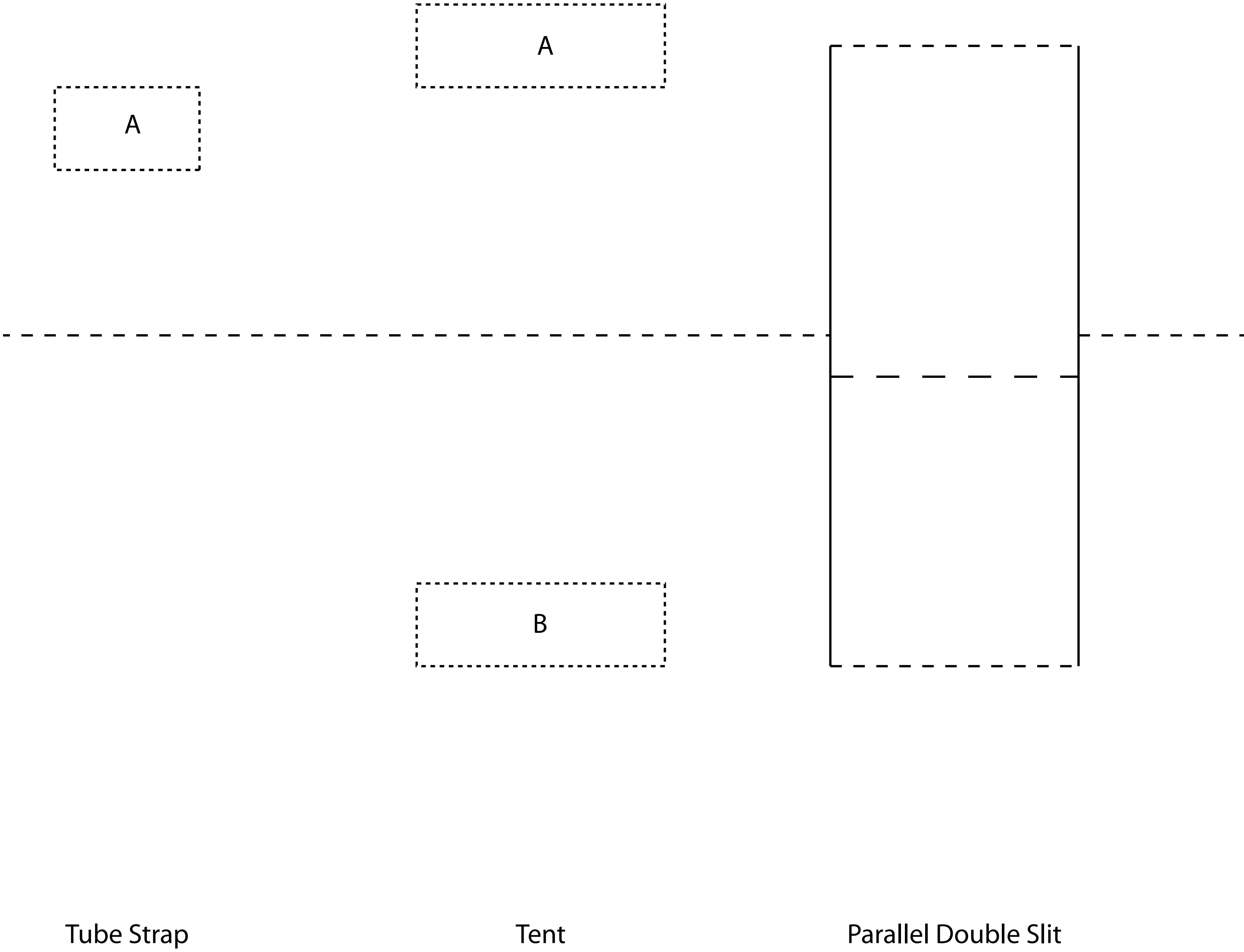


Figure B.3: A parallel double slit, a tent and a tube strap.

The paper cutouts that attach to this page are found on page 107.


A
A
B
Tube Strap
Tent
Parallel Double Slit

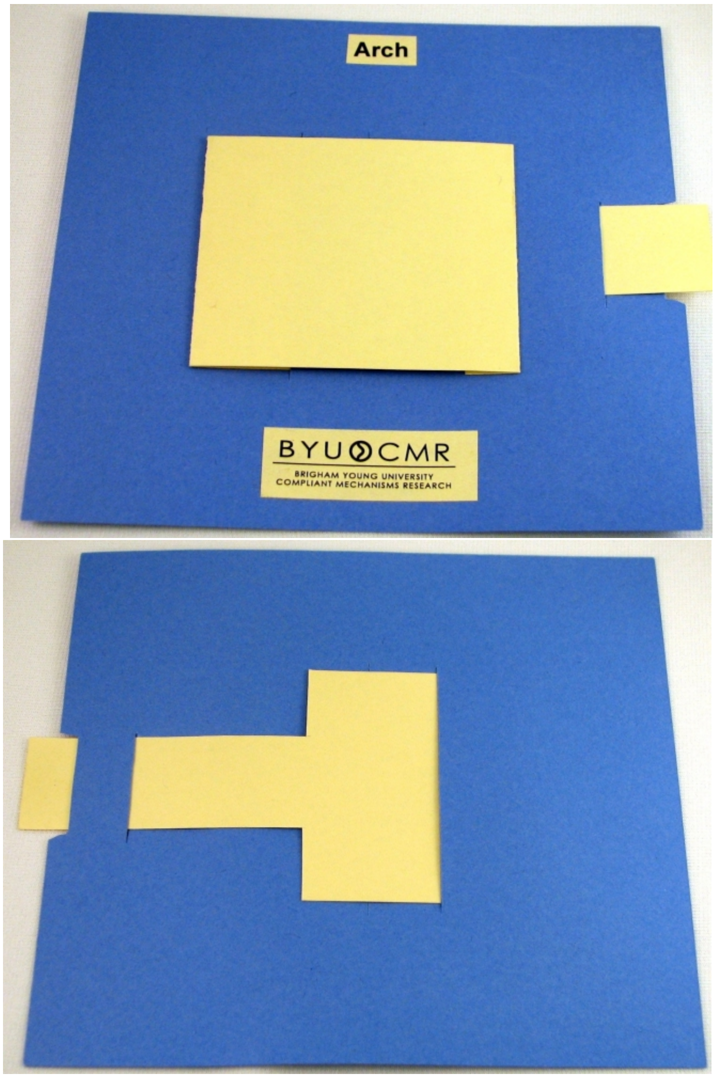
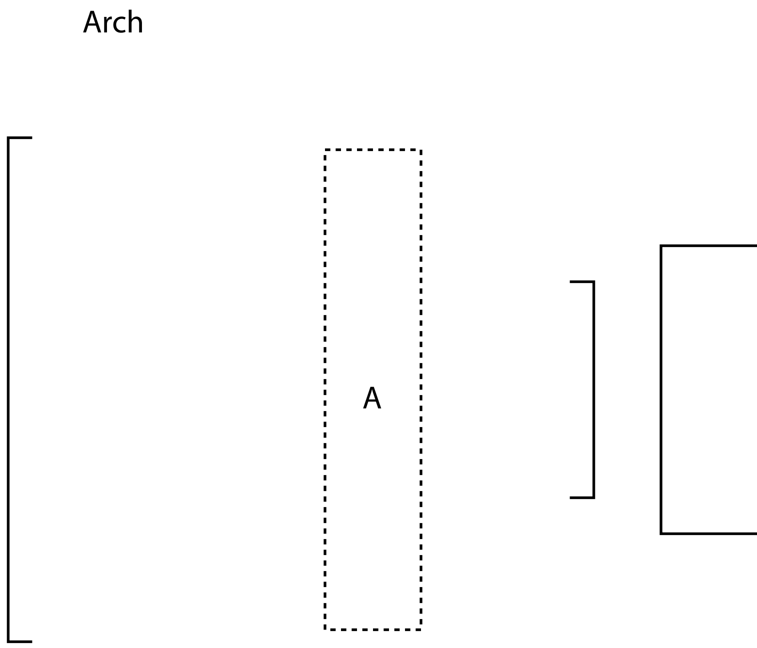


Figure B.4: An arch.

The paper cutout that attaches to this page is found on page 107.


Arch
A

B.3 ANGLE FOLD (SPHERICAL) MECHANISMS

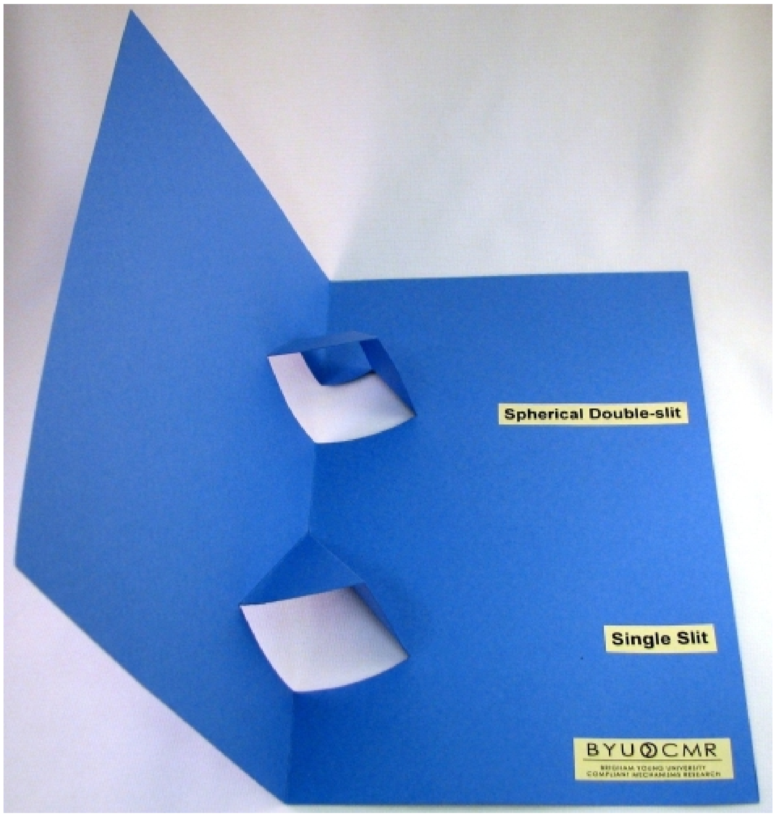
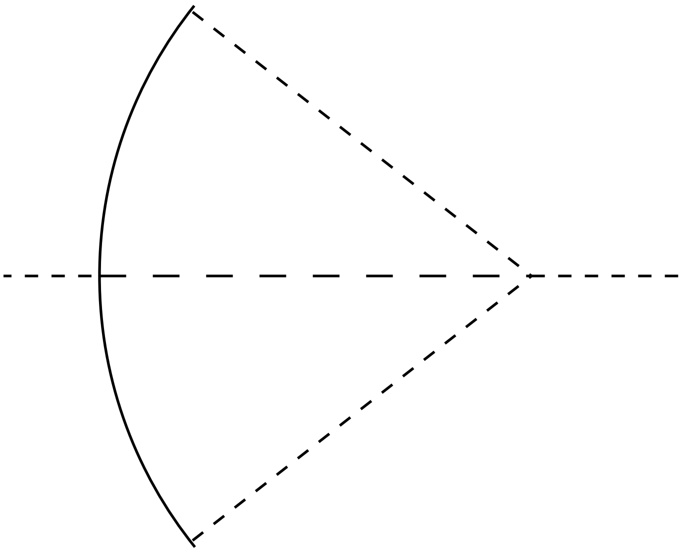
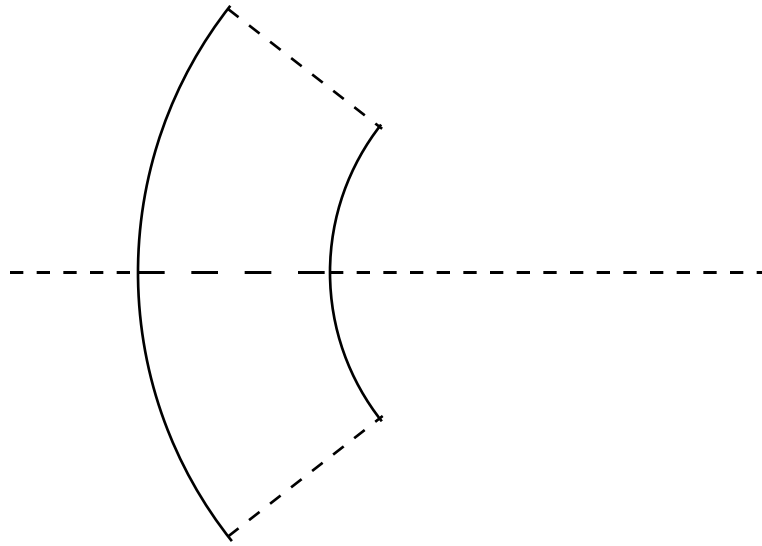


Figure B.5: A single slit and an angle double slit.



Single Slit



Angle Double Slit

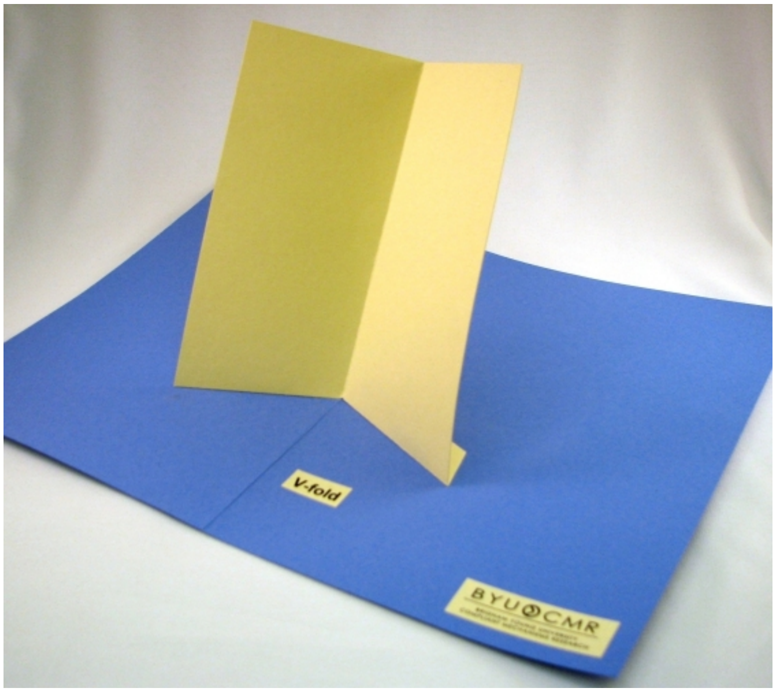
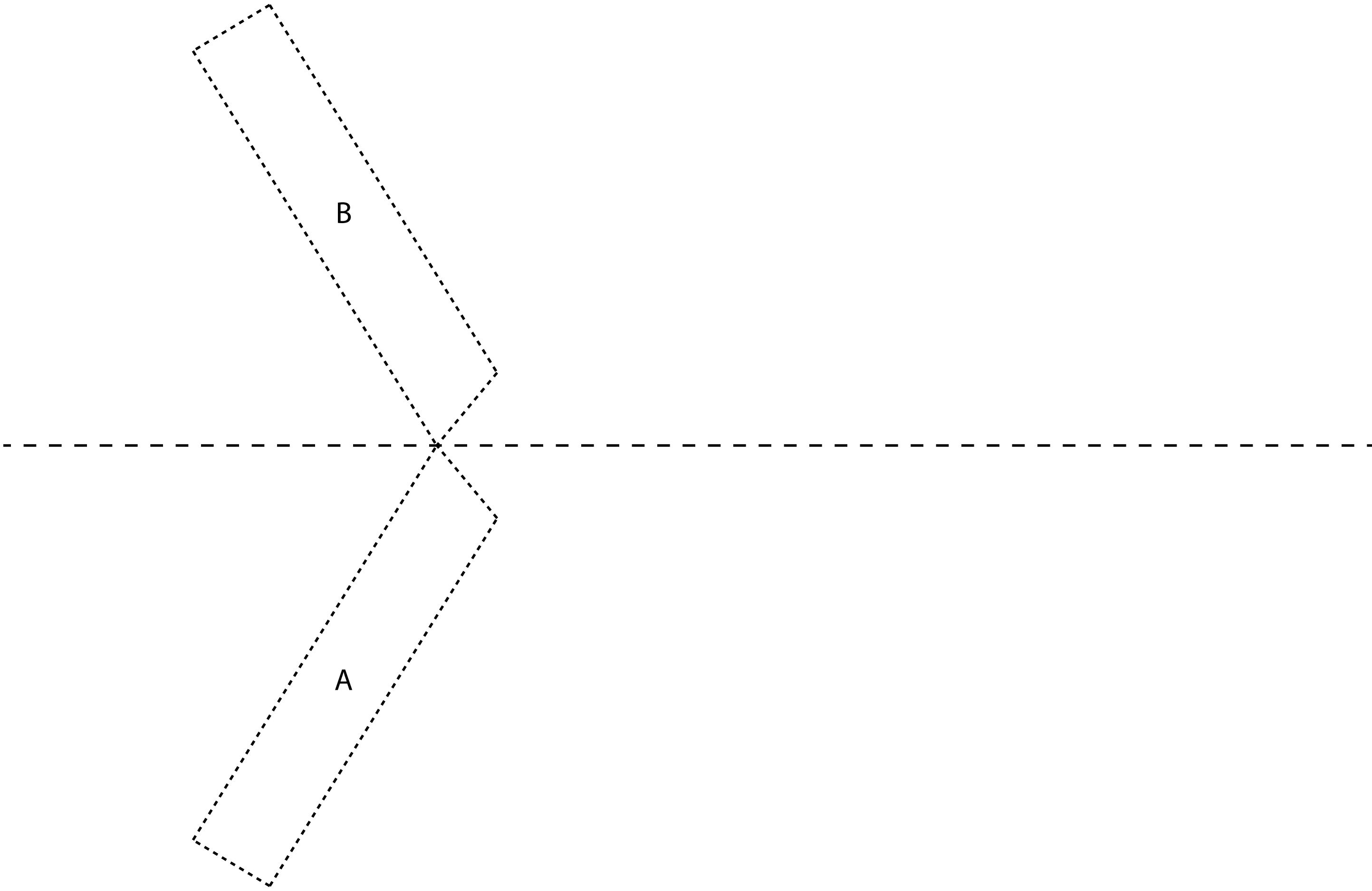


Figure B.6: A V-fold.

The paper cutout that attaches to this page is found on page 108.



V-fold

B.4 COMPLEX MECHANISMS

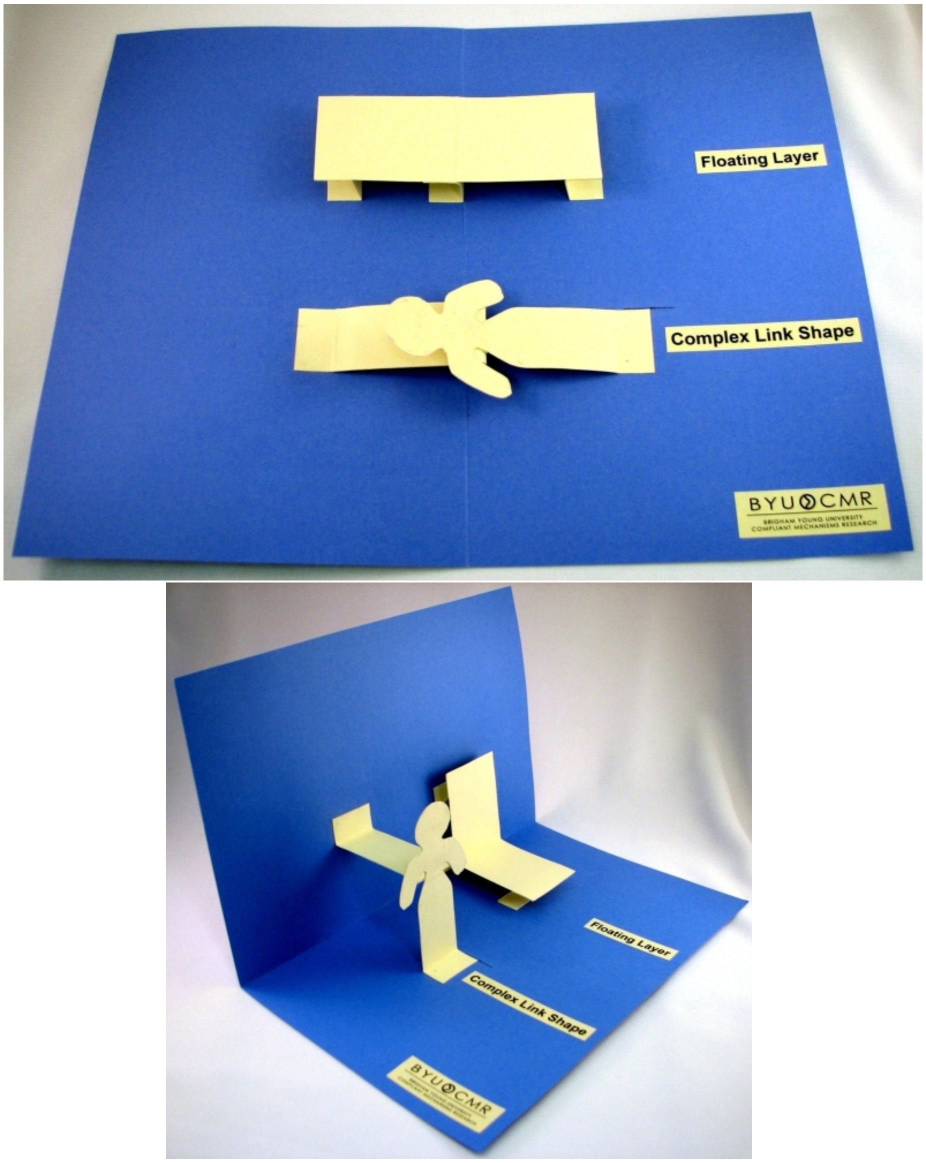
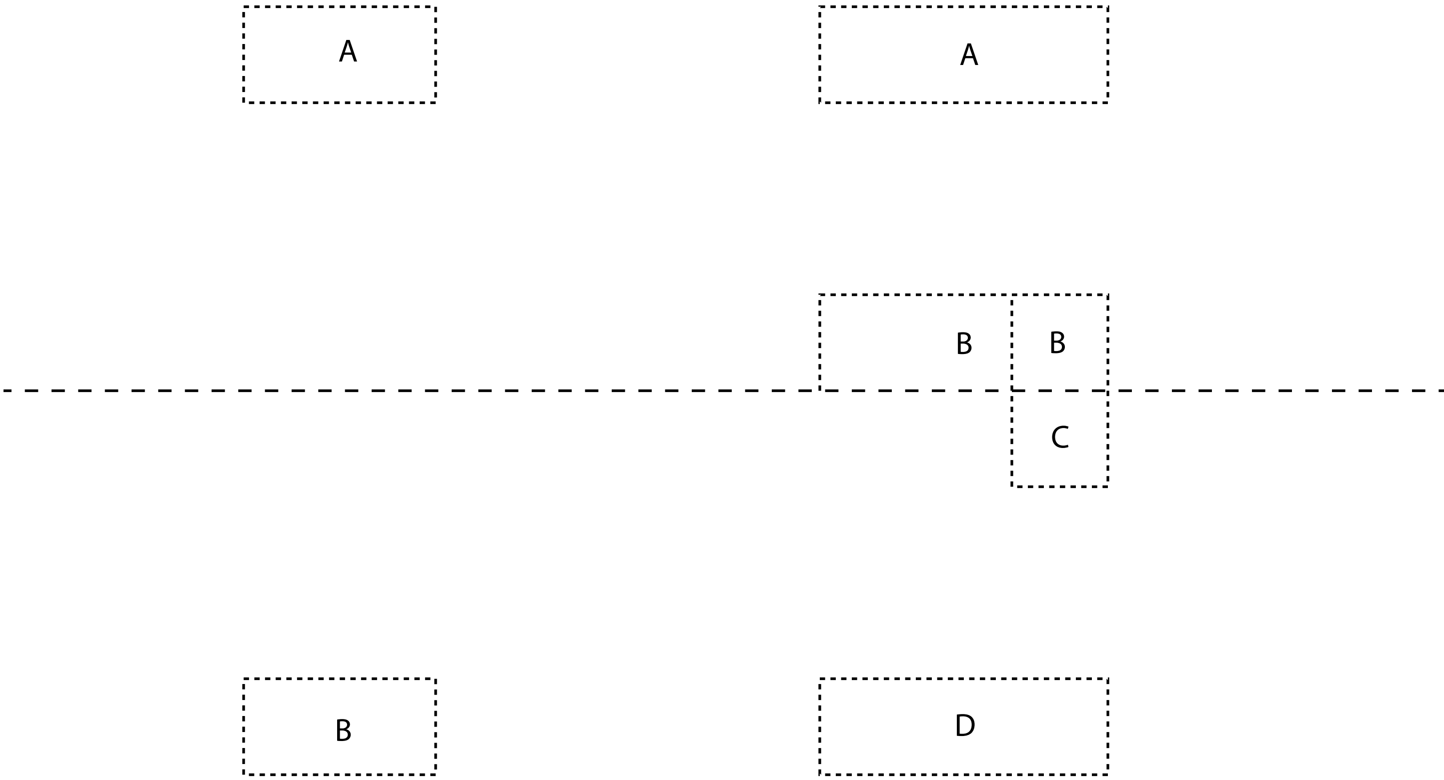


Figure B.7: A floating layer and a complex link shape.

The paper cutouts that attach to this page are found on page 109.



Complex Link Shape

Floating Layer

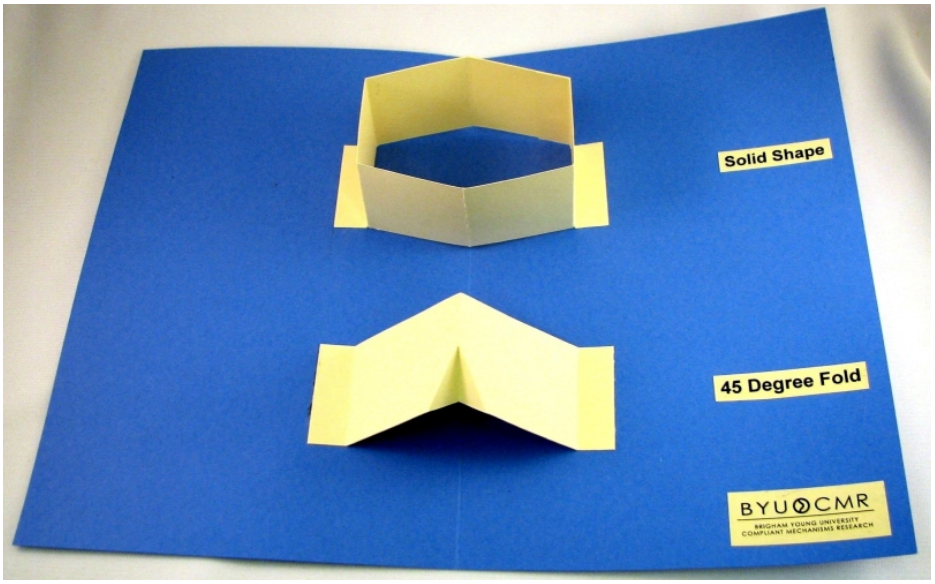


Figure B.8: A solid shape and a 45° fold.

The paper cutouts that attach to this page are found on pages 109 and 110.

A

A

B

B

45 Degree Fold

Solid Shape

B.5 NEW PAPER JOINTS

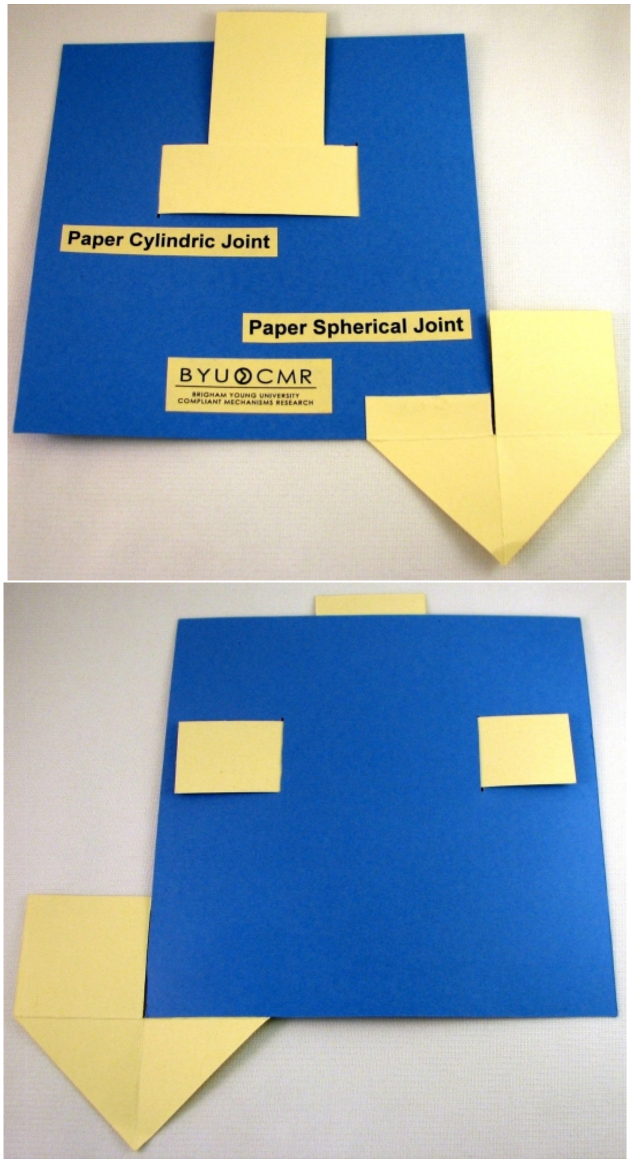
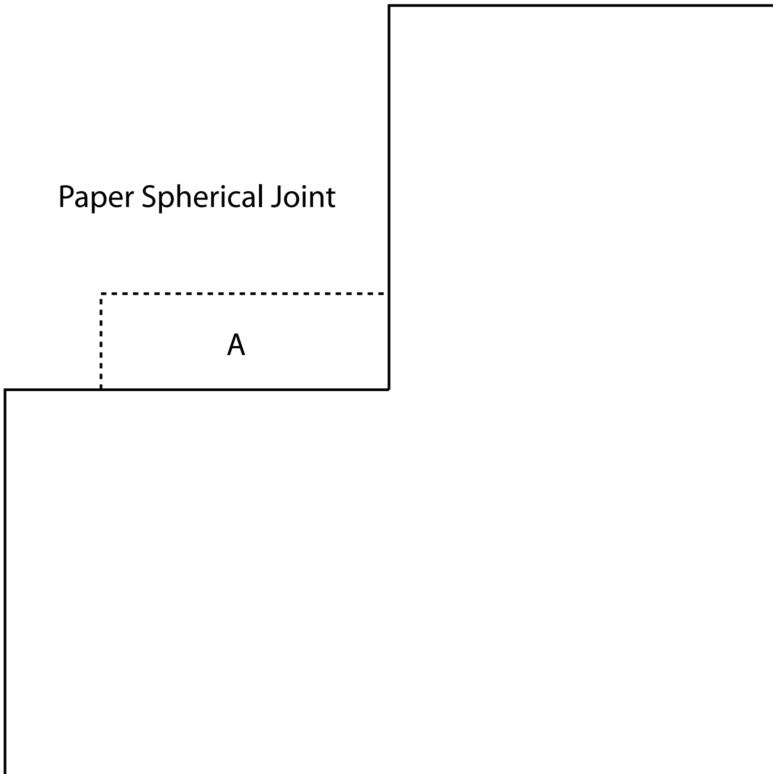


Figure B.9: A paper cylindric joint and a paper spherical joint.

The paper cutouts that attach to this page are found on page 111.

Paper Cylindric Joint

Paper Spherical Joint



B.6 NEW PAPER MECHANISMS

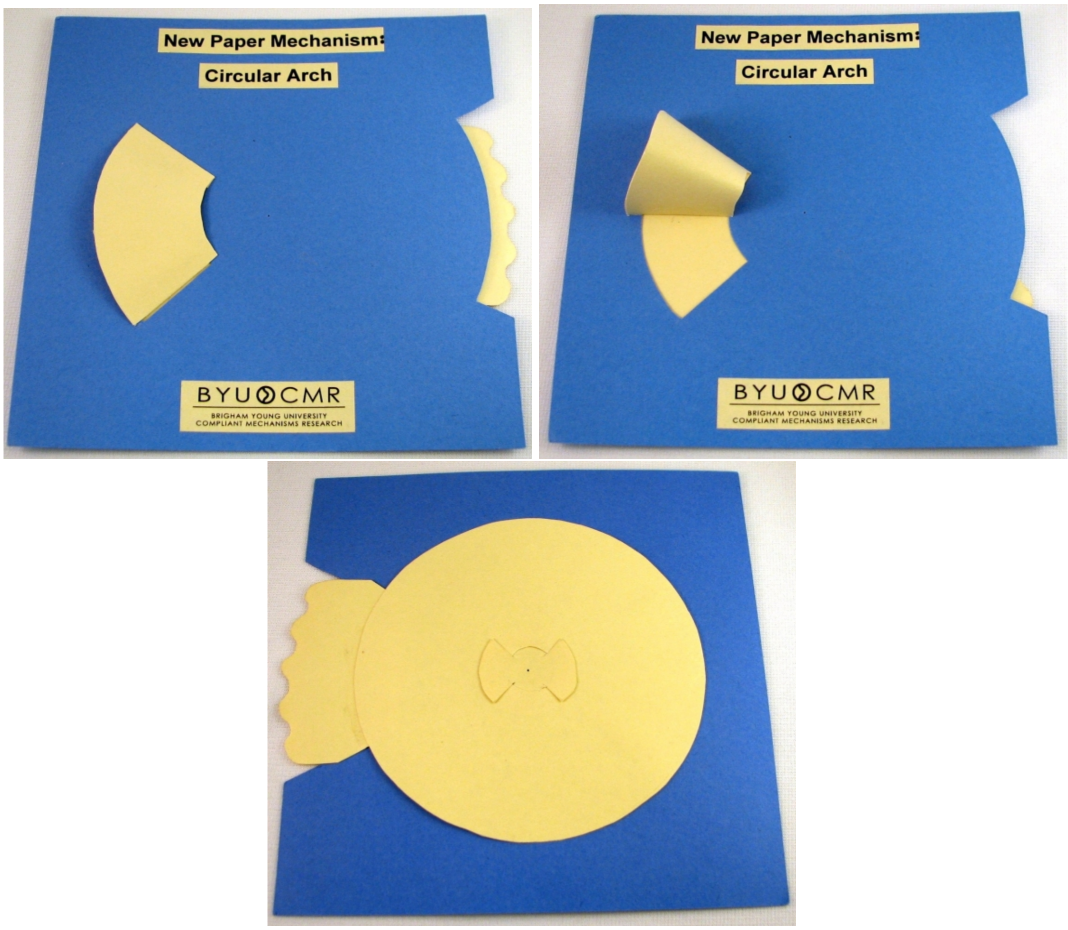
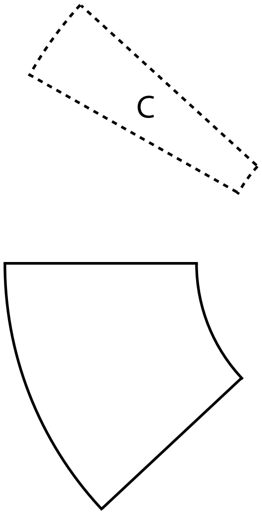
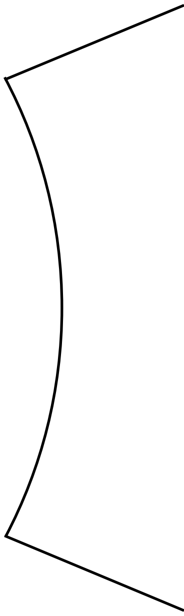
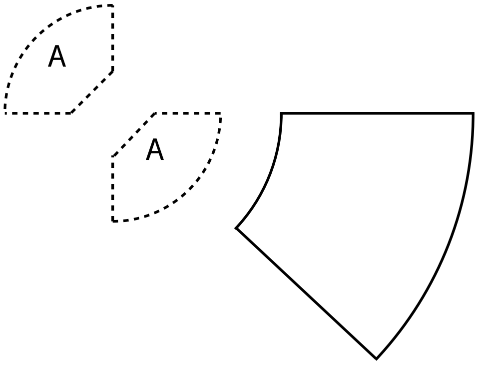


Figure B.10: A circular arch.

The paper cutouts that attach to this page are found on page 103. The wing tabs A on the circular cutout should be inserted through their corresponding hole before being glued to the page.


C




A
A

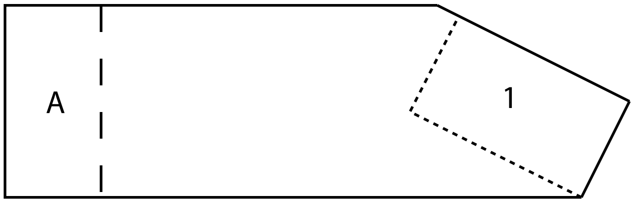
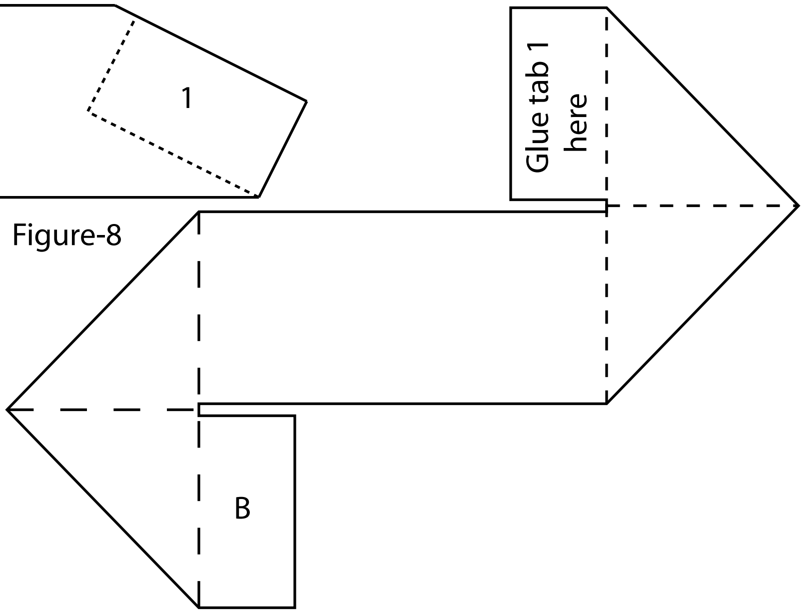
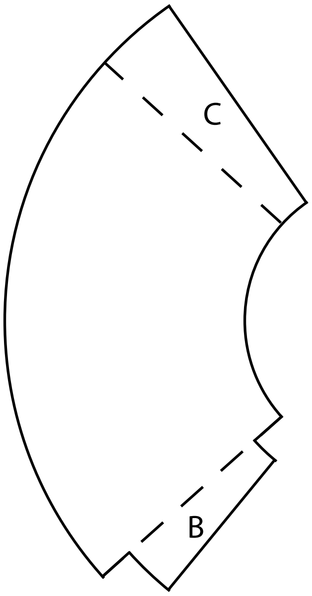
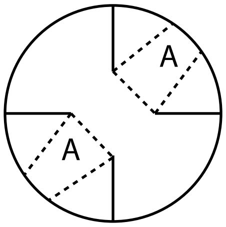


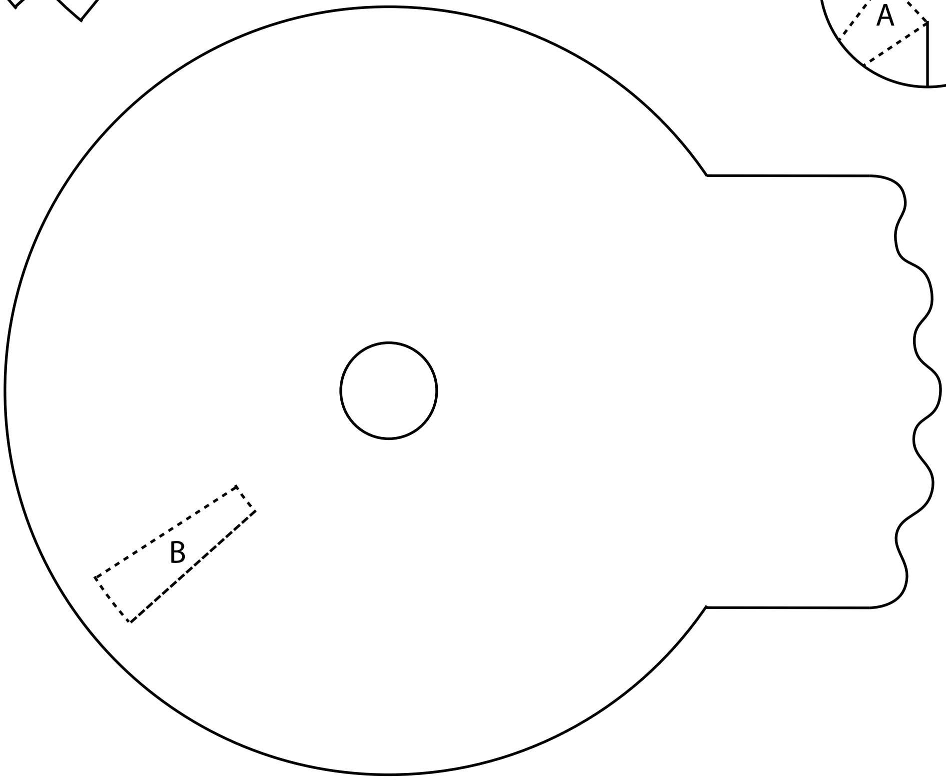
Figure-8





Circular Arch





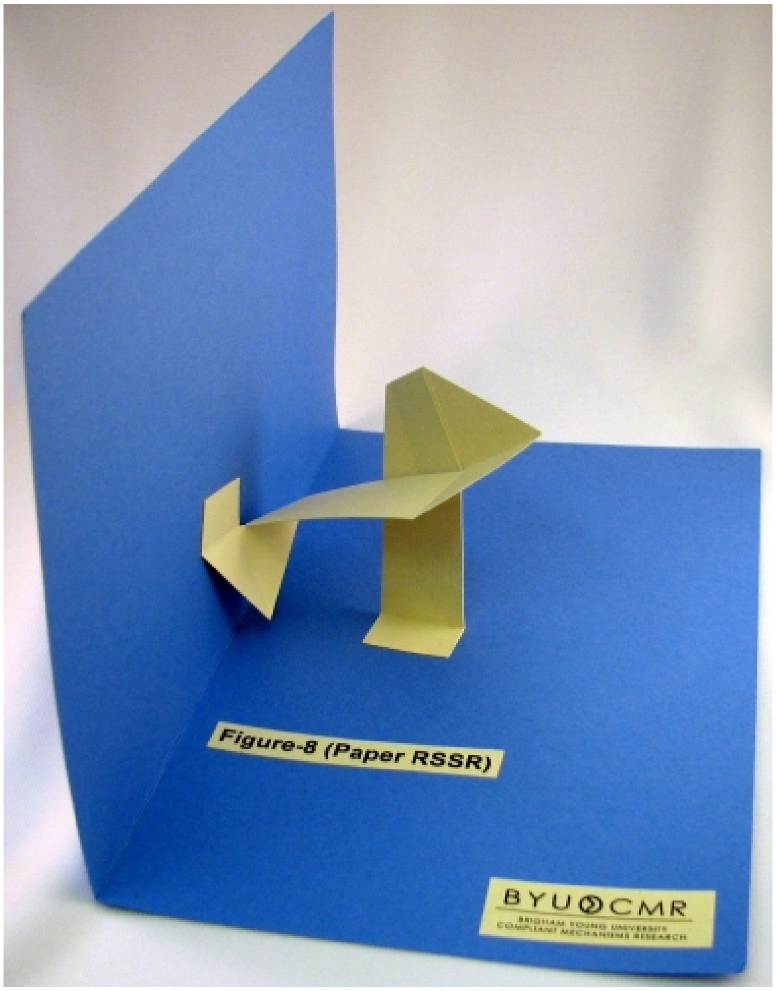


Figure B.11: A figure-8 (paper RSSR).

The paper cutouts that attach to this page are found on page 103.

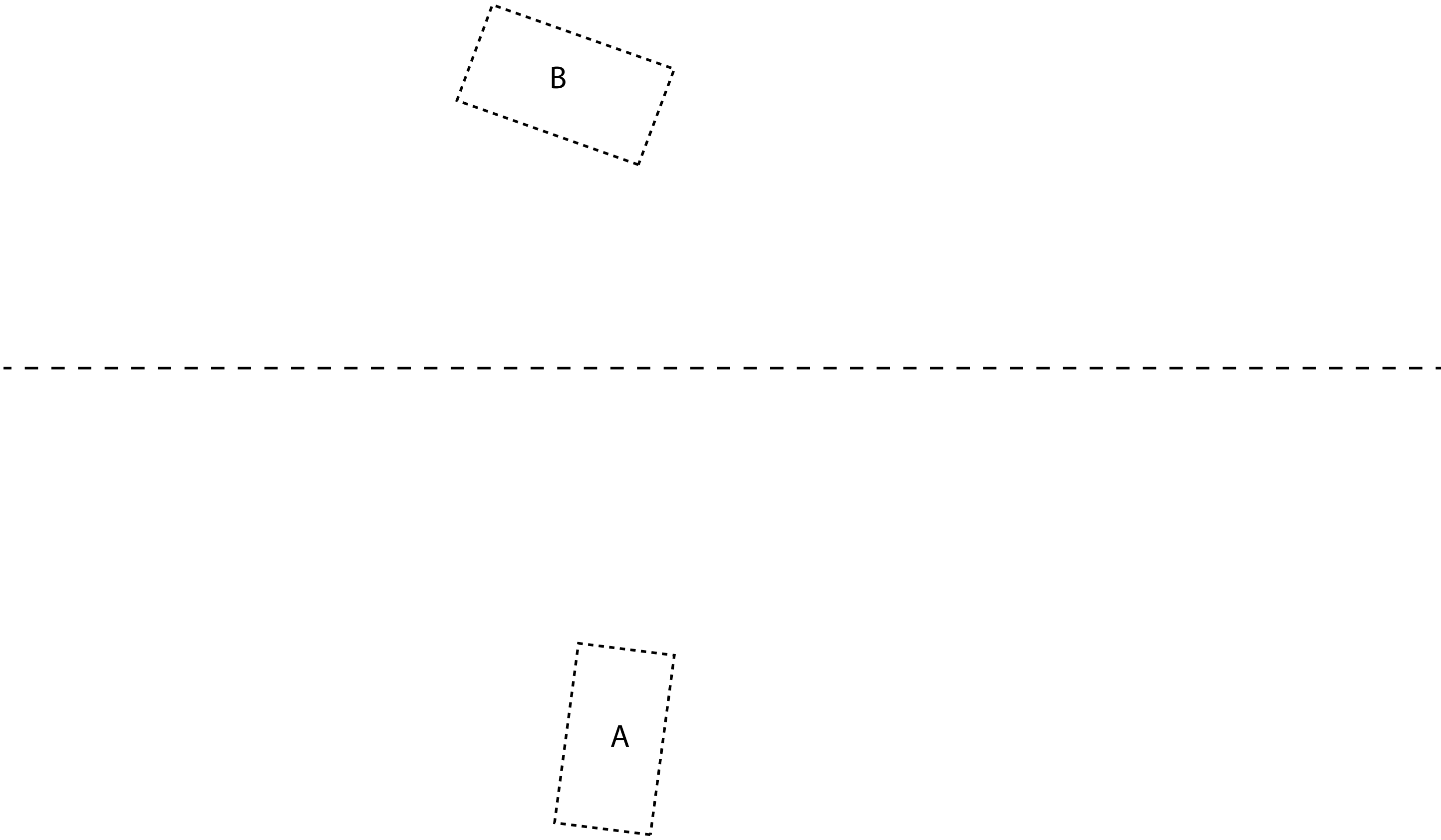
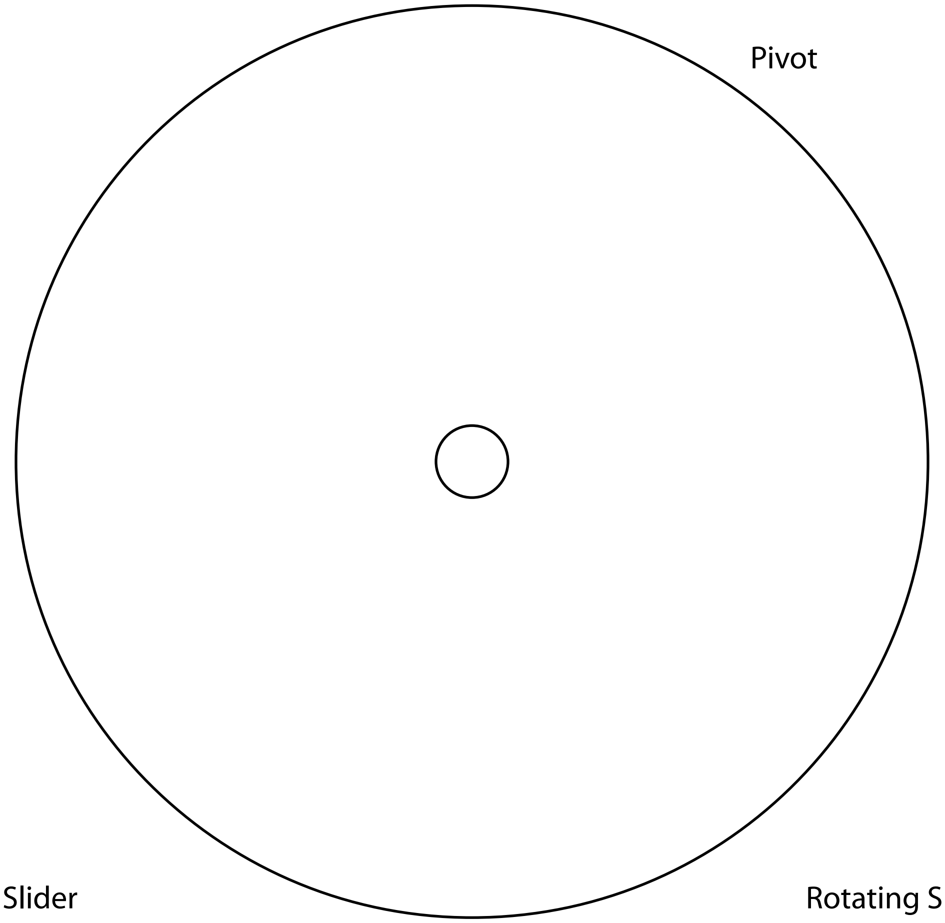
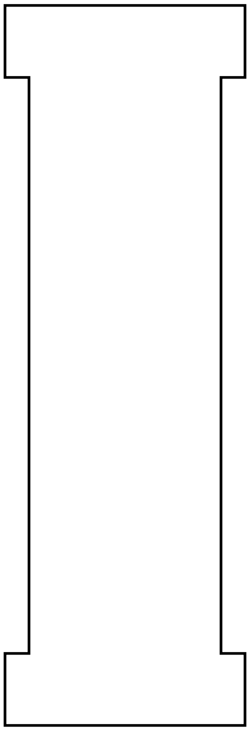


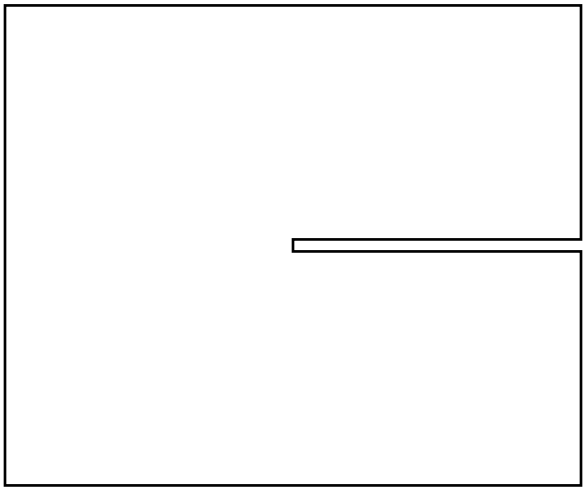
Figure-8

Pivot



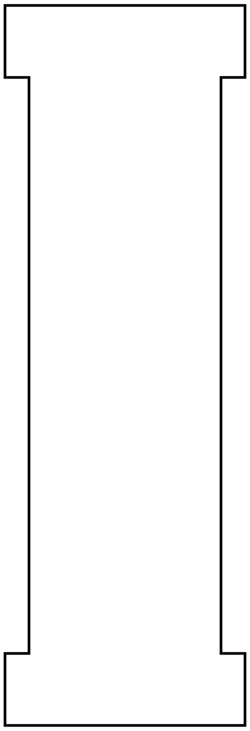
Slider

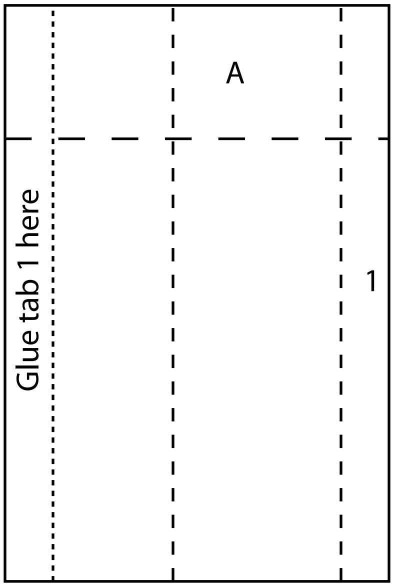




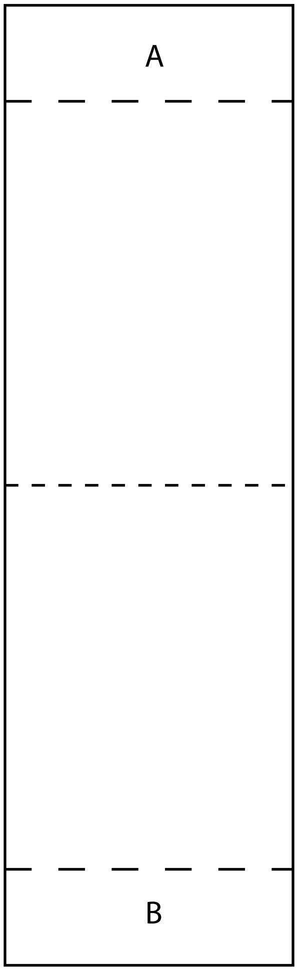
Interlocking Slot

Rotating Slider

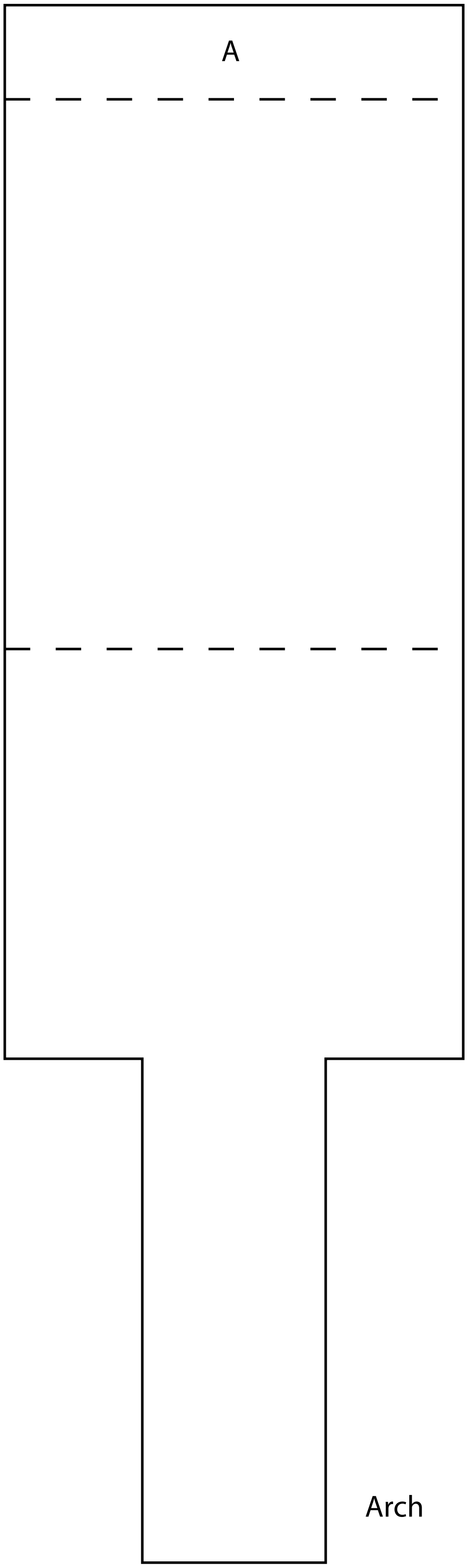




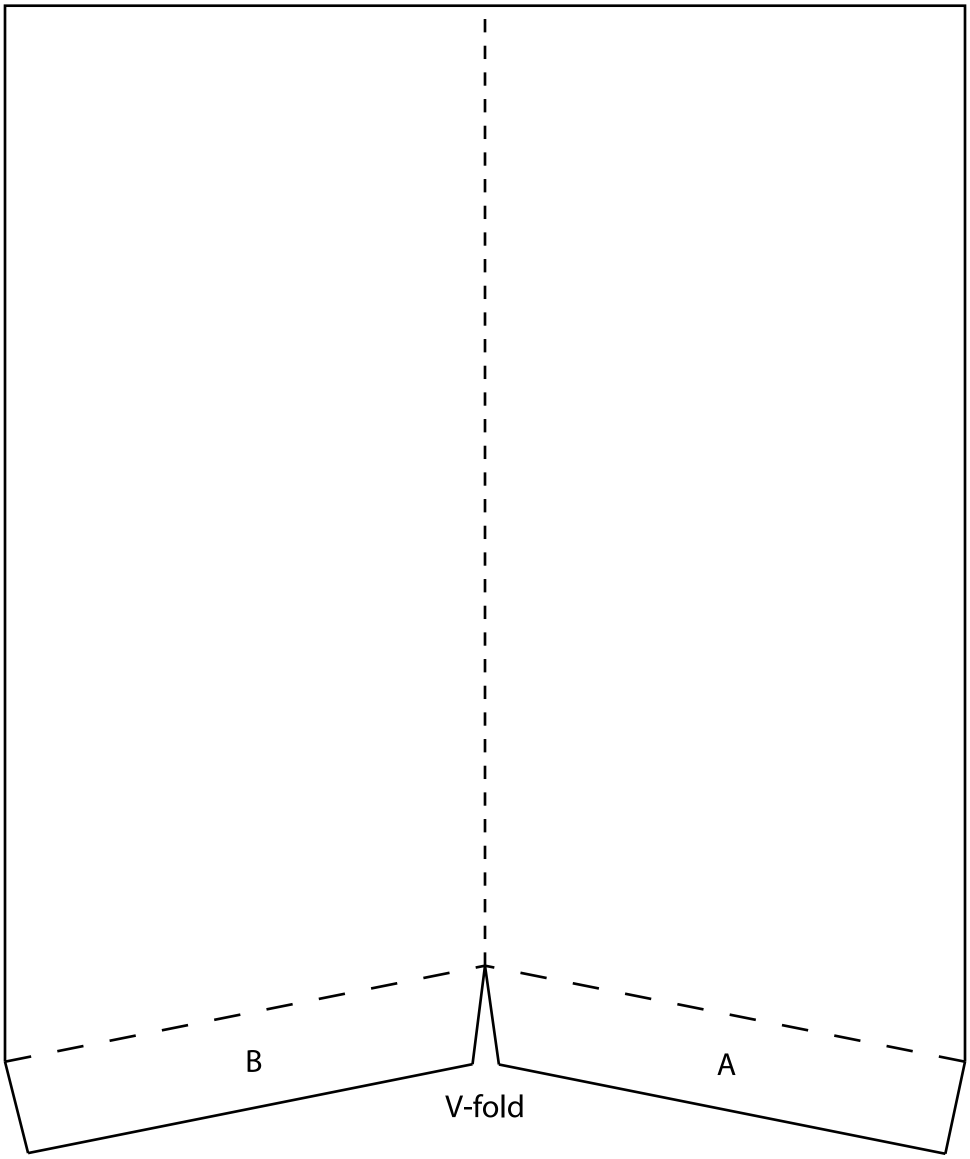
Tube Strap

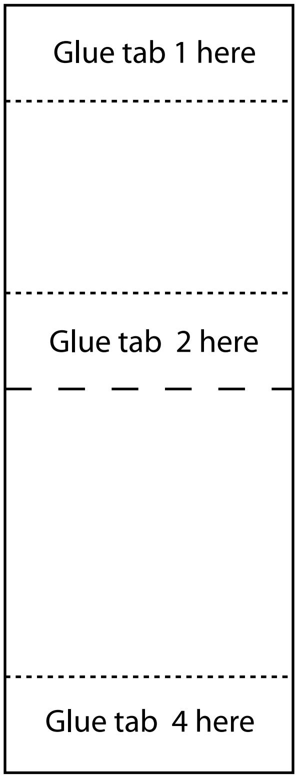


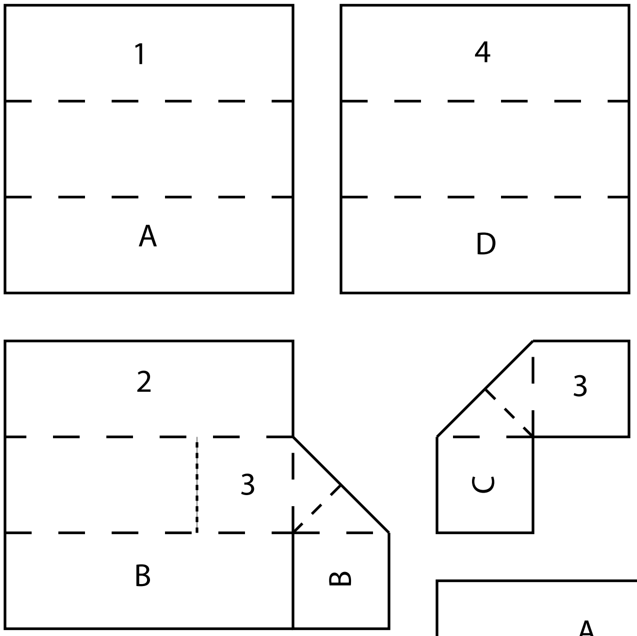
Tent



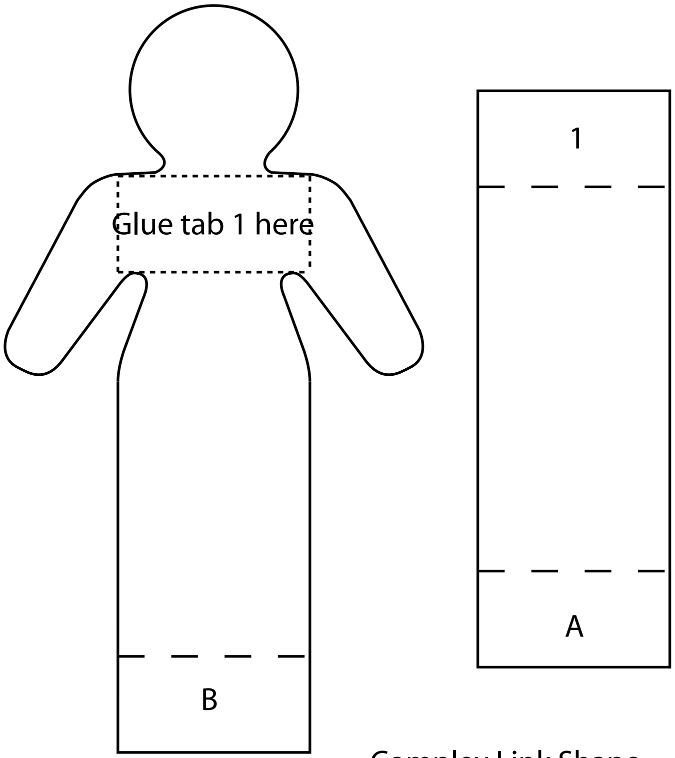
Arch


B
A
V-fold

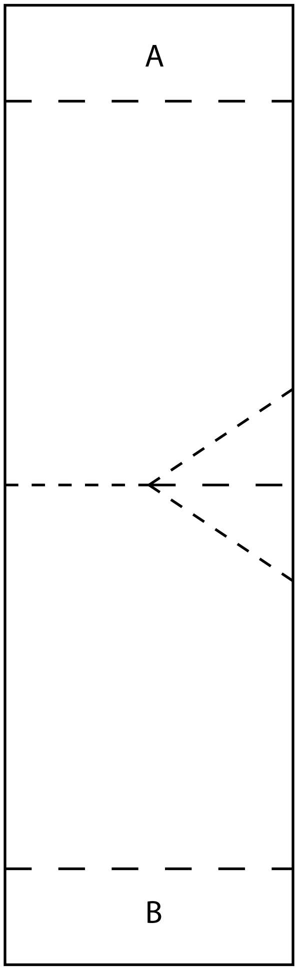




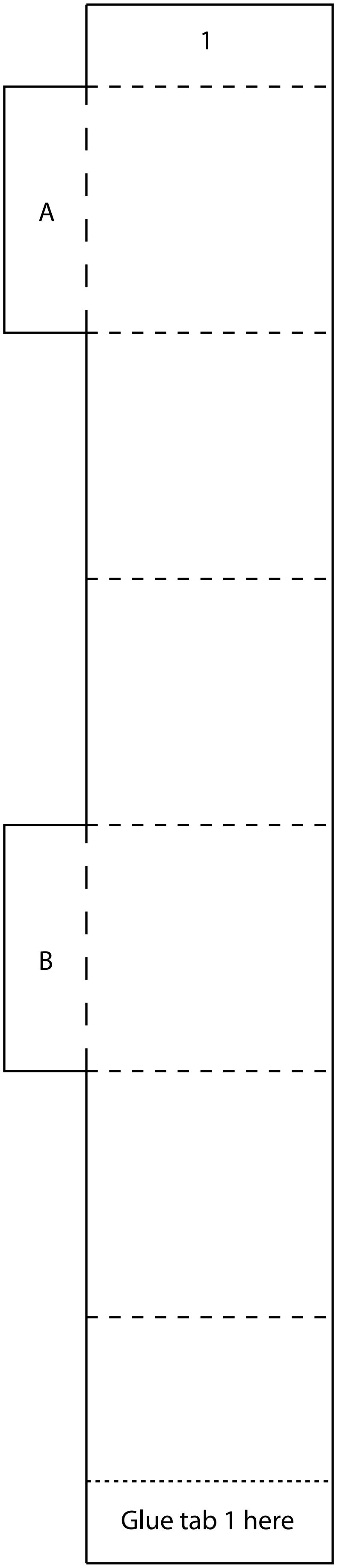
Floating Layer



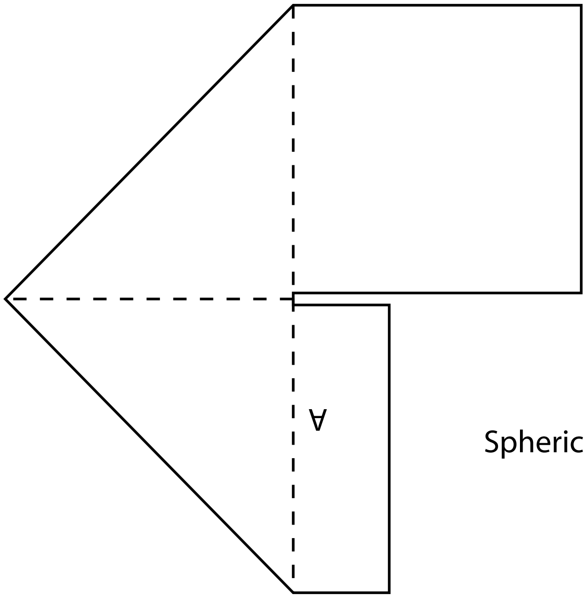
Complex Link Shape



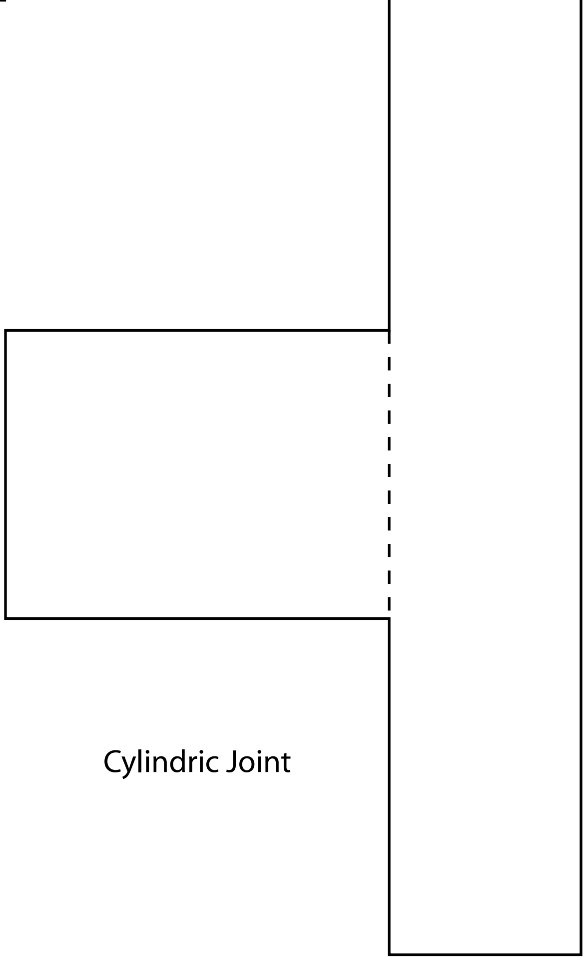
45 Degree Fold



Solid Shape



Spherical Joint



Cylindric Joint