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An Application of Kelvin Wave Expansion to Model Flow Pattern Using in Oil Spill Simulation

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Abstract: In this paper, data of tidal constituents from co-tidal charts are invoked to determine water surface level and velocity. A procedure is generated to be used as a new hydrodynamic calibration tool for estimating the dynamical field and flow pattern in the Persian Gulf. The four main tidal constituents are selected as M_2, S_2, K_1 and O_1. Taking all four constituents together, an approximate flow model is developed to consider the Kelvin wave expansion. Computed values for water surface level and surface layer currents near Kish Island have been compared with measured data from the Iranian Hydrographic center to verify the full hydrodynamic model. Results of the simplified model have been compared with both measurements and a full reference hydrodynamic model to reveal that this calibration approach is a promotion not only towards simplicity, but also towards a speed up of the computation. The final velocity field is composed by flow and wind-induced velocities. A short-term oil spill simulation was undertaken to compare the actual and simulated oil spill drift to interpret the results and prepare data using in risk map analysis.

***Keywords**:* Flow pattern; Kelvin wave expansion; Tidal constituents; Persian Gulf

1. INTRODUCTION

The Persian Gulf, in the southwest Asian region, is an extension of the Indian Ocean located between Iran and the Arabian Peninsula. It lies entirely north of the tropic of Cancer and so is strictly subtropical. The Persian Gulf is a semi-enclosed sea, varying in width from a maximum of 340 km to a minimum of 60 km in the Strait of Hormuz. This basin is entirely located in a shallow continental platform, rarely deeper than 100 m with a mean depth of about 35 m, which at the Strait of Hormuz connects directly with the great depths of the Gulf of Oman by a steep continental slope. The important topographic features are the deeper water near the Iranian coast and the shallow areas in the southwestern parts.

The hypothesis in the present work is that the main tidal motion in the Persian Gulf is due to Kelvin waves (Venkatesh et. al. [1994] and Blain [2000]) and that it must be possible to provide an approximate flow model valid for periods of about 10 days by considering an appropriate Kelvin wave expansion. This paper provides such an expansion and the performance of this expansion is studied in some details. Capturing the main tidal motion using simplified techniques is a field of ongoing activity. A web portal has been made available for describing tidal movement in the Gulf of California (Marinone et. al. [2009]) using a straightforward 14-component tidal Fourier expansion with coefficients extracted from a database generated using a 3D hydrostatic numerical model. A similar approach is followed (Ommundsen

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[2002]) using few Fourier components and this enables the author to capture the main tidal movement in the Vestfjorden Area. Yanagi and Takao [1998] carry out a study, showing that a model with a simple and approximate geometry enables to capture the main tidal characteristics of the Gulf of Thailand.

The oceanography of the Persian Gulf is such that it is an almost isolated system, except for the tide entering from the Gulf of Oman through the Strait of Hormuz as an open boundary. The resulting tidal motion consists of a variety of tidal types with higher amplitudes occurring in the Strait of Hormuz. Analysis of tidal heights from numerous coastal stations around the Gulf has, in Admiralty [2012], led to the publication of comprehensive charts of co-amplitudes and co-phase lines for the principal constituents M_2, S_2, K_1 and O_1 which are used here. This paper is organized as follows: Section 2 gives a brief account on Kelvin waves. The underlying wave phenomenon is well known and has been studied over the last decades. Also in this Section the Kelvin wave expansion adopted in this study is presented. In this expansion it is focused on the main tidal constituents, i.e. M_2 , S_2 ,K_1 and O_1. There are many previous articles in which these four main constituents are referred to as the main tidal constituents in the Persian Gulf (Blain [2000], Elshrbagy et. al. [2006], Sabbagh [2007]). For determining the coefficients in the expansion a fit to the data, such as provided by the admiralty tables, has been introduced. For reasons of comparison, a more elaborated flow model has been calibrated as well and is described in section 3. In this section, it is also shown that the resulting MIKE3-HD model may serve as a reference model in the present study. As a consequence, the final proof of the validity of the present approach needs to be found in the area of oil spill modeling. In order to be able to include a small example related to oil spill modeling as well, a short overview of the spill model is presented after validation of tidal simulation in section 4 leading section 5 discusses the results. The Kelvin wave expansion is compared both with measurements as well as with the MIKE3-HD reference model. Good results are obtained and it can be observed that taking the flow data from the Kelvin wave expansion enables a good simulation of this spill for the first couple of days. Finally, in section 6 some conclusions are presented.

2. APPLICATION OF KELVIN WAVE EXPANSIONS

Kelvin waves are special solutions of the small amplitude frictionless linearized inviscid depth-averaged shallow water equations. In order to determine the Kelvin wave, one may consider the trial solution as follows:

$$\eta = f_1(y)\,e^{\pm i(kx-\sigma t)}\,,\, u = f_2(y)\,e^{\pm i(kx-\sigma t)}\,,\, v = f_3(y)\,e^{\,i(kx-\sigma t)}\,. \tag{1}$$

The functions f_1, f_2, f_3 are determined such that this trial solution solves the differential equations and satisfies the conditions put forward. Some lengthy computations show that $f_1, f_2, f_3 \propto e^{-\beta y}$, with algebraic equations for the parameters implying $f_3 = 0, \sigma^2 = C^2 k^2, \beta = f/C$, where $C = \sqrt{gH}$. Where f is Coriolis factor, k is wave number and u, v are velocity components. As a result, the Kelvin wave, written in its most compact form, reads:

$$\eta = \eta_0\, e^{-\frac{f}{C}y} \cos[k(x \pm Ct) + \varphi], \quad u = \frac{C}{H}\eta, \quad v = 0\,. \tag{2}$$

In practice, Kelvin waves are most important for describing tidal amplitudes in presence of long coastlines. The tidal movement in the Persian Gulf near the Iranian coast stems mainly from Kelvin waves traveling inward from the Strait of Hormuz along the northeast coast. In fact, the plane of the domain is rotated under an angle of $\theta = 25.9^\circ$ in order to fit the domain to the new coordinate system (x', y'), making the coastline nearly parallel to coordinate lines. The mode from (2), traveling inwards, reads:

$$\eta = \eta_0 \, e^{-\frac{f}{C}y'} \cos[k(x' - Ct) + \varphi], \quad u' = \frac{C}{H}\eta, \quad v' = 0 . \tag{3}$$

Here, x', y' are coordinates in a rotated frame and u', v' are the velocity components with respect to the rotated frame. To accomplish expressions in the $x, y-$ coordinate system needs to be observed that:

$$\begin{bmatrix} x \\ y \end{bmatrix} = M \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad \begin{bmatrix} u \\ v \end{bmatrix} = M \begin{bmatrix} u' \\ v' = 0 \end{bmatrix}, \quad M = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{4}$$

In particular, it is known that the waves entering the Persian Gulf at the Strait of Hormuz will carry the associated frequencies of which the data are given in Table 1 for each constituent. In this table T and $k = 2\pi/\lambda = 2\pi/(T\sqrt{gH})$ are the period and the wave number respectively. The four main constituents are numbered with $j = 1, 2, 3, 4$. Here, $j = 1$ corresponds with M_2, $j = 2$ with S_2, $j = 3$ with K_1 and $j = 4$ with O_1. All four constituents introduce a Kelvin wave, traveling into the Persian Gulf with elevation $\eta_j = \eta_{0j} \, e^{-\frac{f}{C}y'} \cos[k_j(x' - Ct) + \varphi_j]$. Taking all four together an approximate flow model is obtained as:

$$\eta = Z_0 + \sum_{j=1}^{4} \eta_j, \quad u' = \frac{C}{h} \sum_{j=1}^{4} \eta_j, \quad v' = 0 . \tag{5}$$

Table 1. Specifying the periods and wave numbers of the main constituents

	M_2	S_2	K_1	O_1
T [hr]	12.42	12.00	23.93	25.85
$k \times 10^6$ $[m^{-1}]$	7.58	7.85	3.94	3.64

It must be remarked that the typical depth parameter H, has been replaced by the still-water depth h. The bathymetry has been provided by the Iranian Hydrographic Center.

As an example, Table 2 presents the amplitudes and phases for a grid point close to Kish Island to use these data for establishing the unknown coefficients in (5). To give an impression on the consistency of the data used, the data from the admiralty tables with data on amplitudes and phases are compared such as presented by the Iranian Hydrographic Center, at four reference locations, i.e. Kish and Siri Islands, Bandar Abbas and Bushehr Port. In Table 3, the maximal and minimal deviations over the four locations are presented. The agreement between the two is reasonable.

Table 2. Phase and amplitude of main constituents for the Kish Island

constituents	M_2	S_2	K_1	O_1
θ : *ph.* [deg]	35.30	69.70	145.31	99.07
μ : *amp.* [m]	0.350	0.140	0.333	0.200

Table 3. Comparison of tidal constituents for 4 locations in the Persian Gulf

Tidal constituents	M_2		S_2		K_1		O_1	
	Ph	Amp.	Ph.	Amp.	Ph.	Amp.	Ph.	Amp.
Max. deviation %	15.2	14.1	21.1	27.5	19.8	18.4	20.7	23.4
Min. deviation %	0.6	0.0	1.3	3.6	0.9	1.8	2.9	2.9

The amplitude and phase taken from these tables with μ_j and θ_j respectively. Starting from μ_j it is possible to define normalized values $\tilde{\mu}_j = \mu_j \Big/ \sum_{j=1}^{4} \mu_{j_0}$. μ_{j_0} referring to coastal amplitudes taken at related coastline positions according to the basic definitions. To obtain this position it is just needed to determine the present position in the (x', y') coordinate system, next traveling towards the coast keeping x' equal. The mean surface level Z_0, has been determined using the relation $Z_0 = 0.15 + \sum_{j=1}^{4} \mu_j$. This relation, valid for the Persian Gulf, concerns the mean sea level and stems from the Iranian Hydrographic Center. It is basically originates from the admiralty tables, where they have added the value 0.15 to account for seasonal streams. Here, calibration has been done by this Center using about 20 years of field data. In fact $\eta_{0j} = \tilde{\mu}_j$, $\varphi_j = \theta_j$ is resulted from a trial and error procedure and showed up to be the best.

Using the charts from the admiralty table, the amplitudes and phases of each of the main constituents are determined in the grid points. These grids are in general obtained by refinement of the coarse grid. In order to obtain η_{0j} and φ_j on the fine grid as well, a simple bilinear interpolation is employed.

3. AN ALTERNATIVE HYDRODYNAMIC MODEL

A full hydrodynamic model, based on the software package MIKE3, is considered as well for reasons of comparison. In the present study, the 3D non-hydrostatic z-layer version of the MIKE3-HD software has been used in combination with $k-\varepsilon$ closure. Here, developments are in line with Elhakeem et. al.[2007]. However, these authors include a more extended physical model taking salinity and temperature effects into account. At Hormuz strait there is an open boundary with prescribed surface elevation fluctuation. Field data at Didamar Island has been obtained from the Iranian Hydrographic Center (see Figure 1). The considered geometry and bathymetry of the Persian Gulf has been provided by the Iranian Hydrographic Center. The prevailing wind direction in the Persian Gulf is from NW to SE. The wind field imposed, consists of a yearly-averaged field, provided by the Iranian bureau of Oceanography and Meteorology and has a velocity with magnitude 5.25 [m/sec] and direction $315°$. The horizontal unstructured grid has 13532 elements with a typical spatial grid size of 5490×4575 [m], which is $3' \times 2.5'$. Later on, this grid and the associated MIKE flow field is used in the particle-based oil spill model.

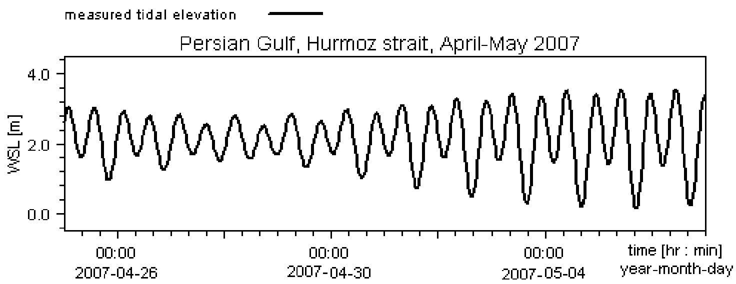

Figure 1. Surface level fluctuations at Hormuz strait imposed as the initial conditions

To verify the full hydrodynamic model, computed values for water surface level and surface layer currents near Kish Island have been compared with measured data from the Iranian Hydrographic Center. From the comparisons it may be concluded that the MIKE3-HD model is suitable for reference purposes (Badri et. al. [2010 a]). Therefore, MIKE3-HD model is calibrated to verify the results. The comparison period was chosen based on the actual available measured data from a current meter at the depth of 10 m near Kish Island. Figure 2 compares the water surface level for the 12-days time interval 25 April–6 May 2007. Comparison of the averaged surface layer currents is done in Figure 3 and shows a fair agreement between measurement data and predicted results obtained by Kelvin model. It is interesting to relate the present comparison to others available in literature. Elshorbagy et al. [2006] and Elhakeem et al. [2007] are considered currents and water levels for a station not too far from Kish Island, with computations carried out on basis of an alternative MIKE3-HD model. In these publications the comparison for the current seems to be a little more favorable than elsewhere. Here, only the main trend is captured for the currents.

4. VERIFICATION OF TIDAL SIMULATION

In order to be able to include a small example of an oil spill, oil spill model is considered. The model is entirely generated by the author of which the details are presented in Badri et al. [2010]. Here its main characteristics are only discussed. The oil spill model is a 3D particle model collecting known formulations for oil drop emergency, surface oil spill spreading, vertical dispersion, evaporation and emulsification. At the mathematical basis of the model, the stochastic differential equation formulation of advection-dispersion with centered Gaussian distributions is considered to model dispersion. The basic ideas have been presented at many places (Venkatesh [1988], Al-Rabeh et. al. [1989] and Wang et. al. [2005]). Although, it is possible to establish stochastic formulations of dispersion using uniform distributions, which is often done in older literature, the more modern formulation using Gaussian (normal) distribution is preferred here. It is well-known that oil spill models contain several empirical parameters. As far as necessary, parameters have been calibrated for application in the Persian Gulf. The final velocity determining the advection is composed of the flow velocities and the wind-induced velocities. Let $\vec{u}=[u,v]^T$ denote the horizontal velocity vector at the surface, then:

$$\vec{u}=\vec{u}_{flow}+k_w(z)\vec{u}_{wind}\ . \tag{6}$$

Here, $\vec{u}_{flow}$ is the velocity vector resulting from the flow model and $\vec{u}_{wind}$ is the wind velocity at 10 meters above the sea level. The function $k_w(z)$ determines the drift resulting from the wind. The influence of wind on oil spills is significant and, therefore, has been determined in quite an advanced manner using analytical wind fields tuned to measurements (Badri et. al. [2010 a, b]). Spatial fluctuations are calculated based on random walk technique. The velocity components u, v and w are determined either using the dynamical field resulting from the Kelvin wave expansion or by applying the full MIKE3-HD model. Independent random variables following reduced centered Gaussian distributions are invoked in random walk technique. The term containing a consequence of oil drop emergence is followed Zheng et al. [2000] to model this term. Another verification of tidal simulation was done related to amphidromic points at the Persian Gulf (Badri et. al. [2010]). It is worth mentioning that an amphidromic point is a point where the tidal fluctuation is almost zero and its occurrence hinges on the interference between Coriolis effects and bathymetry.

5. RESULTS

The dynamical field has been obtained by two different hydrodynamic models. The first uses the Kelvin wave expansion and the second is the full MIKE3-HD model. In this paper both are evaluated. In a first step, the focus is on amphidromic points. A maximum elevation of 5 [cm] is seen near a typical amphidromic point, which is small compared to the maximum tidal water level elevation (of the order of 4 [m], as documented by Rakha et al. [2007]. Thus the Kelvin wave expansion performs well as far as it concerns amphidromic points, which is due to the fact that basic coefficients are directly picked up from field data. Figures 2 and 3 show the measured data near Kish Island and the computed values using the Kelvin wave expansion. A good agreement can be observed. The mean deviation between measured and computed water surface velocity is about 15 per cent better for the Kelvin wave expansion than for the full MIKE3-HD hydrodynamic model. Moreover, CPU time for the MIKE3-HD was found to be about 4–8 hours per run based on the simplest conditions in comparison with 2 minutes for the Kelvin wave expansion, which presents a successful speed-up (Badri et. al. [2010]). Next, a short investigation is done of the oil spill model for the Habash oil spill event. Figure 4 shows the oil slick trajectory for a period of approximately one week using the Kelvin wave expansion. The figure shows also the actual data for the Habash oil spill event and computed results based on the GULFSLIK model (El-Sabh et. al. [1988]). A fair agreement can be observed.

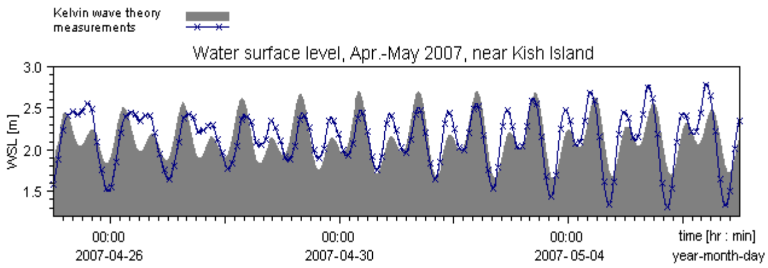

Figure 2. Measured and computed surface level at Kish Island

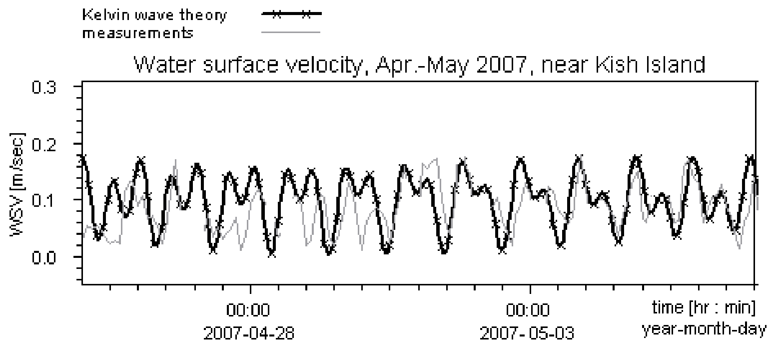

Figure 3. Measured and computed surface velocity at Kish Island

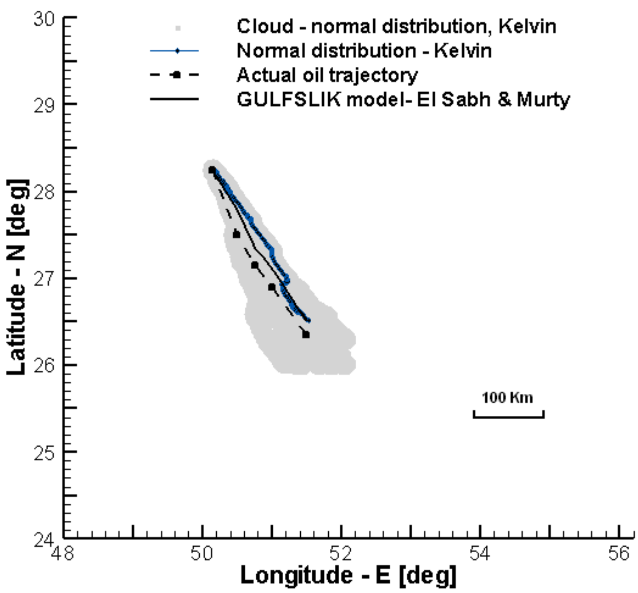

Figure 4. Comparison of the Habash oil spill traj., period of approximately 1 week

6. CONCLUSION

A flow estimation model has been presented to generate the dynamical field in the Persian Gulf. It has been revealed that this new hydrodynamic calibration approach, which has its roots in Kelvin wave theory, presents an alternative hydrodynamic model, such as MIKE3-HD. The improvement is not only towards an estimation of the flow pattern in a simple manner, but also shows a successful speed-up. The model is used for an application involving the prediction of the trajectory of spilled oil in the short term. The final outcome is that it has been established a simple estimation procedure for the flow pattern in order to provide oil spill hazard contour maps in the future.

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