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An Investigation of a Positive Engagement, Continuously Variable Transmission

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AN INVESTIGATION OF A POSITIVE ENGAGEMENT,

CONTINUOUSLY VARIABLE TRANSMISSION

by

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ABSTRACT

AN INVESTIGATION OF A POSITIVE ENGAGEMENT,

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Master of Science

A continuously variable transmission (CVT) is a type of transmission that allows an infinitely variable ratio change within a finite range, allowing the engine to continuously operate in an efficient or high performance range. A brief history of CVTs is presented, including the families under which they can be categorized. A new family of CVTs, with the classification of positive engagement, is presented. Three different published embodiments of CVTs of the positive engagement type are presented describing a meshing problem that exists apparently regardless of the embodiment in this family. The problem is called the non-integer tooth problem and its occurrences are detailed in each of the three embodiments. Specific solutions to the problem, as embodied in each case, are presented. The proposed embodiment of a new, positive engagement, continuouslyvariable transmission is described in detail with the derived general kinematic equations of its motion. The kinematic equations for two variant embodiments

are also derived. The results of the meshing analysis for this new embodiment are given and the non-integer tooth problem is exposed in three different operating conditions of the CVT. Characteristics of a solution to the non-integer tooth problem are then described, which are applicable to positive engagement family in general.

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CHAPTER 1 INTRODUCTION

The primary function of a transmission is to transmit mechanical power from a power source to some form of useful output device. Since the invention of the internal combustion engine, it has been the goal of transmission designers to develop more efficient methods of coupling the output of an engine to a load while allowing the engine to operate in its most efficient or highest power range. Conventional transmissions allow for the selection of discrete gear ratios, thus limiting the engine to providing maximum power or efficiency for limited ranges of output speed. Because the engine is forced to modulate its speed to provide continuously variable output from the transmission to the load, it operates much of the time in low power and low efficiency regimes. A continuously variable transmission (CVT) is a type of transmission, however, that allows an infinitely variable ratio change within a finite range, thereby allowing the engine to continuously operate in its most efficient or highest performance range, while the transmission provides a continuously variable output to the load.

The development of modern CVTs has generally focused on friction driven devices, such as those commonly used in off-road recreational vehicles, and recently in some automobiles. While these devices allow for the selection of a continuous range of transmission ratios, they are inherently inefficient. The reliance on friction to transmit power from the power source to the load is a source of power loss because some slipping is possible. This slipping is also a major contributor to wear, which occurs in these devices.

To overcome the limitations inherent in the current CVT embodiments employing friction, a conceptual, continuously variable, positive engagement embodiment has been proposed for investigation at Brigham Young University. This concept proposes utilizing constantly engaged gears which transmit power without relying on friction. Because the proposed embodiment is new, no engineering analysis has yet been performed to determine its kinematic and meshing characteristics, an understanding of which are necessary to validate the proposed concept as a viable embodiment. This research will investigate both the kinematic and meshing characteristics of this and related concepts.

The objective of this research is also to analyze the family of positive engagement CVTs. Although the CVT embodiment that has been proposed for investigation is new, other embodiments belonging to this family have been developed and published. The embodiments in this family do not rely on friction based power transmission. All embodiments in this family, however, have been based on overcoming a distinct problem which manifests itself seemingly regardless of the embodiment and will hereafter be referred to as the non-integer tooth problem. This research describes the nature of the non-integer tooth problem and details the occurrence of the problem in the proposed concept, as well as three published embodiments, and details solutions to the non-integer tooth problem as embodied in the three published embodiments. The presentation of some published solutions to the non-integer tooth problem clarifies the nature of the non-integer tooth problem, as well as aids in the development of characteristics of a general solution to the non-integer tooth problem applying to all members of the positive engagement CVT family.

Because the intention of this research is to provide greater understanding of the positive engagement CVT family, this research will not focus on the actual design of a positive engagement embodiment. The aim of this research is provide a foundation for future research involving the engineering design of functioning, efficient and robust positive engagement CVT embodiments.

This thesis follows the ensuing organization. Chapter 2 provides a broad review of current CVT designs. This includes a categorization of the types of CVTs and a brief explanation of the principles behind each type. This is presented as background and motivation for the ensuing work relative to the family of positive engagement CVTs, which is presented.

Chapter 3 introduces the new positive engagement CVT embodiment, which was proposed for investigation at Brigham Young University, along with two variants of this embodiment. The introduction to this new embodiment and its two variants includes a description of the operating principles of the embodiments, as well as a derivation of the kinematic equations governing the motion of the three embodiments.

Chapter 4 contains an examination of the family of positive engagement transmissions, focused specifically on the common problem encountered in the family - the non-integer tooth problem. The non-integer tooth problem is defined and is demonstrated in three published embodiments, which illustrate the variations in which the problem can be expressed. This chapter also presents several solutions to the non-integer tooth problem, as contained in the published embodiments, which aid in understanding the non-integer tooth problem, as well as establishing criteria for a solution, which is discussed in Chapter 6. It is important to note that the solutions that are presented from the published embodiments are not ideal solutions, as is be discussed in Chapter 4.

Chapter 5 builds on the understanding of the non-integer tooth problem presented in Chapter 4, and uses the kinematic equations generated in Chapter 3 to perform a meshing analysis of the new positive engagement CVT embodiment presented in Chapter 3. This chapter, based upon the meshing analysis that it describes, also classifies the conditions under which the non-integer tooth problem occurs in the new CVT embodiment.

Chapter 6 makes conclusions about the nature of the non-integer tooth problem based upon the analysis of the new CVT embodiment presented in Chapter 5, as well as the discussion of the non-integer tooth problem as presented in Chapter 4. Based on the nature of the non-integer tooth problem, Chapter 6 also presents the characteristics of a solution to the non-integer tooth problem, applicable to the new embodiment that is presented, as well as to the family of positive engagement CVTs.

Chapter 7 makes conclusion and recommendations for further research.

CHAPTER 2 BACKGROUND

Continuously variable transmissions have been in use for many years. Near the beginning of the twentieth century, cars like the Sturtevant, Cartercar, and Lambert featured friction dependent CVTs (Puttré, 1991). These friction drive CVTs were common in automotive use until engines capable of producing higher torques became common and necessitated the move to geared, fixed-ratio transmissions capable of high torque transfer and having better wear characteristics than friction dependent CVTs. Only in the past few years, with the advent of advanced materials and technology, have friction dependent CVTs returned to commercial application in the automotive industry.

To provide a foundation and motivation for the research presented, this chapter first presents a definition of a continuously variable transmission. For background purposes, a review of the current literature on CVTs is included. The families in which various embodiments can be classified are presented, along with a description of the operating principles in each family. A new family of embodiments of the positive engagement classification is also presented, along with the principles governing this new classification. This research focuses most heavily on embodiments in the final classification.

Definition and Terminology

A transmission is a device which allows the transmission of power from a rotating power source to a rotating load. Conventional transmissions allow for the selection of discrete gear ratios, thus limiting the engine to providing maximum power or efficiency for limited ranges of transmission output speed. A continuously variable transmission, however, is a type of transmission that allows an infinitely variable ratio change within a finite range, thereby allowing the engine to continuously operate in its most efficient or highest performance range.

Beachley and Frank, 1979, present a sub-classification of the continuously variable transmission called the infinitely variable transmission (IVT). While the two terms are often used interchangeably, there is a distinct difference between them. While a CVT allows an infinitely variable ratio change within a finite range, an IVT must be capable of producing an output speed of zero for any input speed, thus giving an infinite speed ratio.

CVT Classifications

There are several classifications of CVTs. The following five are most relevant to the current research: hydrostatic, friction, traction, variable geometry, and electric.

HYDROSTATIC

Hydrostatic transmissions are commonly used in off-road vehicles and agricultural machinery. Many commercial riding lawn mowers commonly employ hydrostatic transmissions in their drivetrains. These transmissions use high-pressure oil, commonly at pressures up to 5000 psi, to transmit power. They are composed of a hydraulic pump and hydraulic motor (see Figure 2.1), which are connected by hydraulic lines (not labeled in Figure 2.1). The hydraulic pump, which is generally driven by the engine, provides power to the hydraulic motor, in the form of high-pressure fluid. The hydraulic motor, in turn, converts the hydraulic power into mechanical power, which is transferred to a load.

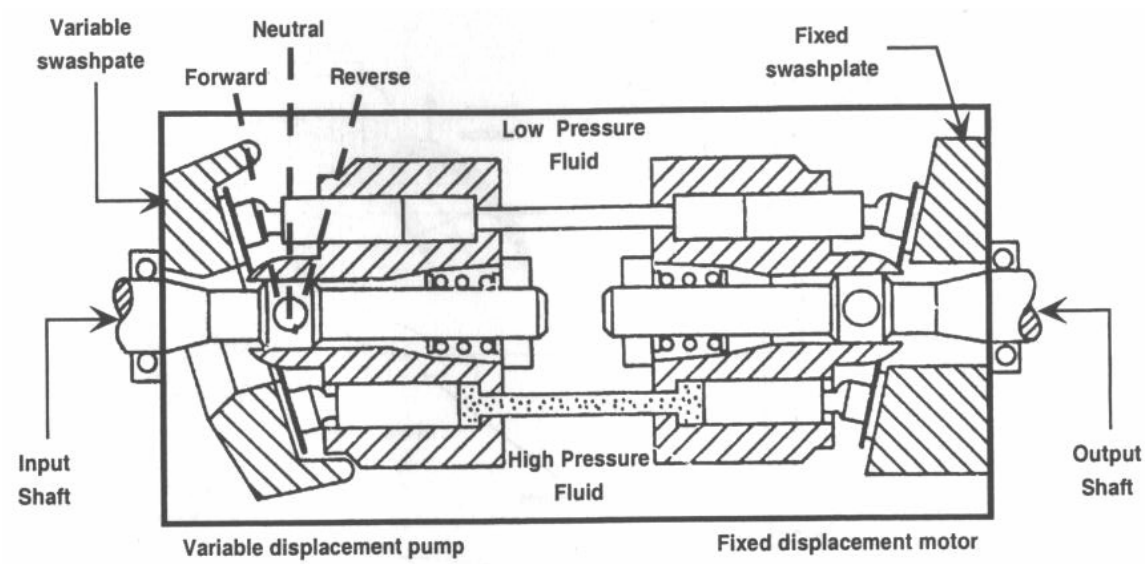


Figure 2.1: Typical Hydrostatic Transmission (Adapted from Beachley and Frank, 1979)

The continuously variable nature of this transmission comes in the ability of the hydraulic pump to adjust the pressure and flow of hydraulic fluid that it supplies to the hydraulic motor by changing its displacement. Hydrostatic transmissions will almost always have a ratio range of infinity, i.e., be IVT's. This is accomplished because the stroke of the pump can be varied from zero to its maximum. Also, because the stroke of the pump can generally be reversed, the hydraulic motor can have both positive and negative rotation, thus providing forward and reverse rotations of the output.

An advantage of the hydrostatic transmission is the ability that it has to transmit high torque from the input to the output, which allows for its application in a wide range of devices. This is enhanced by the ability hydrostatic transmissions have for precise speed control. One major disadvantage of hydrostatic CVTs is their moderate efficiency (between 60 and 80%), which offsets the efficiency gains of allowing the engine to operate in its most efficient regime.

FRICTION

The friction CVT is one of the most common forms of CVTs in use today. These CVTs are characterized by the use of friction to transmit power. Traction drives use a form of friction to transmit power, but are classified separately and will be discussed later. In the friction CVT family, there are several different embodiments. These include rubber V-belts, metallic V-belts, flat rubber belts and chain drives.

The common characteristic of the V-belt drives is the use of a drive and driven sheave, each with variable diameters. The effective diameter of the sheave is adjusted by varying the distance between the two halves of the sheave (see Figure 2.2). Each sheave consists of one mobile and one stationary half, and the two sheaves are positioned at a fixed center distance. As the halves of the sheave move together, the belt is forced up to a larger diameter on the sheave. As the halves of the sheave move apart, the belt returns to a smaller diameter. The ability to continuously vary the diameter of the drive and driven sheaves allows for a continuously varying transmission ratio.

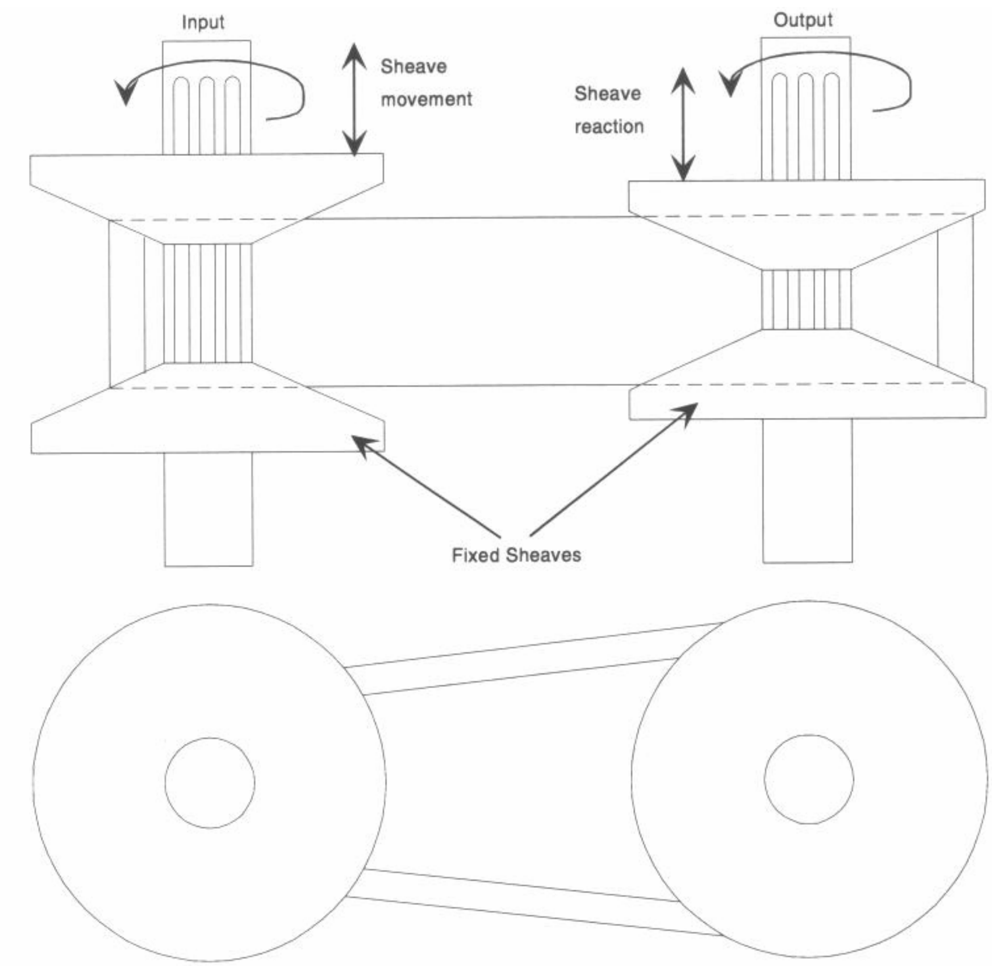


Figure 2.2: Typical Rubber V-belt CVT Configuration

The sheave diameters can be varied in several ways, depending on the type of control desired and the ratio range needed. Figure 2.3 shows a common CVT used in snowmobile and ATV applications. It consists of two sheaves, referred to as the driver or primary clutch, and the driven or secondary clutch, and a composite v-belt. In this application, the control of the CVT is automatic. The primary clutch is actuated by engine rotation, using centrifugal force on flyweights that produce an axial force on the mobile half of the sheave, causing it to move toward the stationary half of the sheave. The secondary sheave is referred to as a torque sensing sheave, and is spring loaded to maintain proper belt tension.

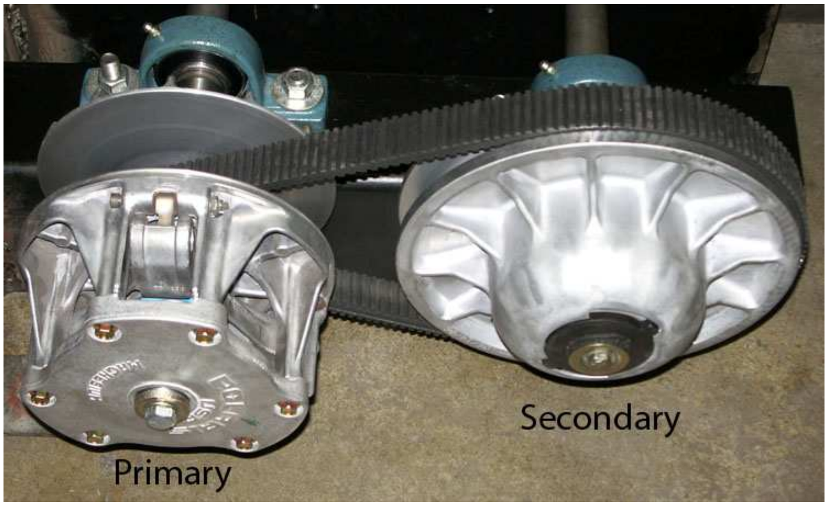


Figure 2.3: Typical ATV or Snowmobile CVT

Rubber V-belt CVTs are also commonly used in machine tools. The control in this case, however, is a mechanical system that determines the spacing of the two halves of one of the sheaves. Because the belt length remains constant, the second pulley must be spring loaded, allowing it to adjust automatically.

It is common for slipping to occur in both rubber V-belt CVT applications presented. This is because the driving force is transmitted through friction between the sides of the V-belt and the inside surfaces of the sheaves. While this negatively affects efficiency, it can have a positive safety effect in machine tools, especially when the machine becomes overloaded.

An advantage of the rubber v-belt CVT is the high ratio range that it can provide, as well as the ability for automatic speed control, which is what makes it so desirable for use in ATVs where an expensive control system is not desirable. Some disadvantages of this type of CVT are its low torque capability and the significant wear that develops due to belt slipping. This wear inhibits the ability of the CVT to shift ratios properly. Belt slipping also contributes to the moderate efficiency of the device, which is usually between 70% and 80%.

Another common belt-type CVT is the metal push belt CVT. This belt driven CVT is different from the previously mentioned rubber belt versions in that power is transmitted through the belt by way of compression. The first company to commercially develop this concept was Van Doorne Transmisse. This metal push belt CVT can transmit more force, and therefore is better suited to the automotive industry. Figure 2.4 shows the XTRONIC CVT, developed by Nissan, which employs a metal push belt.

Figure 2.4: Nissan XTRONIC CVT

The construction of the metal push belt is shown in Figure 2.5. The belt consists of thin, high-strength, segmented steel blocks that are held together by stacked bands of steel. The bands are stacked into slots on both sides of the blocks, and help maintain the shape of the belt as it passes through the sheaves. Kluger and Fussner, 1997, stated that the load path is dependent on the complex interaction and friction between the bands and block slots, the adjacent blocks, and the block sidewalls and the faces of the sheaves.

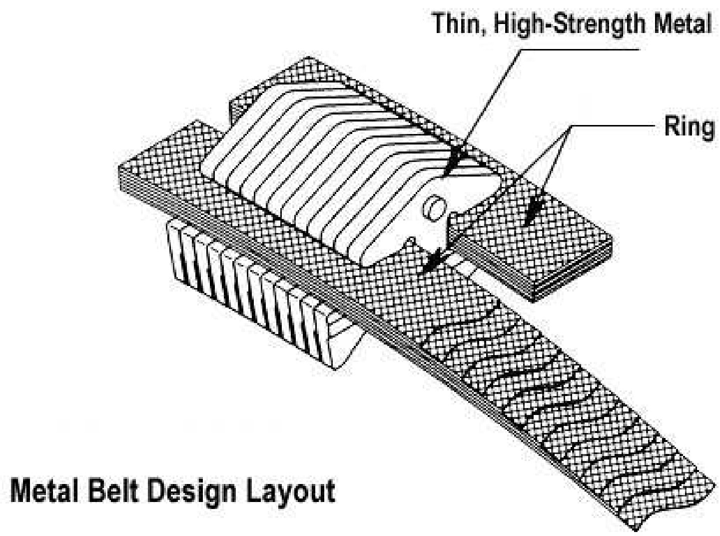


Figure 2.5: Metal Push Belt Design Layout (Taken from http://www.insightcentral.net/ encyclopedia/encvt.html, March, 2007)

The advantage of the metal push belt over the rubber v-belt is its ability to transmit higher torque, usually up to 350 N-m, which, as stated previously, makes it more useful in higher torque situations, like in automobiles. It is also more efficient - between 80% and 90% - than the rubber v-belt, due to the reduced amount of slipping that it allows. A disadvantage of the metal push belt CVT is the high contact stresses in the sheaves, which requires special materials and special controls to minimize belt slip, which would otherwise rapidly wear the sheaves.

A third type of friction CVT is the flat belt CVT. Kluger and Fussner, 1997, state that flat belts are more efficient for transmitting power because more of the allowable belt tension can be used for transmitting power rather than producing belt to sheave forces. Developed originally by Kumm Industries, the flat belt CVT is composed of a flat elastomer belt and two pulleys. The two pulleys are composed of two guideway discs on each side. These guideway discs have logarithmic spiral guideway slots which support the ends of the belt drive elements. The set of guideways in one disc have clockwise curvature and the slots in the opposing disc have counterclockwise curvature (see Figure 2.6).

Actuation and control of the flat belt CVT is accomplished by means of a hydraulic actuator in each of the two pulleys. This actuator rotates the inner set of discs of each pulley relative to the outer set of discs. This causes the belt drive elements to be positioned at a desired diameter (see Figure 2.7). Pressure is set in the hydraulic actuator to generate the required belt tension at the desired speed ratio.

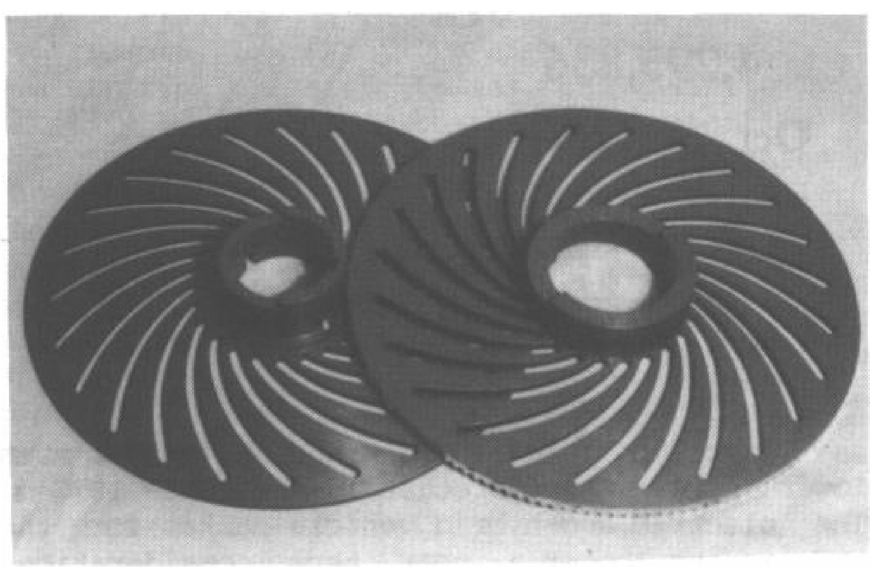


Figure 2.6: Guideway Discs (From Kumm and Kraver, 1985)

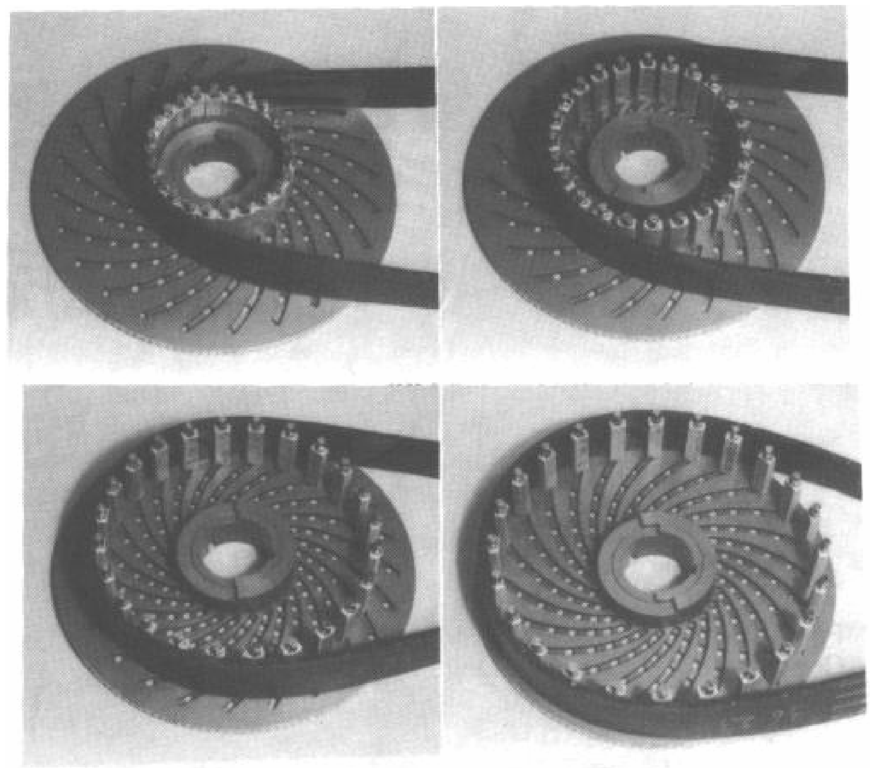


Figure 2.7: Radial Positioning of Belt by Drive Elements in Guideway Slots (From Kumm and Kraver, 1985)

Like the rubber v-belt and metal push belt CVTs, the flat belt CVT is capable of providing a high ratio range, but its efficiency approaches 95%, which is higher than the other belt type CVTs. One disadvantage that flat belt CVTs have is that they require complex controls to maintain belt tension, but when belt tension is maintained properly, there is little wear in the CVT and its torque transmission capability approaches 450 N-m.

The final friction CVT that will be considered in this work is the steel chain drive CVT. The steel chain drive CVT, also referred to as the PIV chain drive, is similar to the rubber V-belt CVT. It also contains a drive and driven sheave composed of a stationary and mobile half, which halves are moved relative to each other to adjust the effective diameter of the sheaves. Power is transmitted from the drive to the driven sheaves through a steel chain, which like the rubber V-belt, transmits power in tension. Figure 2.8 shows an example of the pulleys and chain used in this transmission.

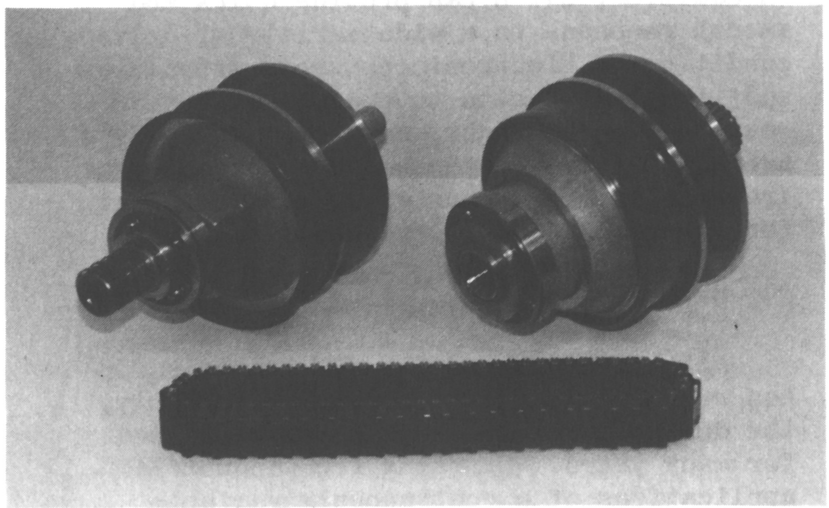


Figure 2.8: Example of PIV Steel Chain Drive CVT (Taken from Avramadis, 1986)

Because the contact between the sheaves and chain is metal-to-metal, special precautions must be taken to ensure the elimination of slipping. Any slipping would reduce power transmission efficiency and accelerate wear. Since force is transmitted from the sheaves to the chain through compressive forces on the sides of the chain, a special hydraulic-mechanical torque sensing device must be employed to regulate the clamping force, especially during torque spikes (Avramadis, 1986).

When chain slipping is eliminated, the steel chain drive CVT is about 90% efficient, and is capable of providing a high ratio range. A disadvantage that the chain drive CVT has is the noise it generates due to the cyclic interaction of successive links of the chain with the sheaves.

TRACTION

Traction drives were one of the earliest forms of CVT concepts ever developed. A traction drive is a transmission that transmits power through rolling contact. The 1906 Cartercar, powered with a 12-hp engine, was developed with just such a transmission. Many current applications employ traction drives. These include applications such as machine tools, low-power yard equipment, and recently some automotive applications. Figure 2.9 shows several transmissions of the traction type.

There are some discrepancies in the definition of traction drives. Some authors, such as Beachley and Frank (1979), and Chana (1986), categorize V-belt drives and traction drives together in the friction drive category. Hewko (1986), however, and Singh and Nair (1992), categorize them separately. For the purposes of this work, we will consider the definition of a traction drive presented by Hewko (1986). A generic traction drive is "a power transmission device which utilizes hardened, metallic, rolling bodies for transmittal of power through an elastohydrodynamic fluid film." This definition distinguishes traction drives from variable-sheave drives because sheave contact is static, while traction drives employ rolling contact. This means that sheave contact does not exhibit significant elastohydrodynamic fluid film phenomena.

In properly designed traction drives, power is transferred from the driving roller to the driven roller through the shearing of the fluid film between them, not through body-to-body contact. This happens because the contact between the rolling bodies, which generally happens over a finite area in the shape of an ellipse, traps the fluid and subjects it to extreme compressive stress, usually on the order of 100,000 to 500,000 psi. This extreme stress increases the instantaneous viscosity of the fluid by several orders of magnitude, thereby increasing its shear strength. It is the shear strength of the fluid at this increased viscosity that determines the amount of torque that can be transmitted.

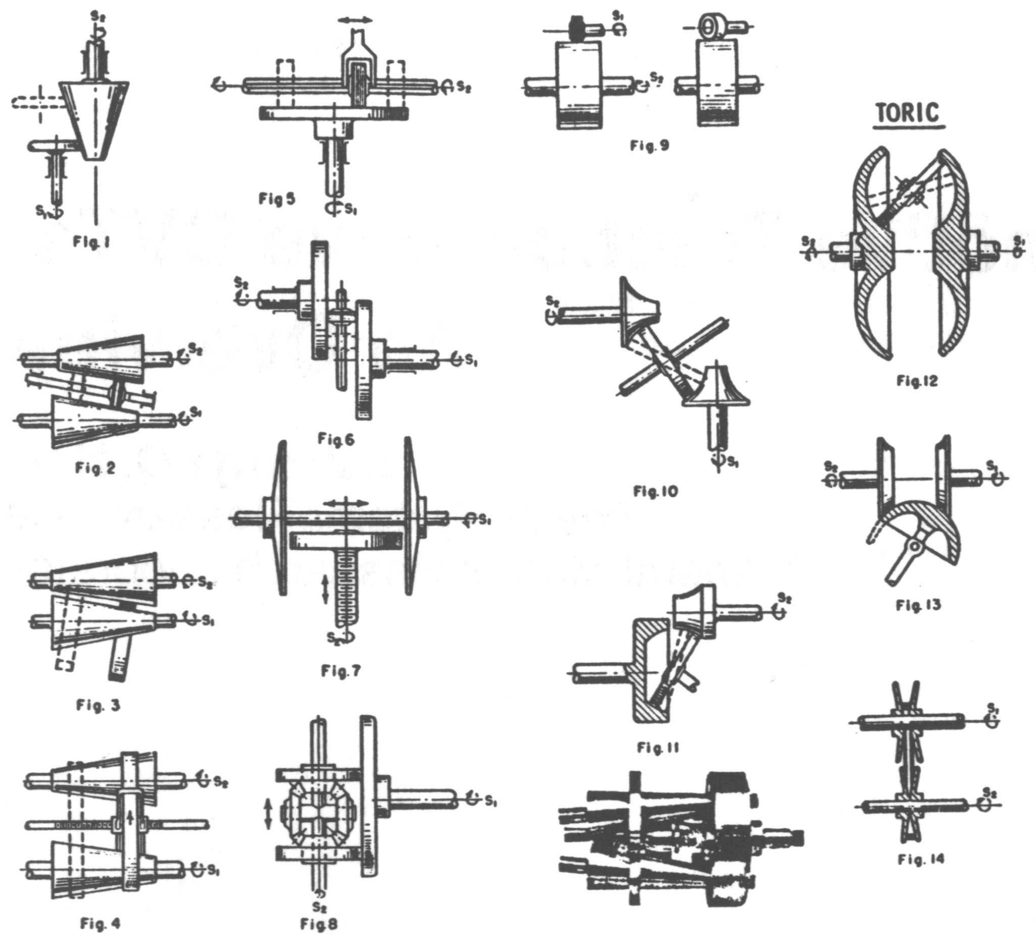


Figure 2.9: Examples of Traction Drive CVTs (Taken from Loewenthal, 1983)

The toric CVT (one form of traction drive) has been the subject of extensive research in order to adapt it to automotive applications. Figure 2.10 illustrates a single toric drive. In this arrangement, one race serves as the input and the other race as the output. To transmit rotation and torque from the input race to the output race, three rolling discs are placed between them in the toroidal cavity at 120° intervals. To ensure proper contact between the races and the rollers, the outside diameter of the rollers must be equal to the transverse diameter of the torus, while the center of the rollers are located on its pitch diameter (Hewko, 1986). The ratio, in the arrangement shown in Figure 2.10, is adjusted by changing the inclination of the rollers in the toroidal cavity.

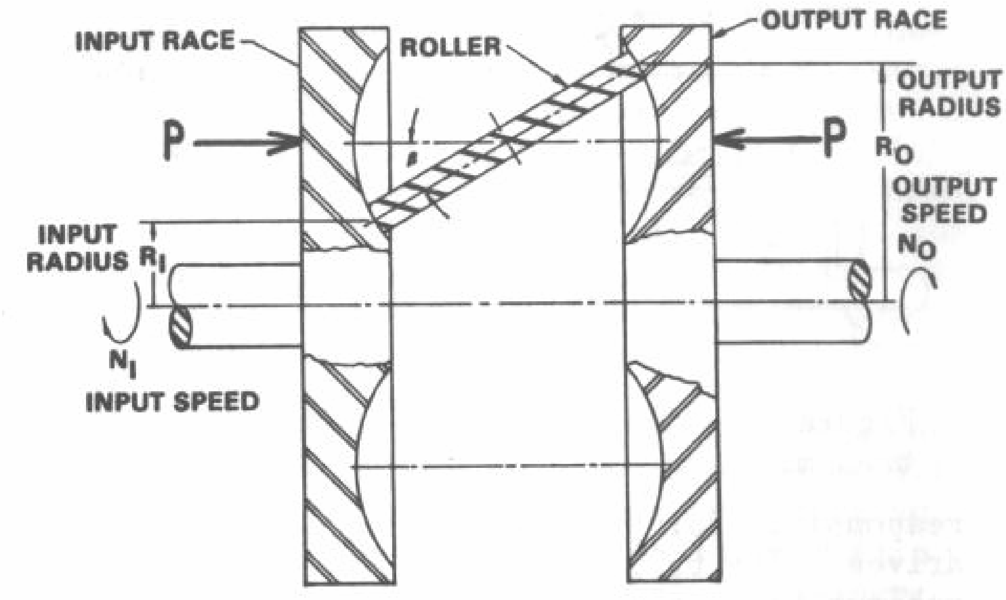


Figure 2.10: Typical Single Unit Toric Drive (Taken from Hewko, 1986)

Traction drive CVTs have become more common in automotive applications during recent years due to their ability to transmit a moderate amount of torque and their good efficiency (between 85 and 90%). Some major disadvantages they have over belt type designs, however, are their higher weight and size, as well as their complexity. They require high dimensional precision to maintain proper contact, and also require special lubricants and high quality materials to resist the contact stresses they generate.

Variable Geometry

Variable geometry describes a group of CVTs that use epicyclic motion and the ability to change the mechanism geometry to continuously vary the speed ratio of the transmission. These devices usually generate some form of oscillatory output for a constant input velocity, and thus employ one-way, or overrunning, clutches to correct the negative portion of the oscillatory movement (Benitez et al., 2004). An overrunning clutch is a device that allows torque to be transmitted in one direction, but freewheels if torque is applied in the opposite direction.

One of the simpler variable geometry CVTs, shown in Figure 2.11, uses overrunning clutches and kinematic linkages as mechanical diodes. This device is called the Zero-Max, and would normally be constructed of six to eight of the linkages shown in Figure 2.11. Each of the links is connected to the input shaft through an eccentric. This causes the power link to oscillate, which motion is transferred to the output shaft through the overrunning clutch. The magnitude of the oscillation, and thus the output rotation, is adjusted by moving point A on the control link.

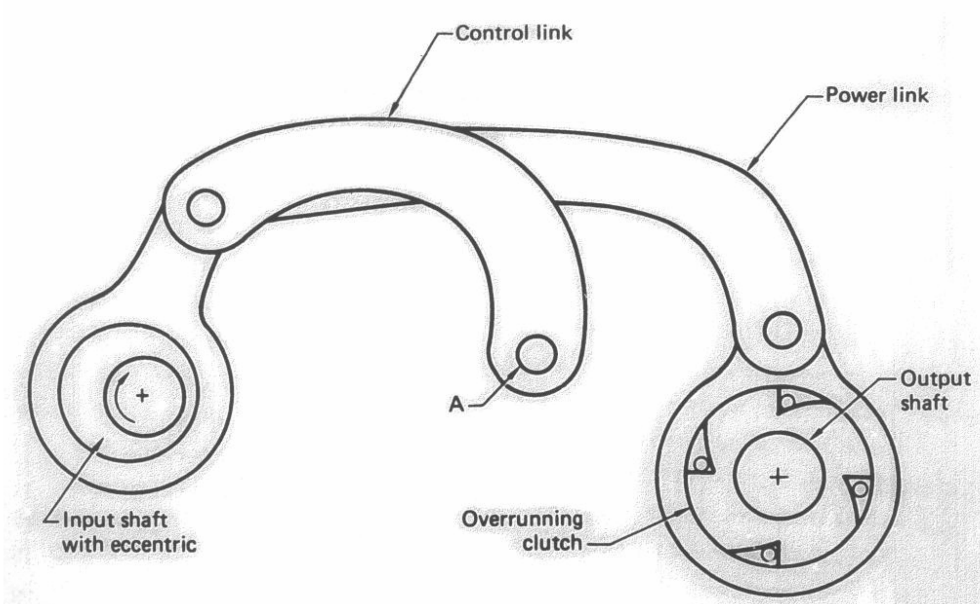


Figure 2.11: Single Linkage of the Zero-Max CVT (Taken from Beachley and Frank, 1979)

Another CVT that operates under similar principles is that developed by Benitez et al, 2004. This transmission is actually classified as an IVT because of its ability to obtain a speed ratio of zero. Figure 2.12 show the design of the IVT. This device is composed of two subsystems: the variator unit and the differential unit. The variator unit is responsible for the variation of the transmission ratio, and is composed of the input shaft, a control plate, five main planet shafts and a sun gear. The planet gears are mounted through overrunning clutches on the planet shafts. The planet shafts are held in position and caused to orbit the sun gear by the control plate, and the planet gears are maintained in mesh with the sun gear.

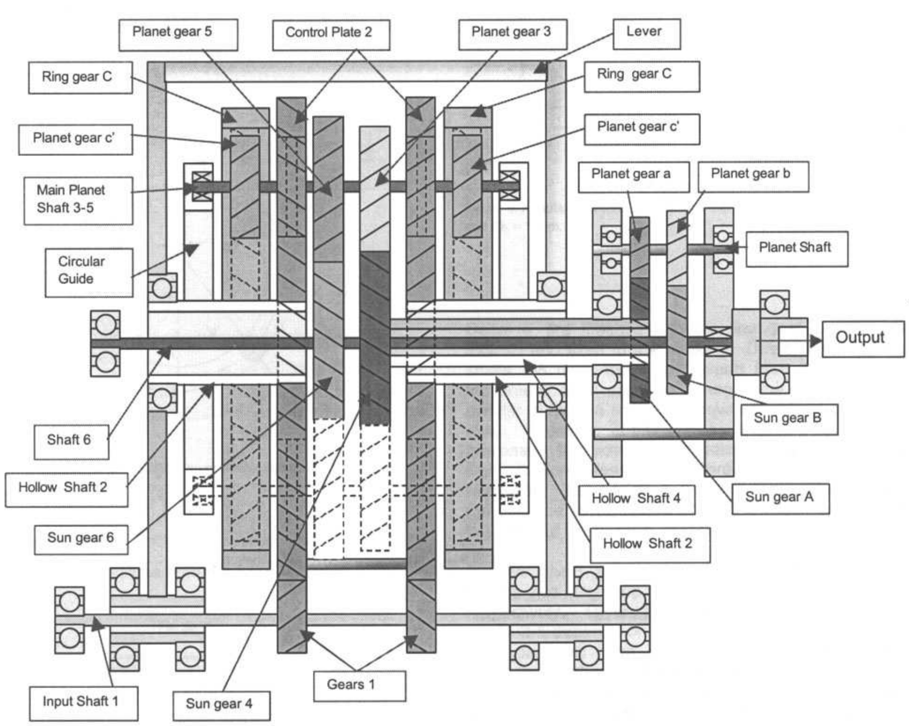


Figure 2.12: IVT Design (Taken from Benitez et al., 2002)

To enable the ability to adjust the ratio of the transmission, the control plate is able to shift radially relative to the sun gear. When the axes of the control plate and the sun gear are collinear, the circumferential spacing of the planet gears about the sun gear is equal and the angular velocities of their orbits about the sun gear are equal. When the control plate is shifted radially, relative to the sun gear, and the planets maintain mesh with the sun gear, the circumferential spacing of the planet gears about the sun gear is not equal, and the angular velocities of their orbits about the sun gear are not equal. This is shown in Figure 2.13. In this arrangement, only the planet gear whose orbit has the highest angular velocity about the sun gear transmits torque to the sun gear. The other planet gears freewheel by means of the overrunning clutch.

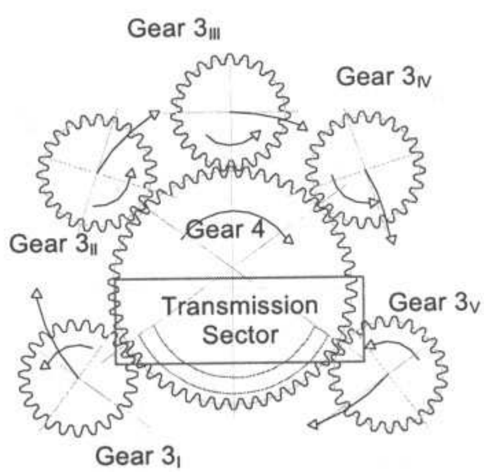


Figure 2.13: Angular Section of Torque Transmission (Taken from Benitez et al., 2002)

In the device developed by Benitez et al., as seen in Figure 2.12, there are two variator units, whose outputs are their respective sun gears. These sun gears function as the inputs to the second subsystem of the transmission, the planetary differential. The differential compounds the two inputs from the sun gears and produces a final output.

A major advantage of variable geometry CVTs is that they provide positive engagement of the input and output, which translates into higher torque capabilities than any other CVT design. This also allows for good efficiencies, approaching 95%. The major disadvantage of the variable geometry CVT is the oscillating output that it produces, which greatly limits the applications in which it can be used.

ELECTRIC

The electric motor combination shown in Figure 2.14 creates a CVT that is analogous to the hydrostatic CVT. The generator converts mechanical power in the form of rotational velocity and torque to electrical power in the form of voltage and current. This electrical power is passed through the control circuitry and then fed to the electric motor, which converts the electrical power back to mechanical power. In this arrangement, because there is no rigid connection between the generator and motor, the speed ratio from input to output can be continuously varied. Also, it is possible to achieve a ratio of infinity, making this arrangement an IVT.

A major disadvantage of the electric CVT design is its inefficiency at speeds other than the motor's design speed, which occurs because DC motors operate most efficiently over a narrow range of speeds. It is, however, able to transmit high torques, as demonstrated by its use in diesel locomotives, and is capable of precise speed control.

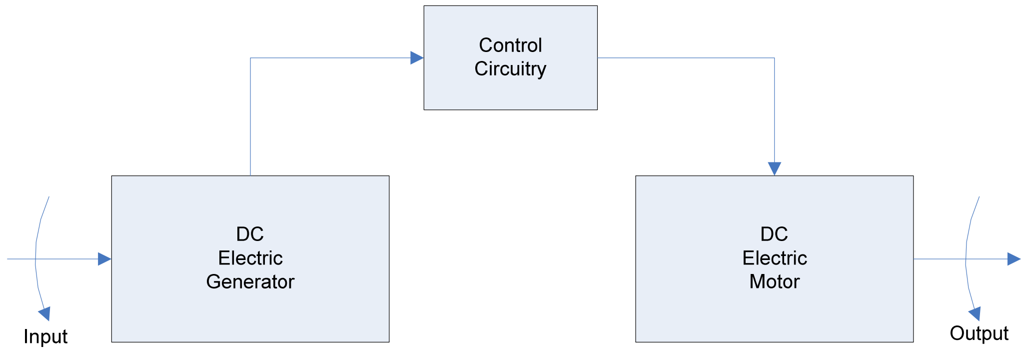


Figure 2.14: Example of a Simple Electric CVT

CVT Comparison

Table 2.1 shows a comparison of the previously described transmission types based on five important characteristics describing their performance under normal operating conditions. It is significant to note that the variable geometry type transmission is the only one of the types presented that has an oscillating output, which is generally undesirable in transmissions.

Table 2.1: CVT Comparison Chart

Transmission Type	Characteristics				
	Torque Capability	**Wear**	**Output**	**Complexity**	**Ratio Range**
Hydrostatic	High	Low	Non-Oscillating	Low	High
Friction	Low	High	Non-Oscillating	Low	Moderate
Traction	Moderate	Moderate	Non-Oscillating	Low/Moderate	High
Variable Geometry	Moderate	Low	Oscillating	Moderate	Moderate
Electric	High	Low	Non-Oscillating	Low	High

Table 2.2 shows a comparison of the advantages and disadvantages of the CVT types described in this chapter. It is interesting to note that the modern development of CVTs for automotive use has focused on metal push belt CVTs and traction drive CVTs.

Table 2.2: Comparison of CVT Advantages and Disadvantages

Transmission Type	Advantages	Disadvantages
Hydrostatic CVT	High Torque Transmission	Moderate Efficiency (60-80%)
	Precise Speed Control	
Rubber V-Belt CVT	Good Ratio Range	Moderate Efficiency (70-80%)
	Automatic Speed Control	Low Torque Capability
		Significant Wear
Metal Push Belt CVT	Good Ratio Range	High Contact Stresses in Sheaves
	Moderate Efficiency (80-90%)	Limited Torque Capability (Max 350 N-m)
	Highest Commercially Available Torque Capability	Highly Sensitive to Wear
		Requires Special Controls to Limit Belt Slip During Torque Spikes
Flat Belt CVT	High Ratio Range	Limited Torque Capability (~ 450 N-m)
	Good Efficiency (90-95%)	Requires Special Controls to Maintain Belt Tension
	Long Belt Life	
	Better Torque Capability Than V-belt CVTs	
Steel Chain Drive CVT	High Ratio Range	High Contact Forces at Sheave
	Good Efficiency (90%)	Higher Noise than Belt Drive CVTs
		Highly Sensitive to Wear
Traction Drive CVT	High Ratio Range	High Contact Forces Between Elements
	Good Efficiency (85-95%)	Higher Weight and Size than Other Designs
	High Rate of Ratio Change	High Dimensional Precision
	High Torsional Damping	Requires Special Lubricants
		Requires Special High Quality Steel
		Highly Sensitive to Wear
Variable Geometry CVT	Positive Engagement of Input and Output	Oscillating Output
	High Torque Capability	Rely on One-Way Clutches
	Good Efficiency (90-95%)	
Electric CVT	High Torque Capability	Complex Control System
	Precise Speed Control	Inefficient Power Transmission

POSITIVE ENGAGEMENT CVT

Positive engagement describes a family of CVTs that couple the input power source and the output in a positive manner, as occurs in a simple gear pair found in a positive engagement, discrete ratio transmission. While this is frequently the case in embodiments in the variable geometry CVT family, positive engagement CVTs do not generate an oscillatory output, which is the major distinction between the two families. Because families in this embodiment have high torque capabilities due to the positive engagement of input and output, and also have high efficiency due to not relying on friction for power transmission (as is done in many CVTs), positive engagement CVTs are the ideal CVT. Figure 2.15 shows that the positive engagement transmission is the intersection of the positive engagement transmission with the continuously variable transmission, thereby providing the benefits of both classes of transmission. The embodiments presented in the ensuing chapters are classified in the positive engagement CVT family.

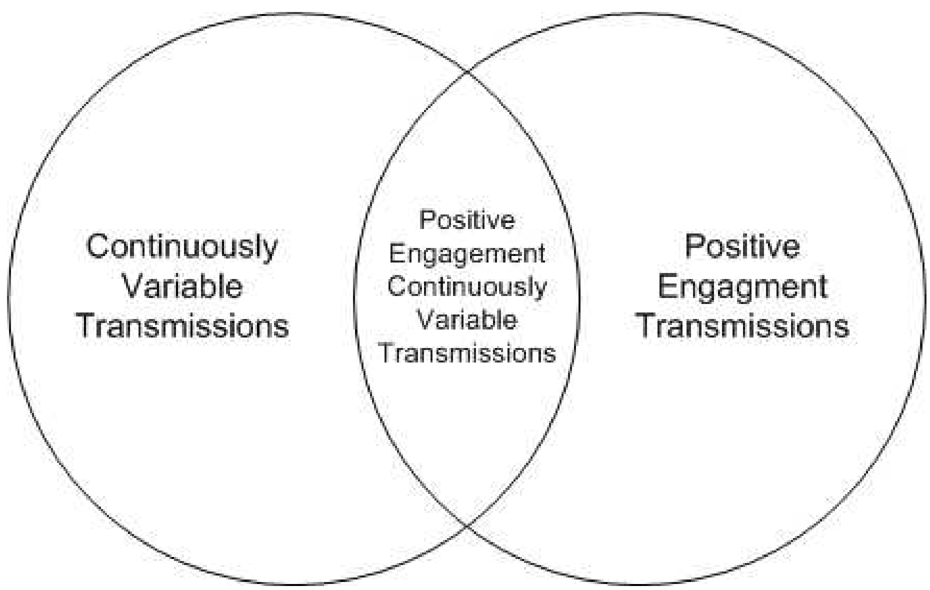


Figure 2.15: Transmission Family Intersections

It is important to note that while a positive engagement CVT would have higher power transmission efficiency than a friction dependent CVT, the calculation of this efficiency is only for the operation of the transmission in a power transmitting ratio. It is not the overall efficiency, which would incorporate losses during the starting of the power source, which may require disconnecting the transmission from the power source through a clutch. Also, while standard positive engagement transmissions are highly efficient at transmitting power, their overall efficiency is actually lower than the positive engagement CVT would be due to power losses during shifting.

PLANETARY GEAR TRAIN

Because the new PECVT embodiments that will be presented and analyzed in Chapter 3 are similar to planetary gear trains, it is helpful to first present a discussion of common planetary gear trains. A planetary gear train, as shown in Figure 2.16, consists of a central sun gear, a carrier arm which supports several planet gears in mesh with and rotation about the sun gear, and an outer ring gear which meshes with the planets.

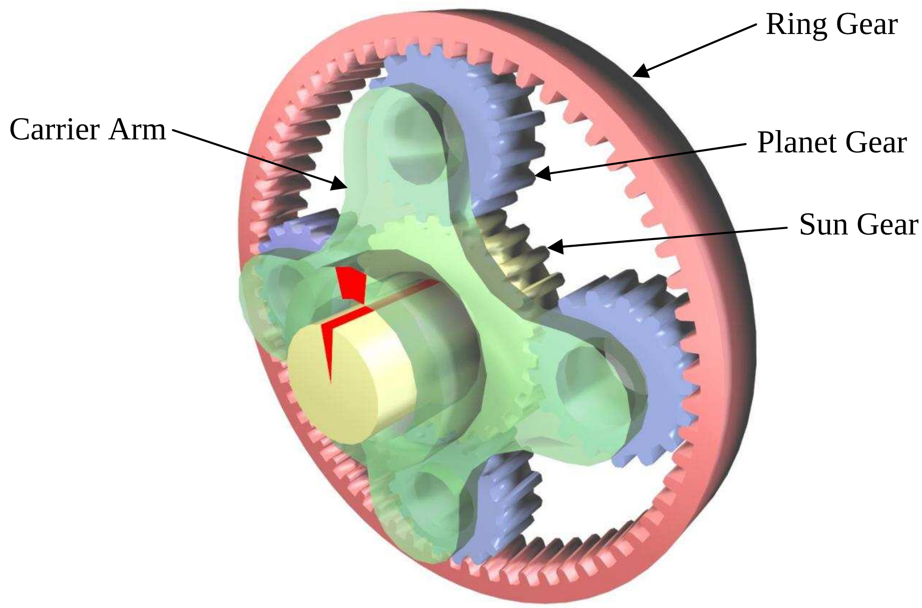


Figure 2.16: Planetary Gear Train (Taken from http://en.wikipedia.org/wiki/Image: Epicyclic_ gear_ratios.png, June, 2007)

The gear ratio of a planetary gear train is somewhat non-intuitive, and is dependent upon the relationship of two inputs to the gear train. Because a planetary gear train has two degrees of freedom, it requires two inputs to produce a single output. It is often the case that one of the inputs involves holding either the sun gear, carrier arm, or the ring gear stationary. The following two equations are the general equations for calculating the transmission ratio for a planetary gear train:

$$(2+n)\omega_a + n\omega_s - 2(1+n)\omega_c = 0 \tag{2.1}$$

$$N_s + 2N_p = N_a \tag{2.2}$$

Where:

ω_a = Ring Gear RPM
ω_s = Sun Gear RPM
ω_c = Carrier RPM
$n = N_s / N_p$
N_s = Number of Teeth on the Sun Gear
N_p = Number of Teeth on the Planet Gear
N_a = Number of Teeth on the Ring Gear

It is important to note the significance of Equation 2.2. Because gears can only have integer numbers of teeth, there are only discrete sizes of planetary gear trains that are possible.

While it is common for one of the inputs to the planetary gear train to be stationary, it is also possible to provide two rotational inputs to the train to get a combined output. This application is discussed in the following section.

CVT Applications

While continuously variable transmissions have many desirable characteristics, current CVT configurations can be very complex and costly. Also, some of the configurations presented can have undesirably low efficiencies, like the hydrostatic and electric CVT designs.

In order to combat the low efficiencies in some CVT configurations, and to gain the benefits of a positive engagement CVT using CVTs from other classifications, many researchers have begun exploring the power-split principle. The benefit of this principle is a gain in efficiency by passing only some of the power through the continuously variable unit. The remainder of the power is passed through a fixed ratio mechanical unit (like a planetary gear train) with high efficiency, and combined with the variable input from the continuously variable unit. Figure 2.17 shows a schematic of such a device, where the continuously variable unit is the hydrostatic transmission discussed previously. Power from the engine is used to turn the hydrostatic unit, which produces a continuously variable output. Power from the engine is also supplied directly to the mechanical differential, which combines this input with the continuously variable input that it receives from the hydrostatic unit to produce a continuously variable output. This embodiment has been employed commercially by John Deere (see Figure 2.18) to overcome the inherent inefficiencies in the hydrostatic unit. Although efficiency is gained through this approach, a reduction in ratio range occurs.

The application of continuously variable transmissions in many fields of power transmission is rapidly expanding as continuing research improves their functionality and efficiency. The application of CVTs, however, in high torque situations is still not common. This is due to the reliance on friction in most embodiments as a means of power transfer. The CVT that has been proposed for investigation at Brigham Young University, as well as the published embodiments of the positive engagement CVT family that will be described later, are an attempt to produce a CVT that does not rely on friction for power transfer, but instead provides a mechanical gear train with positive engagement of the input and output of the drive. Such a transmission would provide for high torque transfer in a highly efficient, continuously variable manner, which would provide ideal power transmission.

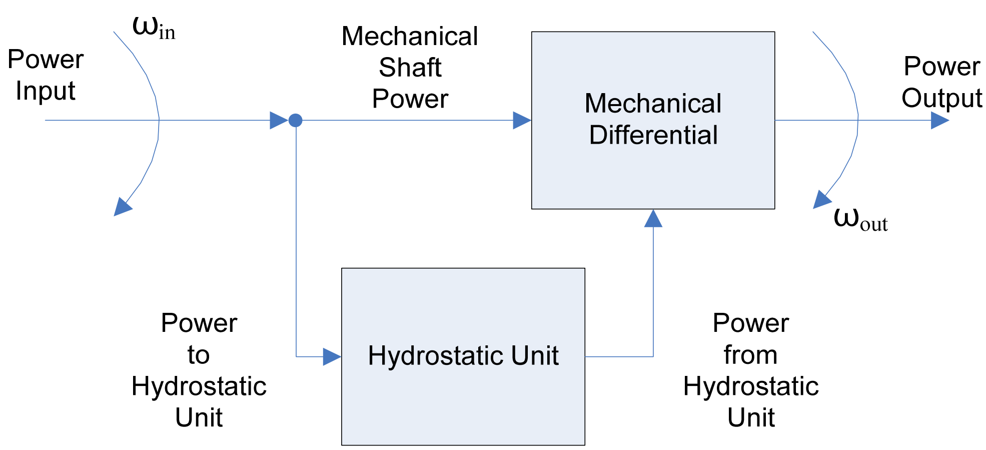


Figure 2.17: Schematic of the Power Split Principle

Figure 2.18: John Deere IVT (Taken from http://www.deere.com/en_AU/equipment/ag/tractors /8030_series/transmission.html)

CHAPTER 3 NEW CVT CONCEPT

ORIGINAL EMBODIMENT

Figure 3.1 shows the general embodiment of the proposed new continuously variable transmission. The embodiment (hereafter referred to as embodiment 1) consists of a central reference gear (A) whose axis is co-axial with the major axis of the transmission. The reference gear (A) also acts as the output of the transmission. An input arm, or drive gear carrier (B), is connected to the axis of the reference gear (A), allowing it to rotate around the axis of, and relative to, the reference gear (A). The input arm (B) is the input to the transmission from an external power source. Connected to the input arm is a drive gear (C), which the input arm (B) causes to orbit about the reference gear (A). The drive gear (C) is connected to the reference gear (A) through a gear pair relationship, which means that the rotation of the drive gear (C) about its axis has a fixed relationship to the rotation of the reference gear about its axis (this would be accomplished through a gear set between the reference gear (A) and the drive gear (C), as represented by the idler gears in Figure 3.1). The idler gears shown in Figure 3.1 also show the rotational direction relationships between each of the labeled gears, but do not show how a gear set between the reference gear (A) and the driven gear (E) would actually be arranged (ensuing figures do not show the idler gears to simplify the illustrations). Also connected to the axis of the reference gear (A) is a stationary arm (D), which remains fixed (does not rotate) and supports a driven gear (E). The reference gear (A) rotates relative to this stationary arm (D). The driven gear (E) is also connected to the reference gear (A) through a gear pair relationship (represented by the idler gears shown in the Figure 3.1), in the same way that the drive gear is connected to the reference gear, as described above.

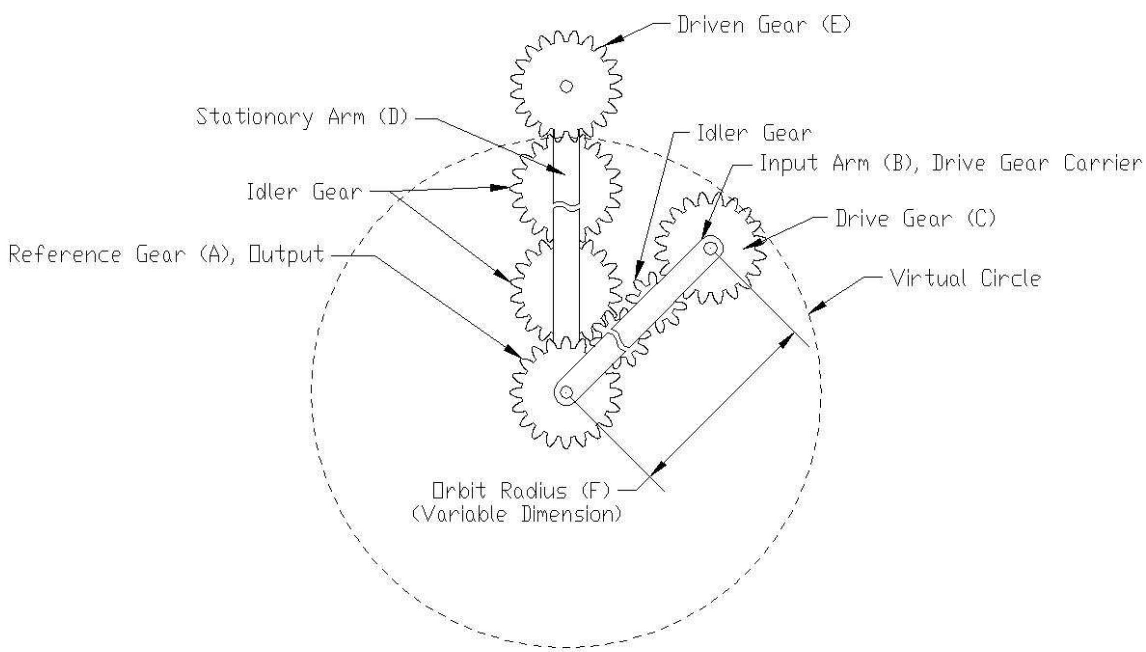


Figure 3.1: Basic Embodiment of the Proposed CVT

When the input arm (B) is rotating about the axis of the reference gear (A), the drive gear (C) *orbits* around the reference gear (A) at an angular velocity equal to that of the input arm (B), and hence the input of the transmission. This orbiting motion also causes the drive gear (C) to *rotate* about its own axis in the opposite direction of the orbit motion, given that the reference (output) gear (A) is rotating in the same direction as, and faster than the input. The angular velocity at which the drive gear (C) rotates, relative to its orbit, is also dependent on the gear ratio of the gear set connecting the reference gear (A) and the drive gear (C).

The drive gear (C) shown in Figure 3.1 connects the input portion of the transmission to the driven portion of the transmission. This is accomplished as the drive gear (C) orbits past and meshes with the driven gear (E). The contact and meshing of the drive (C) and driven (E) gears are what cause rotation of the reference (output) gear (A), again through a gear set (not shown) between the drive gear (E) and the reference or output gear (A).

The orbit radius (F) shown in Figure 3.1 is the controlling dimension of the transmission. As the drive gear (C) orbits the reference gear (A) at a certain orbit radius (F), it traces out a virtual circle, as seen in Figure 3.1. As the orbit radius (F) is varied, the diameter of the virtual circle is also changed to remain tangent to the drive gear (C). The radius at which the driven gear (E) is held is also adjusted as the orbit radius (F) of the drive gear (C) changes such that the driven gear (E) remains tangent to the virtual circle.

Because the drive gear (C) has an orbiting angular velocity in one direction and a rotational angular velocity in the opposite direction, the resulting tangential (pitch line) velocity of the drive gear (C) at the point of tangency with the virtual circle, relative to the central axis of the transmission, is dependent upon the orbit radius (F) noted in Figure

3.1. As the orbit radius is varied, the resulting pitch line velocity can vary at values greater than the input. Because the drive gear (C) must mesh with the driven gear (E), the driven gear must have a pitch line velocity equal to the resultant pitch line velocity of the drive gear (C) at the virtual circle.

While Figure 3.1 shows only one driven and one drive gear, an embodiment with only one driven and one drive gear would not maintain constant engagement. This is because the drive and driven gears would only maintain engagement for a short portion of the drive gear's orbit. The functioning embodiment would thus be composed of a plurality of both drive and driven gears. The embodiment could consist of an equal number of drive and driven gears (see Figure 3.2), or a differing number of drive and driven gears in a so called "Vernier relationship" (see Figure 3.3). The advantage of the Vernier relationship between driving and driven portions of the transmission is the ability to ensure constant engagement between driving and driven portions with a minimum of drive and driven gears. In other words, an embodiment having a Vernier relationship with five driven and four drive gears (see Figure 3.3) will have an engagement point of a drive gear with a driven gear every eighteen degrees of rotation of the input, whereas an embodiment as shown in Figure 3.2, with four driven and four driving gears, will have an engagement point of a drive gear with a driven gear every ninety degrees of rotation of the input.

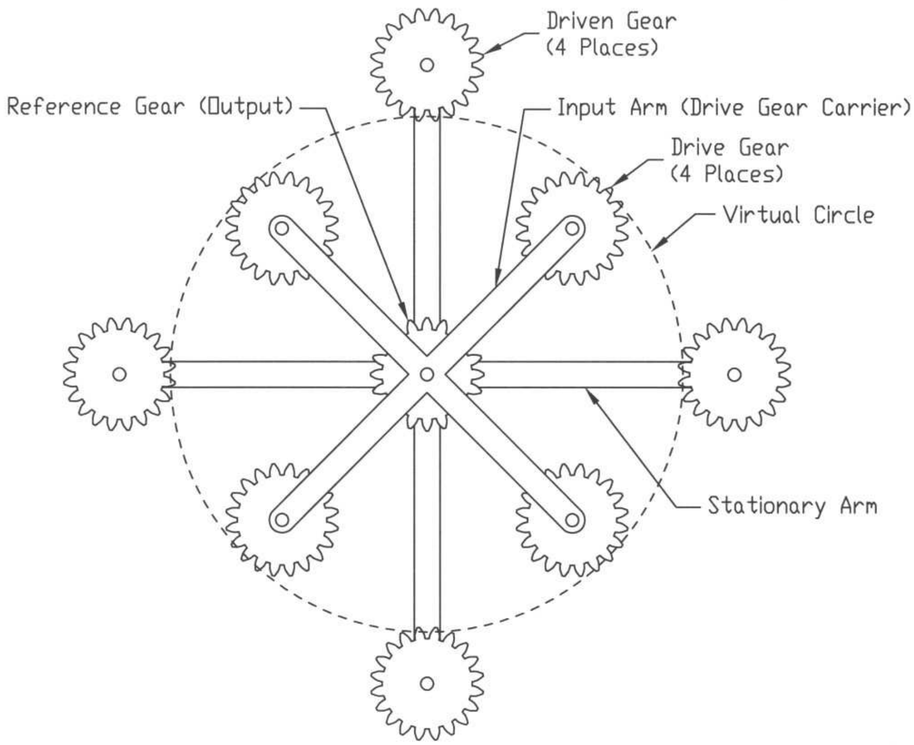


Figure 3.2: Embodiment with Equal Number of Drive and Driven Gears

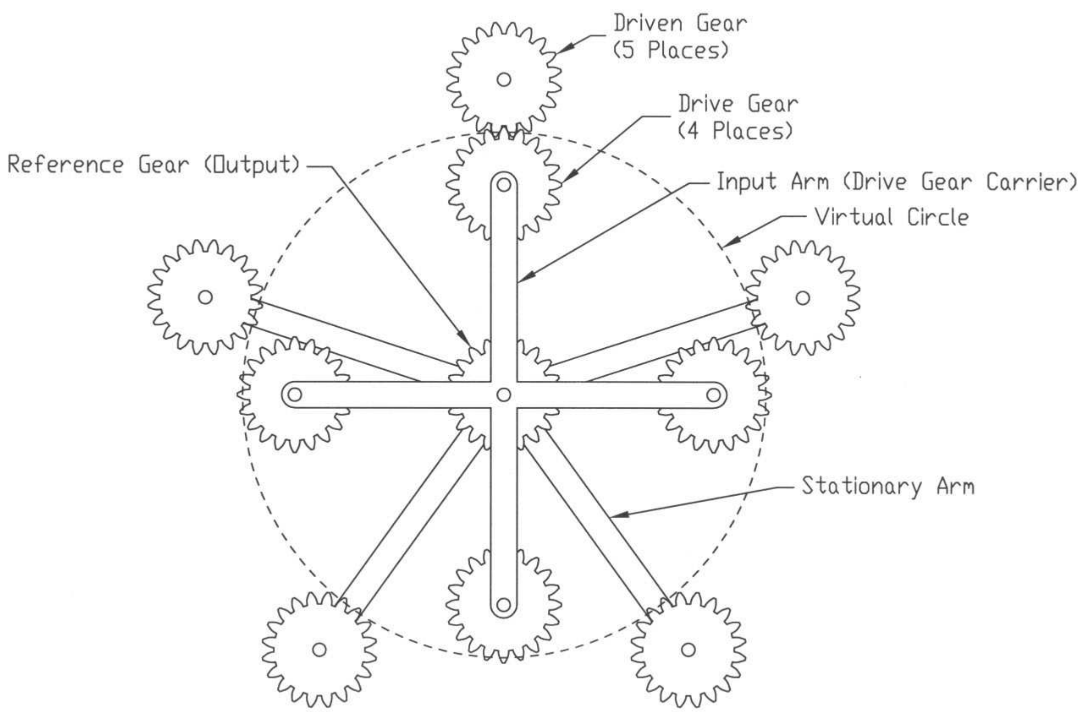


Figure 3.3: Embodiment with Vernier Relationship Between Drive and Driven Gears

<u>KINEMATIC ANALYSIS FOR EMBODIMENT 1</u>

This section details the derivation of the kinematic equations governing the function of the transmission.

The resultant pitch line velocity of the drive gear at the point of tangency with the virtual circle ($V_{PL,drive}$) is a combination of the pitch line velocity due to its rotation about its own axis ($V_{PL,RO}$) and its translation (orbit) about the central axis of the transmission ($V_{PL,T}$).

$$V_{PL,drive} = V_{PL,RO} + V_{PL,T} \tag{3.1}$$

The drive gear pitch line velocity due to rotation ($V_{PL,RO}$) is:

$$\omega_{drive} = (\omega_{in} - \omega_{ref})\frac{r_{ref}}{r_{drive}} \tag{3.2}$$

$$V_{PL,RO} = \omega_{drive} r_{drive} \tag{3.3}$$

$$V_{PL,RO} = (\omega_{in} - \omega_{ref})\frac{r_{ref}}{r_{drive}} \cdot r_{drive} \tag{3.4}$$

$$V_{PL,RO} = (\omega_{in} - \omega_{ref}) r_{ref} \tag{3.5}$$

where:

ω_{drive} = Angular Velocity of Drive Gear
ω_{in} = Angular Velocity of Input Arm
ω_{ref} = Angular Velocity of Reference Gear
r_{ref} = Radius of Reference Gear

r_{drive} = Radius of Drive Gear

The drive gear pitch line velocity due to translation ($V_{PL,T}$) is:

$$V_{PL,T} = \omega_{in}(r_{orbit} + r_{drive}) \tag{3.6}$$

where:

r_{orbit} = Orbit Radius of the Drive Gear

The resultant Drive Gear Pitch-Line Velocity ($V_{PL,drive}$), as seen at the point of tangency with the virtual circle, substituting Equations 3.5 and 3.6 into Equation 3.1, and combining terms, is:

$$V_{PL,drive} = V_{PL,T} + V_{PL,RO} \tag{3.1}$$

or,

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive}) + (\omega_{in} - \omega_{ref})r_{ref} \tag{3.7}$$

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive} + r_{ref}) - \omega_{ref}\, r_{ref} \tag{3.8}$$

The Driven Gear Pitch-Line Velocity ($V_{PL,driven}$) is:

$$V_{PL,driven} = \omega_{driven} r_{driven} \tag{3.9}$$

where:

ω_{driven} = Angular Velocity of Driven Gear
r_{driven} = Radius of Driven Gear

Because the Driven Gear has a gear pair relationship with the Reference Gear, and because the angular velocity of the Reference gear is the desired output of the final equation:

$$V_{PL,driven} = \omega_{driven} r_{driven} = \omega_{ref} r_{ref} \tag{3.10}$$

where:
ω_{ref} = Angular Velocity of Reference Gear
r_{ref} = Radius of Reference Gear

The Final Equation of Motion relating the pitch line velocities of the drive gear at its point of tangency with the virtual circle, and the driven gear at this same point (since they must be equal for proper meshing), using Equations 3.8 and 3.10 is:

$$V_{PL,drive} = V_{PL,out} \tag{3.11}$$

$$\omega_{ref} r_{ref} = \omega_{in} (r_{orbit} + r_{drive} - r_{ref}) + \omega_{ref} r_{ref} \tag{3.12}$$

$$\omega_{ref} = \frac{\omega_{in}}{2 \cdot r_{ref}} (r_{orbit} + r_{drive} + r_{ref}) \tag{3.13}$$

Equation 3.13 relates the input angular velocity of the transmission (ω_{in}) to the output velocity of the transmission (ω_{ref}).

OTHER POSSIBLE EMBODIMENTS

During the course of this research, two alternative embodiments employing the same principles as the embodiment just discussed, have been investigated. The following sections will describe the embodiments, including a derivation of their kinematic equations.

First Alternative Embodiment – Output Rings

The first alternative embodiment (hereafter known as embodiment 2) that was investigated in this research is shown in Figure 3.4. This embodiment consists of a carrier arm (A) that is mounted on and free to rotate about the central axis (D) of the transmission. The carrier arm is the input to the transmission. The embodiment is also composed of a reference gear (not shown), which is mounted on the central axis (D) of the transmission, and is fixed such that it does not rotate. On the opposing ends of the carrier arm are held two drive gears (A), which are permitted to rotate about their own axes relative to the carrier arm (F). The drive gears (A) are connected through a gear train (not shown) to the reference gear (not shown, and rotationally fixed) such that when the carrier arm rotates in one direction about the central axis, the drive gears (A) rotate in the opposite direction about their own axes. The carrier arm (F) causes the drive gears to orbit the central axis (D) of the transmission. The orbit radius (E) defines the length of the carrier arm (F) from the central axis (D) of the transmission to the axis of the drive gear (A). This length is variable, and is the controlling dimension governing the transmission ratio, which will be discussed later. The virtual circle (C) is a circle which is defined as being tangent to the pitch circles of the drive gears (A).

The transmission is also composed of driven gears (B), which in this embodiment are ring gears. These driven gears (B) are located radially about the central axis (D) of the transmission, and positioned radially such that their pitch circles are tangent to the virtual circle. These driven gears (B) are the output gears of the transmission.

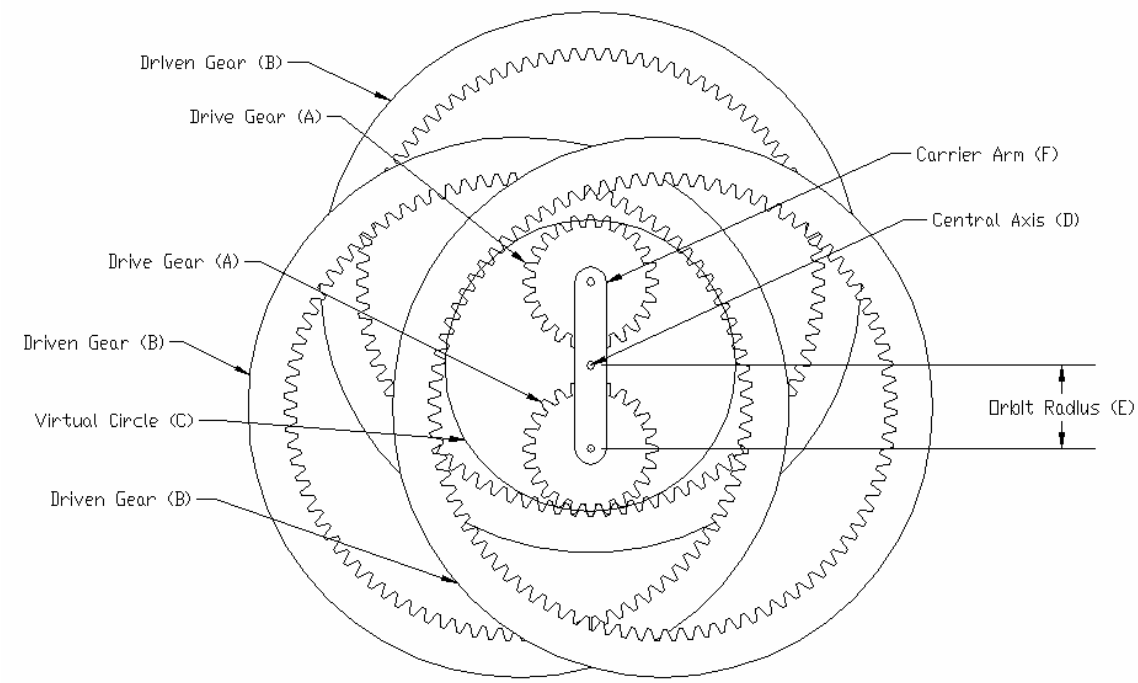


Figure 3.4: First Alternative Embodiment

Figure 3.4 shows the described embodiment having three driven gears (B) and two drive gears (A). This is purposely done to create a Vernier relationship between the drive and driven portions of the transmission, which allows for maintaining constant engagement of the drive and driven portions of the transmission with minimal part count. This embodiment parallels the original embodiment presented at the beginning of this chapter, with the exception that the driven gears, which were external spur gears in the original embodiment, have been replaced with internal ring gears.

KINEMATIC ANALYSIS OF EMBODIMENT 2

The resultant pitch line velocity of the drive gear at the point of tangency with the virtual circle ($V_{PL,drive}$) is a combination of the pitch line velocity due to its rotation about its own axis ($V_{PL,RO}$) and its translation (orbit) about the central axis of the transmission ($V_{PL,T}$).

$$V_{PL,drive} = V_{PL,T} - V_{PL,RO} \tag{3.14}$$

The drive gear pitch line velocity due to rotation ($V_{PL,RO}$) is:

$$\omega_{drive} = \omega_{in} \frac{r_{ref}}{r_{drive}} \tag{3.15}$$

$$V_{PL,RO} = \omega_{drive} r_{drive} \tag{3.16}$$

$$V_{PL,RO} = \omega_{in} \cdot \frac{r_{ref}}{r_{drive}} \cdot r_{drive} \tag{3.17}$$

$$V_{PL,RO} = \omega_{in} \cdot r_{ref} \tag{3.18}$$

where:

ω_{drive} = Angular Velocity of Drive Gear
ω_{in} = Angular Velocity of Input Arm
r_{ref} = Radius of Reference Gear
r_{drive} = Radius of Drive Gear

The drive gear pitch line velocity due to translation ($V_{PL,T}$) is:

$$V_{PL,T} = \omega_{in} (r_{orbit} + r_{drive}) \tag{3.19}$$

where:

r_{orbit} = Orbit Radius of the Drive Gear

The resultant Drive Gear Pitch-Line Velocity ($V_{PL,drive}$), as seen at the point of tangency with the virtual circle, substituting Equations 3.18 and 3.19 into Equation 3.14, and combining terms, is:

$$V_{PL,drive} = V_{PL,T} - V_{PL,RO} \tag{3.14}$$

or,

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive}) - (\omega_{in} \cdot r_{ref}) \tag{3.20}$$

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.21}$$

The Driven Gear Pitch-Line Velocity ($V_{PL,driven}$) is:

$$V_{PL,driven} = \omega_{driven} r_{driven} \tag{3.22}$$

where:

ω_{driven} = Angular Velocity of Driven Gear
r_{driven} = Radius of Driven Gear

The Final Equation of Motion relating the pitch line velocities of the drive gear at its point of tangency with the virtual circle, and the driven gear at this same point (since they must be equal for proper meshing), using Equations 3.21 and 3.22 is:

$$V_{PL,out} = V_{PL,drive} \tag{3.23}$$

$$\omega_{driven} r_{driven} = \omega_{in}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.24}$$

$$\omega_{driven} = \frac{\omega_{in}}{r_{driven}}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.25}$$

Equation 3.25 relates the input angular velocity of the transmission (ω_{in}) to the output velocity of the transmission (ω_{driven}).

<u>SECOND ALTERNATIVE EMBODIMENT – FIXED REFERENCE GEAR</u>

The second alternative embodiment that was investigated in this research is shown in Figure 3.5. The embodiment (hereafter referred to as embodiment 3) consists of a central reference gear (A) whose axis is co-axial with the major axis of the transmission. The reference gear (A) is fixed so that it cannot rotate. An input arm, or drive gear carrier (B), is connected to the axis of the reference gear (A), allowing it to rotate about the axis of, and relative to, the reference gear (A). The input arm (B) is the input to the transmission from an external power source. Connected to the input arm is a drive gear (C), which the input arm (B) causes to orbit about the reference gear (A). The drive gear (C) is connected to the reference gear (A) through a gear pair relationship, which means that the rotation of the drive gear (C) about its axis has a fixed relationship to the rotation of the input arm (B) about its axis (this would be accomplished through a gear set between the reference gear (A) and the drive gear (C), not shown in Figure 3.5). Also connected to the axis of the reference gear (A) is a stationary arm (D), which remains fixed (does not rotate) and supports a driven gear (E), which is the output of the transmission.

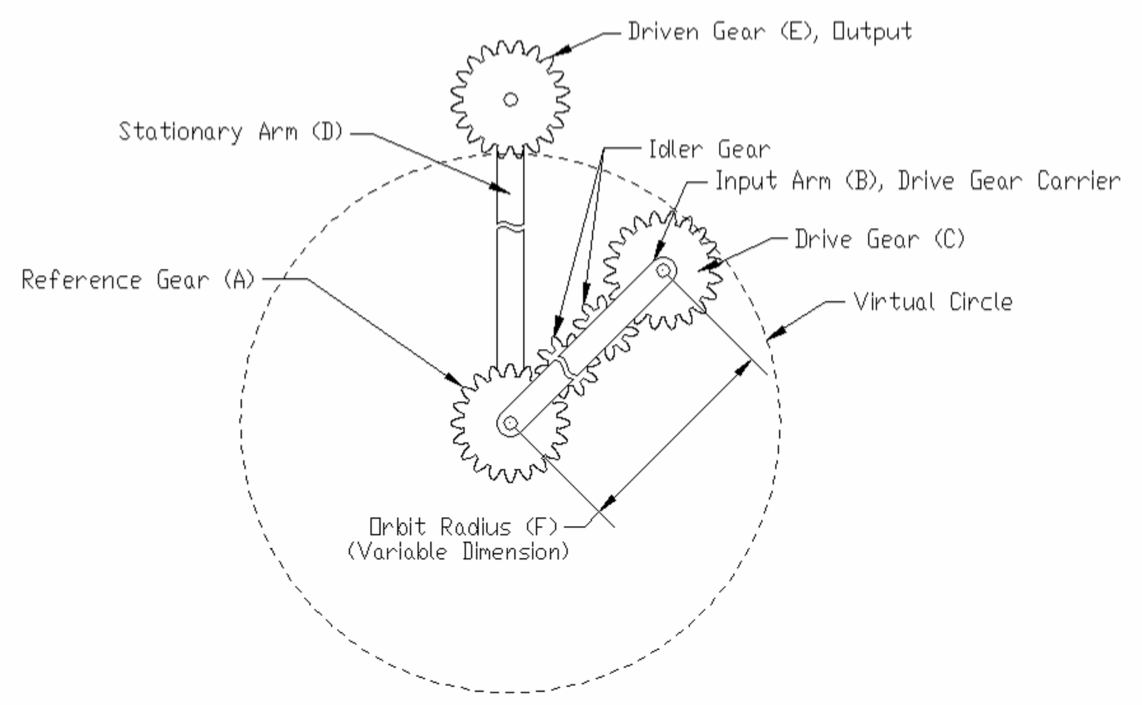


Figure 3.5: Second Alternative Embodiment

When the input arm (B) is rotating about the axis of the reference gear (A), the drive gear (C) *orbits* around the reference gear (A) at an angular velocity equal to that of the input to the transmission. This orbiting motion also causes the drive gear (C) to *rotate* about its own axis in the opposite direction of the orbit motion. The angular velocity at which the drive gear (C) rotates, relative to its orbit, is dependent on the gear ratio of the gear set connecting the reference gear (A) and the drive gear (C).

The drive gear (C) shown in Figure 3.5 connects the input portion of the transmission to the driven portion of the transmission. This is accomplished as the drive gear (C) orbits past and meshes with the driven gear (E). The contact and meshing of the drive (C) and driven (E) gears are what cause rotation of the driven (output) gear (E).

The orbit radius (F) shown in Figure 3.5 is the controlling dimension of the transmission. As the drive gear (C) orbits the reference gear (A) at a certain orbit radius (F), it traces out a virtual circle, as seen in Figure 3.5. As the orbit radius (F) is varied, the diameter of the virtual circle is also changed to remain tangent to the drive gear (C). The radius at which the driven gear (E) is held is also adjusted as the orbit radius (F) of the drive gear (C) changes such that the driven gear (E) remains tangent to the virtual circle.

Because the drive gear (C) has an orbiting angular velocity in one direction and a rotational angular velocity in the opposite direction, the resulting tangential (pitch line) velocity of the drive gear (C) at the point of tangency with the virtual circle, relative to the central axis of the transmission, is dependent upon the orbit radius (F) noted in Figure 3.5. Because the drive gear (C) must mesh with the driven gear (E), the driven gear must have a pitch line velocity equal to the resultant pitch line velocity of the drive gear (C) at the virtual circle.

While Figure 3.5 shows only one driven and one drive gear, an embodiment with only one driven and one drive gear would not maintain constant engagement. This is because the drive and driven gears would only maintain engagement for a short portion of the drive gear's orbit. The functioning embodiment would thus be composed of a plurality of both gears. The embodiment could consist of an equal number of drive and driven gears (see Figure 3.6), or a differing number of drive and driven gears in a so called "Vernier relationship" (see Figure 3.7). The advantage of the Vernier relationship between driving and driven portions of the transmission is the ability to ensure constant engagement between driving and driven portions with a minimum of drive and driven gears. In other words, an embodiment having a Vernier relationship with five driven and four drive gears (see Figure 3.7) will have an engagement point of a drive gear with a driven gear every eighteen degrees of rotation of the input, whereas an embodiment as shown in Figure 3.6, with four driven and four driving gears, will have an engagement point of a drive gear with a driven gear every ninety degrees of rotation of the input.

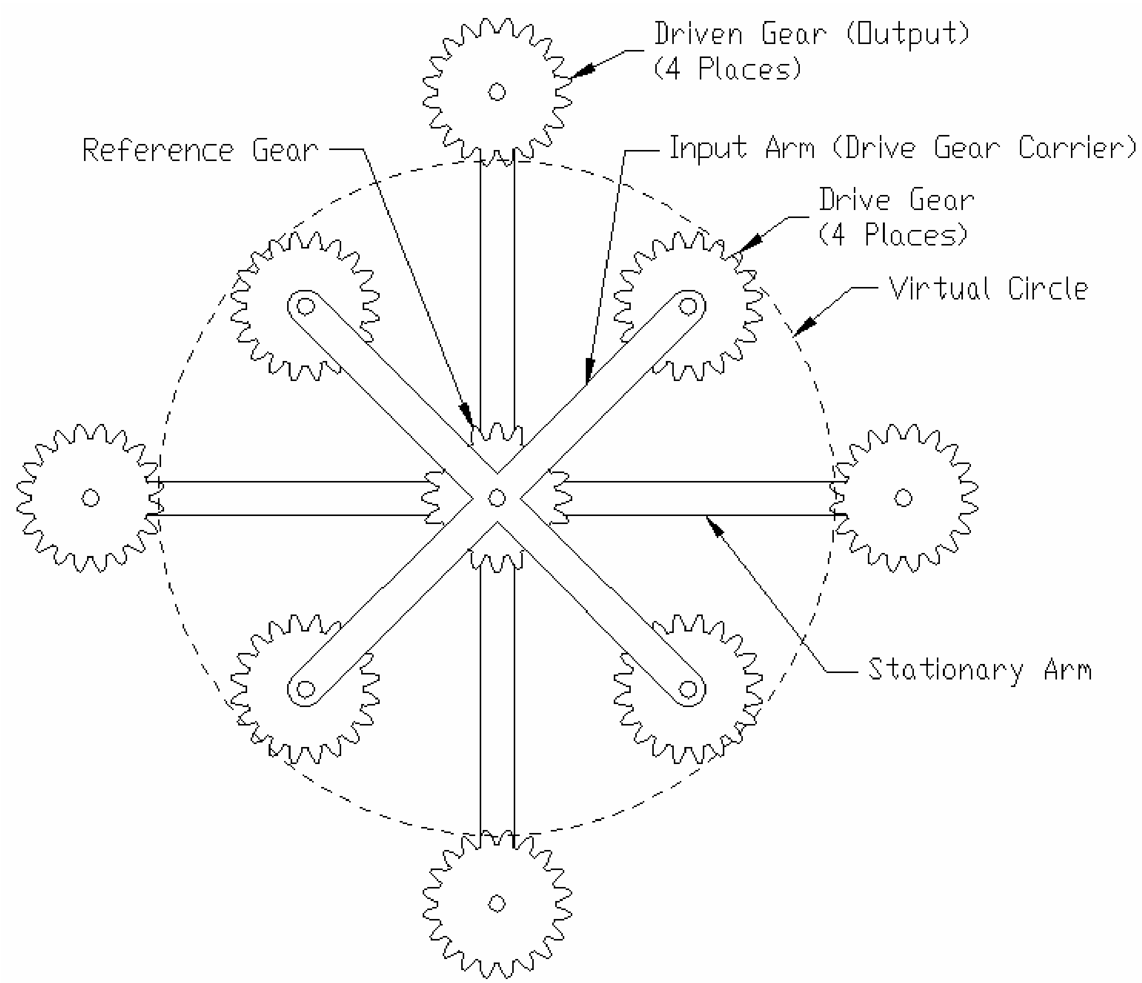


Figure 3.6: Embodiment 3 with Equal Number of Drive and Driven Gears

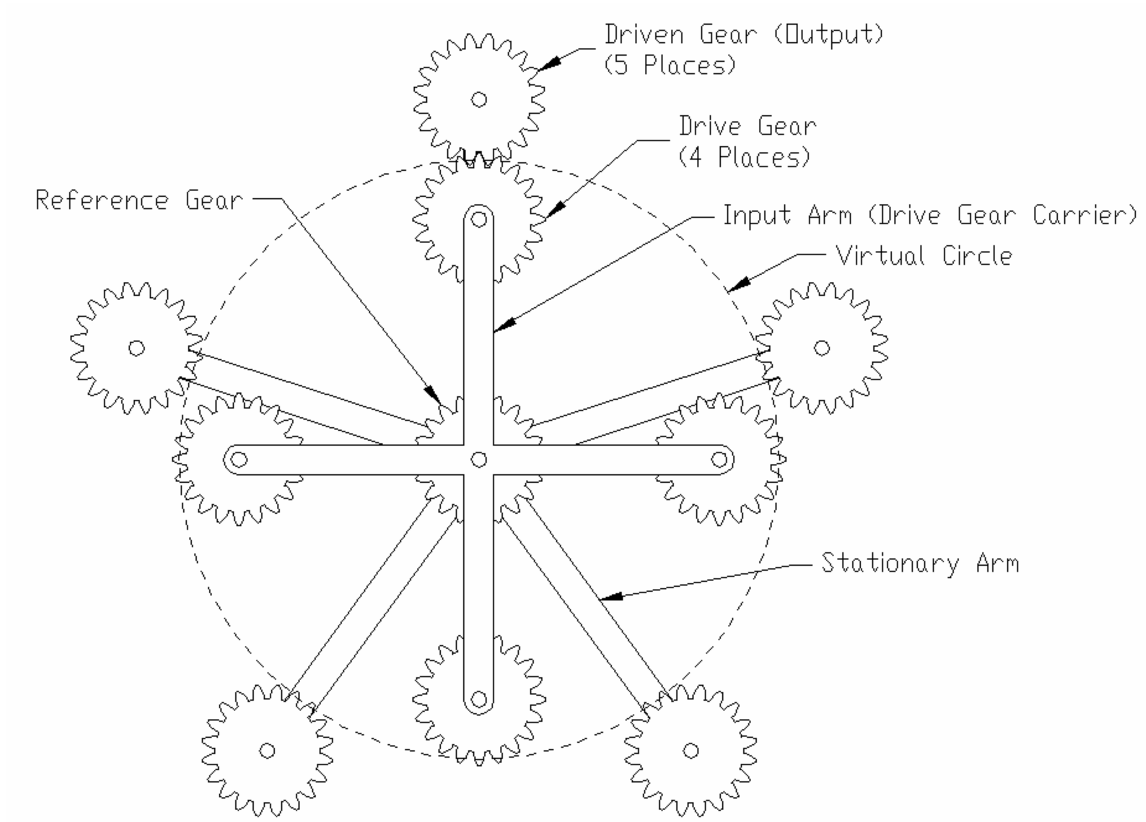


Figure 3.7: Embodiment 3 with Vernier Relationship between Drive and Driven Gears

KINEMATIC ANALYSIS OF EMBODIMENT 3

The resultant pitch line velocity of the drive gear at the point of tangency with the virtual circle ($V_{PL,drive}$) is a combination of the pitch line velocity due to its rotation about its own axis ($V_{PL,RO}$) and its translation (orbit) about the central axis of the transmission ($V_{PL,T}$).

$$V_{PL,drive} = V_{PL,T} - V_{PL,RO} \tag{3.26}$$

The drive gear pitch line velocity due to rotation ($V_{PL,RO}$) is:

$$\omega_{drive} = \omega_{in} \frac{r_{ref}}{r_{drive}} \tag{3.27}$$

$$V_{PL,RO} = \omega_{drive} r_{drive} \tag{3.28}$$

$$V_{PL,RO} = \omega_{in} \cdot \frac{r_{ref}}{r_{drive}} \cdot r_{drive} \tag{3.29}$$

$$V_{PL,RO} = \omega_{in} \cdot r_{ref} \tag{3.30}$$

where:

ω_{drive} = Angular Velocity of Drive Gear
ω_{in} = Angular Velocity of Input Arm
r_{ref} = Radius of Reference Gear
r_{drive} = Radius of Drive Gear

The drive gear pitch line velocity due to translation ($V_{PL,T}$) is:

$$V_{PL,T} = \omega_{in}(r_{orbit} + r_{drive}) \tag{3.31}$$

where:

r_{orbit} = Orbit Radius of the Drive Gear

The resultant Drive Gear Pitch-Line Velocity ($V_{PL,drive}$), as seen at the point of tangency with the virtual circle, substituting Equations 3.30 and 3.31 into Equation 3.26, and combining terms, is:

$$V_{PL,drive} = V_{PL,T} - V_{PL,RO} \tag{3.26}$$

or,

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive}) - (\omega_{in} \cdot r_{ref}) \tag{3.32}$$

$$V_{PL,drive} = \omega_{in}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.33}$$

The Driven Gear Pitch-Line Velocity ($V_{PL,driven}$) is:

$$V_{PL,driven} = \omega_{driven} r_{driven} \tag{3.34}$$

where:

ω_{driven} = Angular Velocity of Driven Gear
r_{driven} = Radius of Driven Gear

The Final Equation of Motion relating the pitch line velocities of the drive gear at its point of tangency with the virtual circle, and the driven gear at this same point (since they must be equal for proper meshing), using Equations 3.33 and 3.34 is:

$$V_{PL,driven} = V_{PL,drive} \tag{3.35}$$

$$\omega_{driven} r_{driven} = \omega_{in}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.36}$$

$$\omega_{driven} = \frac{\omega_{in}}{r_{driven}}(r_{orbit} + r_{drive} - r_{ref}) \tag{3.37}$$

Equation 3.37 relates the input angular velocity of the transmission (ω_{in}) to the output velocity of the transmission (ω_{driven}).

COMPARISON OF THE THREE EMBODIMENTS

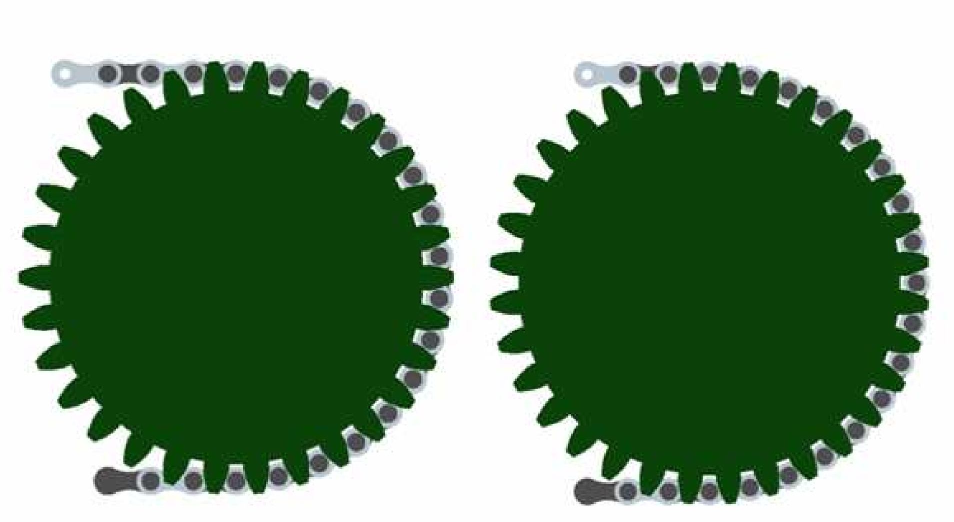
Embodiments 1, 2 and 3 are all similar in form. In each of the embodiments, the radial position of the drive gear relative to the central axis of the transmission is called the orbit radius, which is the controlling dimension of the transmission ratio. Also, in each case, a virtual feature, called the virtual circle, is used to describe the diameter at which the drive and driven portions of the transmission mesh. The purpose for generating two variant embodiments as part of this research was to better understand the meshing characteristics of the proposed concept, which characteristics will be discussed later.

CHAPTER 4 THE NON-INTEGER TOOTH PROBLEM

The three conceptual embodiments that have been described in the previous chapter can be classified as positive engagement, continuously variable transmissions. Positive engagement refers to the condition of the input being positively engaged with the output, as in the meshing of a gear pair, such that the transmission of power is not accomplished through the use of friction. The positive engagement condition must be met during the traversing of the ratio range of the transmission for it to be classified as positive engagement.

Other transmissions of the positive engagement, continuously variable classification currently exist, several of which will be described briefly later. The examination of these published embodiments reveals a significant meshing problem between the driving and driven portions of the transmission at certain transmission ratios, which meshing is critical for positive power transmission. This meshing problem is called the non-integer tooth problem.

The generic non-integer tooth problem is best understood when considering the case of the rear sprocket cluster of a multi-speed bicycle. Each of the sprockets has an equal pitch, which allows them to all mesh with a chain of the same pitch, but each sprocket has a different pitch diameter. These pitch diameters are specifically set such that the resulting circumference is divisible by the circular pitch, resulting in the sprocket having an integer number of teeth. Figure 4.1 shows two sprockets with equal pitch, but with different pitch diameters, and thus different numbers of teeth. Because each sprocket has a different pitch diameter, and thus a different number of teeth, each provides a different gear ratio when driven by another sprocket.



(a) (b)

Figure 4.1: (a) 30 Tooth Sprocket and (b) 32 Tooth Sprocket (Both Have Diametral Pitch = 16)

To allow a bicycle to have infinitely incremented gear ratios, there would need to be an infinite number of sprockets, each with a different pitch diameter, and thus infinitely different numbers of teeth. If a sprocket is created, however, with a pitch diameter whose resulting circumference is not divisible by the pitch of the chain, a non-integer number of teeth would result on the sprocket, as shown by the overlapping teeth in Figure 4.2, thereby ensuring at least one place on the sprocket where the chain will not mesh properly.

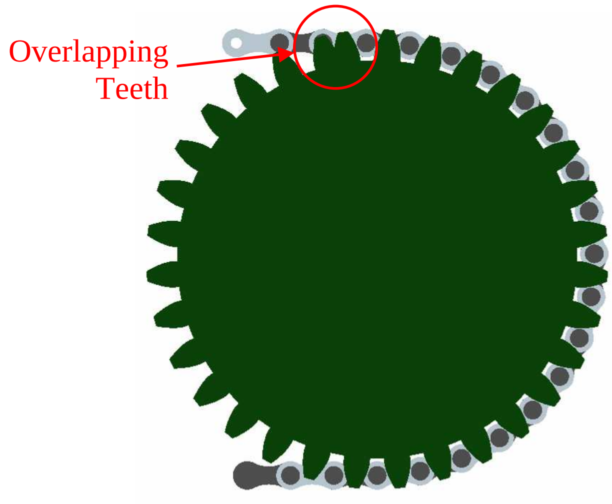


Figure 4.2: Sprocket with a Non-Integer Number of Teeth (diametral pitch = 16)

Standard bicycles overcome the non-integer tooth problem by having a finite number of sprockets, both driving and driven, which all have the same diametral pitch. Each of these sprockets has an integer number of teeth, which allows them to mesh properly throughout their complete rotation. This limits the bicycle, however, to a finite number of discrete gear ratios, with no ability to continuously vary the power transmission ratio.

PUBLISHED EMBODIMENTS

The generic non-integer tooth problem occurs in all attempts to produce a positive engagement, continuously variable transmission. Three published embodiments will now be discussed, especially with regards to the occurrence of the non-integer tooth problem in each case. Because a discussion of the non-integer tooth problem in the published embodiments is better understood when also considering the methods of rectifying the problem, a discussion of the embodied solutions will also be presented.

PIVOT-ARM CVT

The pivot-arm CVT, originally developed by Mortensen, 2000, and later analyzed and modified by Christensen, 2002, at Brigham Young University, is an embodiment that employs compliant members to provide a mechanism that will change its active diameter to create a continuous range of mechanical advantage. Figure 4.3 shows a specific design of the pivot-arm embodiment, meant for application in a bicycle drive train (see Figure 4.4). The design consists of seven arms, called pivot arms, which are allowed to rotate about their connection to a common carrier about which they are attached. The pivot arms are connected to each other through compliant members which resist the rotation of the arms, and which cause all of the arms to rotate the same amount. As the arms rotate, causing the compliant members to deflect, the effective diameter of the CVT (front sprocket) is changed.

Like CVT's used in other applications, the pivot arm CVT does not operate at discrete increments of ratio change. This means that the distance between the ends of the pivot arms, where the chain is driven, changes continuously rather than incrementally as the effective diameter of the CVT is adjusted. At this point the non-integer tooth problem expresses itself. If fixed sprockets were attached and rotationally fixed at the end of the pivot arms to drive the chain, slack would occur in the chain between the sprockets as the effective radius of the CVT decreased, and the chain would skip off the sprockets when the effective radius increased. This would occur because the distance between the fixed sprockets would only be divisible evenly by the pitch of the chain at certain effective radii of the CVT.

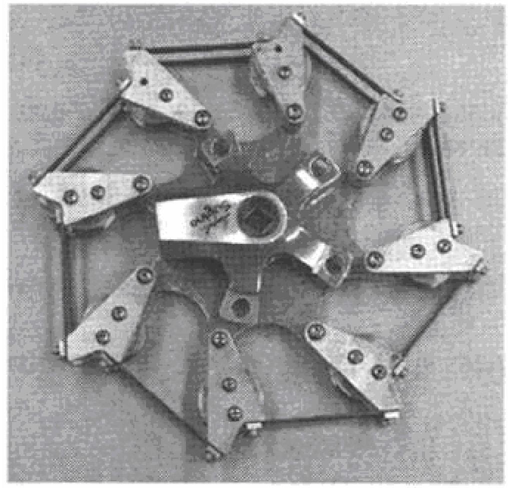


Figure 4.3: Pivot-Arm CVT (Taken from Christensen, 2002)

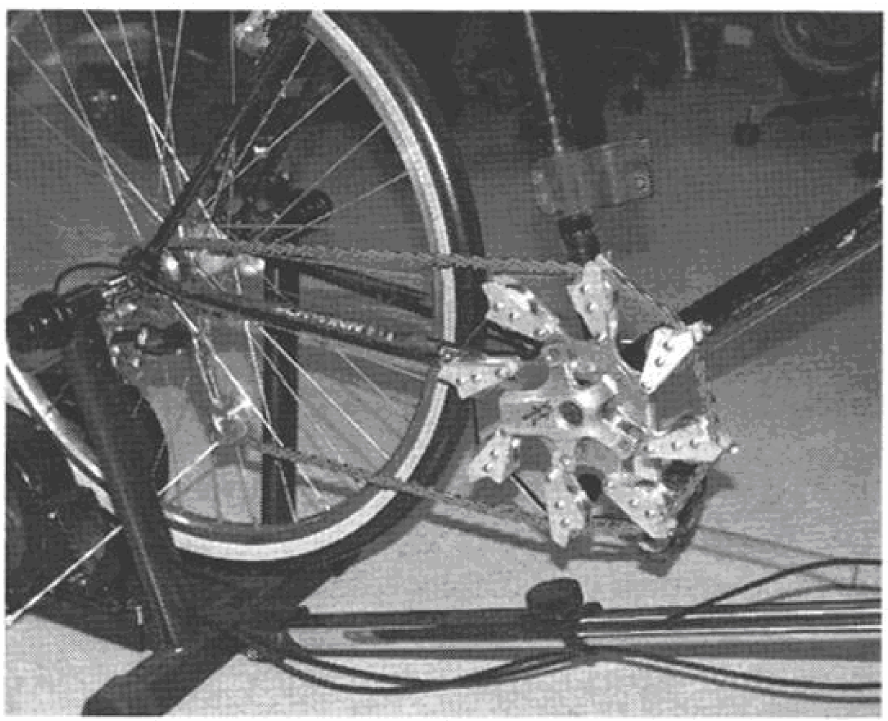


Figure 4.4: Application of the Pivot-Arm CVT in a Bicycle Drivetrain (Taken from Christensen, 2002)

To overcome this problem, Christensen discusses the use of a V-belt and sheaves to replace the chain and sprockets, respectively, which would solve the problem by allowing the belt to slip as needed. Slipping of the belt, however, greatly increases the losses in the system, and reduces the amount of torque that can be transferred. Thus to improve efficiency, Christensen states that the ideal configuration would use a chain and sprockets, as described before, but still accommodate the continuous nature of the CVT. Christensen shows that this can be accomplished by the implementation of a one-way clutch on the axle of each of the sprockets. The clutch should be oriented such that it will transmit power when torque is applied through the input, but will freely allow the chain to move in the opposite direction. In this manner, the CVT is able to release excess chain or prevent skipping while transversing all desired ratios.

FIXED-PITCH CVT

The fixed-pitch CVT (see Figure 4.5), developed by Kenneth B. Hawthorn, 2006, operates on principles similar to the pivot-arm CVT. It contains sprockets held by a carrier mechanism that controls the radial position of the sprockets, thus allowing the effective radius of the CVT to be changed, and in so doing the transmission ratio. The carrier mechanism functions the same as the flat belt CVT described in Chapter 2. Each of the two pulleys is composed of discs with guideway slots that support the shafts upon which the sprockets are held. When the discs are rotated relative to each other, the shafts which hold the sprockets move radially.

The design also has only two points of contact per chain, one on the power side and one on the load side. This, Hawthorn states, allows for ratio shifts while maintaining positive engagement. Multiple chains are incorporated (three are shown in Figure 4.5) to provide multiple contact points which maintain engagement between the power and load sides through their rotations.

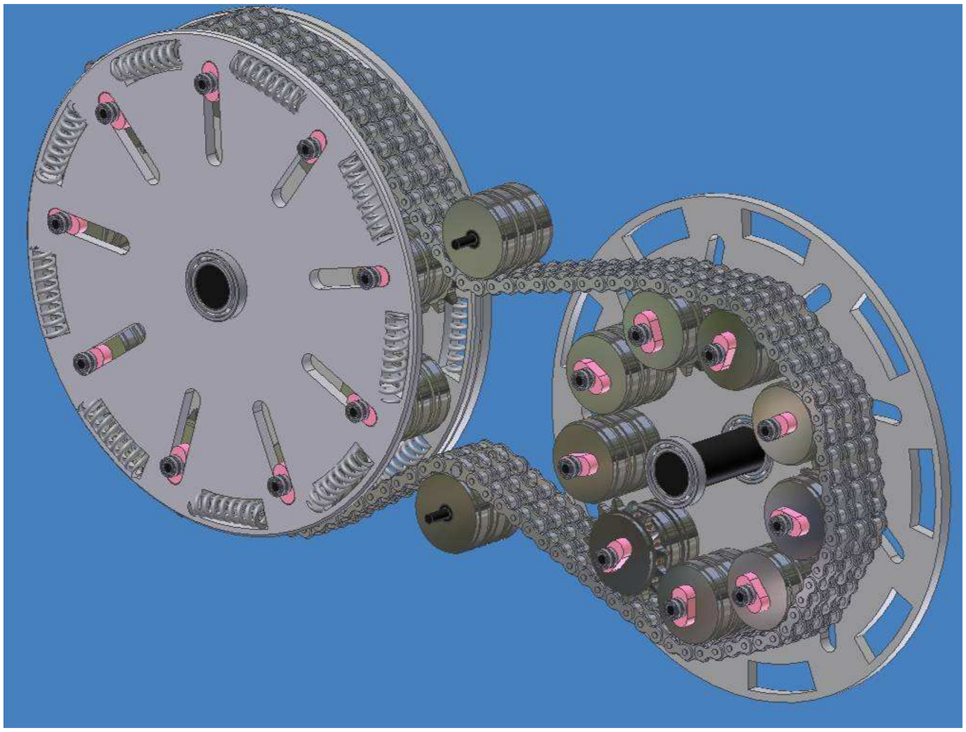


Figure 4.5: Fixed-Pitch CVT (Taken from Hawthorn, 2006)

This embodiment is likewise subject to the non-integer tooth problem, in that the distance between the sprockets can change in a continuous manner. This continuous change would allow the distance between the sprockets, specifically where they would mesh with a chain (assuming the sprockets could not rotate), to assume values not divisible evenly by the pitch of the chain. As in the previous embodiment, this would cause slack to occur in the chain as the effective radius of the CVT decreased, and the chain would skip off the sprockets when the effective radius increased.

To overcome the non-integer tooth problem, Hawthorn proposes a different method of allowing reorientation of the sprockets than the one-way clutches proposed by

Christensen. He instead proposes a specially designed sprocket (see Figure 4.6), called the power sprocket, that is able to freely rotate upon its supporting shaft, thereby allowing the chain to engage properly with the sprocket. Once the chain becomes fully seated on the sprocket, however, it causes the sprocket to lock on its supporting shaft, thereby eliminating the sprocket's rotation, allowing it to transmit torque. By allowing the sprocket to adjust its orientation when not transmitting power, the CVT is able to release excess chain or prevent skipping while transversing all desired ratios.

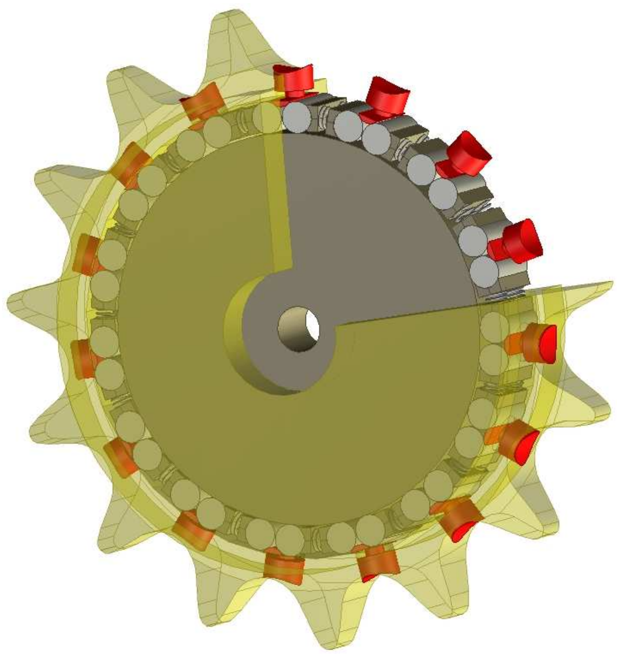


Figure 4.6: Power Sprocket (Taken from Hawthorn, 2006)

ANDERSON CVT

The Anderson CVT (Anderson, 2003) is a positive engagement, dual cone type CVT, that uses two cones positioned with their axes parallel, but with the larger end of each one alongside the smaller end of the other (see Figure 4.7). In the arrangement, one cone would act as the driving sprocket, and the other as the driven sprocket. In the embodiment, a chain winds around the cones to allow for the positive transmission of power from the drive cone to the driven cone. The ratio of the transmission is determined by the axial position of the chain along the cones. The axial position of the chain defines the diameter of the driving and driven cones that are employed to transmit power. The chain can be shifted in infinitely variable increments to provide continuously variable ratios.

To allow the chain to engage the cones in a positive manner, the cones have sprocket bars that act as teeth, similar to a sprocket (see Figure 4.7). Because the circumferential distance between the sprocket bars is different at each axial position along the cone, there are positions at which the circumferential distance between the sprocket bars is not divisible by the pitch of the chain with which they are to engage. In other words, because the number of teeth on the cones is constant along the length, and the diametral pitch is also constant, the circular pitch varies continuously from one end of the cone to the other. In this arrangement, this is the expression of the non-integer tooth problem.

To overcome the non-integer tooth problem, the inventor proposes a method of allowing the sprocket bars to adjust their position, thereby adjusting their effective circular pitch to match that of the chain. As shown in Figure 4.8, the sprocket bars on the cones are allowed to float, meaning that they can move radially and adjust a limited amount along the circumference of the cone. It is the adjustment along the circumference of the cone that allows the adjustment of the effective circular pitch. Because of the ability to float, the distance between the sprocket bars can adjust to accommodate a chain of fixed pitch.

Figure 4.7: Anderson CVT (Taken from Anderson, 2007)

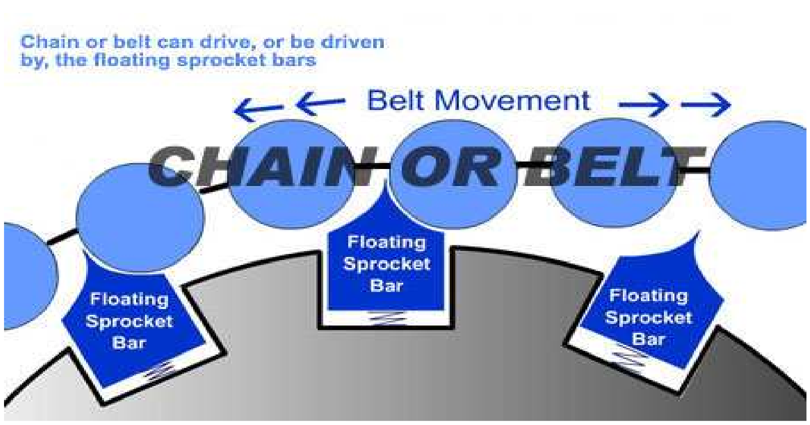


Figure 4.8: Floating Sprocket Bars (Taken from Anderson, 2007)

CHAPTER 5 MESHING ANALYSIS OF THE PROPOSED CVT

Because of the expression of the non-integer tooth problem in the three published embodiments of positive engagement, continuously variable transmissions that have been discussed, and because the meshing characteristics of the proposed embodiment have not been previously investigated, this chapter presents an analytical analysis of the meshing characteristics of the proposed embodiment. This chapter shows that the non-integer tooth problem is present in the proposed embodiment, and describes the conditions under which it occurs.

When the orbit radius of the positive displacement, continuously variable transmission, previously called embodiment 1, is held constant, it functions as a standard epicyclic gear train, and therefore must follow the established geometric condition for the assembly of epicyclic gears. This condition states that for an epicyclic gear train composed of N equally spaced planet gears (this applies to both drive and driven gears in the described embodiment), an annulus (the virtual circle in the described embodiment) with X number of teeth, and a sun gear (the reference gear in the described embodiment) with S number of teeth:

$$\frac{(X+S)}{N} = Integer \tag{5.1}$$

For the proposed transmission, the number of planets (drive or driven gears) and their spacing is constant, as is the number of teeth on the sun gear (reference gear). However, the number of teeth on the annulus (virtual circle) is not constant. This is because the virtual circle of the proposed transmission embodiment (see Figure 5.1) has a diameter that is a function of the orbit radius, which is infinitely variable. Because the diameter of the virtual circle can change in infinitely small amounts, its virtual number of teeth can take on values that are not integers. Therefore, Equation 5.1 can take on values that are not integers, meaning that the drive and driven gears will not mesh properly without some correction in their alignment, or orientation with respect to one another.

The assembly condition equation (Equation 5.1) indicates that proper meshing will only occur at specific orbit radii of the drive gear, thus eliminating the ability of the transmission to transverse infinite ratios.

It has been considered that a possible solution to the non-meshing problem at orbit radii that do not satisfy Equation 5.1 would be to allow the driven gears to be rotated relative to each other by some amount that would correct for their misalignment. To facilitate the investigation of a proposed correction, a spreadsheet was created that provided for analyzing the kinematic motion of the transmission over time. The results of this spreadsheet were also verified with a Matlab program. The spreadsheet was specifically set up to track the orientation of each of the gears in the transmission, as well as the meshing of the drive and driven gears in question. The spreadsheet allowed for changing the ratio (by changing the orbit radius) of the transmission over time, as well as allowing it to be fixed at any desired ratio.

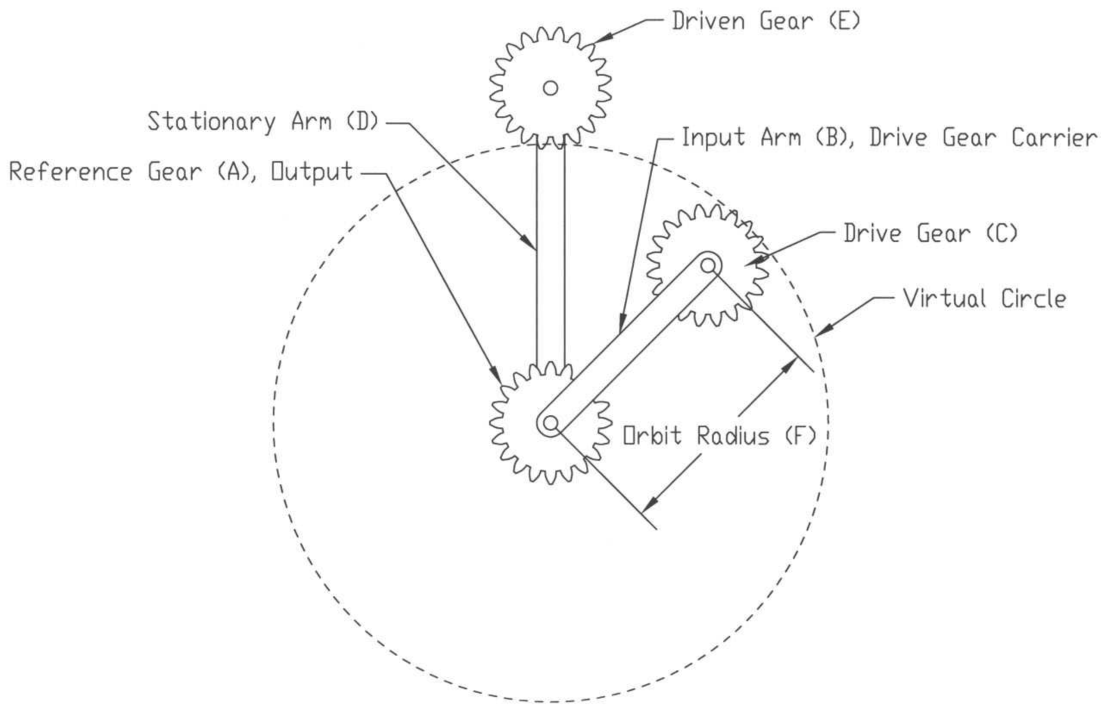


Figure 5.1: Basic Embodiment of the Proposed CVT

The analytical investigation yielded three cases:

Case 1: When the transmission operates under the following conditions:

1. The orbit radius creates a virtual circle with an integer number of teeth.

2. The number of teeth on the virtual circle satisfies the geometric condition for the assembly of a planetary gear train (Equation 5.1) when the denominator of Equation 5.1 is the number of drive gears, and the equation produces an even integer.

3. The number of teeth on the virtual circle satisfies the geometric condition for the assembly of a planetary gear train (Equation 5.1) when the denominator of Equation 5.1 is the number of driven gears, and the equation produces an even integer.

Under these operating conditions, no correction in the alignment of the driven gears is needed.

Case 2: When the transmission operates under the following conditions:

1. The orbit radius creates a virtual circle with an integer number of teeth.

2. The number of teeth on the virtual circle satisfies the geometric condition for the assembly of a planetary gear train (Equation 5.1) when the denominator of Equation 5.1 is the number of drive gears, and the equation produces an even integer.

3. The denominator of Equation 5.1 is the number of driven gears, and the equation produces an odd integer or non-integer.

Under these operating conditions, the amount of correction is known, and the correction must only occur once at that particular orbit radius.

Case 3: When the transmission operates under the following conditions:

1. The denominator of Equation 5.1 is the number of drive gears, and the equation produces an odd integer or non-integer.

2. The denominator of Equation 5.1 is the number of driven gears, and the equation produces an odd integer or non-integer.

Under these operating conditions, the amount of correction is known, but the correction must occur at each engagement of the driven gear with each drive gear. Table 5.1 provides a summary of the three cases that have been described.

Table 5.1: Cases Summary

$\frac{(X+N)}{Drive}=$	$\frac{(X+N)}{Driven}=$	# of Teeth on the Virtual Circle =	Correction Necessary	Continuous Correction	Case
Even Integer	Even Integer	Integer	No	No	1
Even Integer	Odd Integer	Integer	Yes	No	2
Even Integer	Non-Integer	Integer	Yes	No	2
Odd Integer	Odd Integer	Integer	Yes	Yes	3
Odd Integer	Even Integer	Integer	Yes	Yes	3
Odd Integer	Non-Integer	Integer	Yes	Yes	3
Non-Integer	Odd Integer	Integer	Yes	Yes	3
Non-Integer	Even Integer	Integer	Yes	Yes	3
Non-Integer	Non-Integer	Integer	Yes	Yes	3
Non-Integer	Non-Integer	Non-Integer	Yes	Yes	3

For the transmission to mesh properly, the drive and driven gears must enter meshing at the same relative alignments for each orbit. This means that when the drive and driven gears approach meshing, the teeth of each gear must be aligned relative to the other gear to ensure proper engagement. The reason that a correction must occur in cases 2 and 3 is to ensure the proper alignment of the drive and driven gears, as without correction the teeth of the gears would not be properly aligned, and thus would not correctly mesh.

The difference in the amount of correction required under each operating condition of the transmission can be understood by examining the relationship between the orbit of the drive gears about the reference gear and the rotation of the drive and driven gears about their own axes (see Figure 5.1). It is important to understand certain meshing characteristics of the current embodiment to more fully understand these relationships. One of the characteristics of the proposed embodiment corresponds to how many degrees the input arm must rotate between engagements of a drive gear with sequential driven gears. These points of engagement occur at equal intervals of degrees of input arm rotation, as described by the following equation:

$$\frac{360}{Drv * Drn} = \text{Degrees between Engagements} \qquad (5.2)$$

Where:

Drv= the number of drive gears
Drn= the number of driven gears

These gear relationships are described in the following three cases that correspond to the three operating conditions described in the preceding conclusions:

Case 1: For each orbit of the drive gear about the reference gear, the drive and driven gears will rotate about their axes an angular amount that is divisible evenly by the angular spacing between their teeth. This will cause the same orientation of drive and driven gear teeth to exist, relative to each other, at the same point on each orbit. The alignment of the teeth will also be correct at the points where a drive and driven gear must mesh (Equation 5.2). Figure 5.2 shows a plot of the misalignment of the teeth of the driven gear relative to the drive gear under case one at each angle of the input arm for three revolutions of the input arm (the plot only shows the range from 0 to 90 degrees of input arm position for ease of examination). It can be seen that the plot exactly overlaps for sequential orbits, which demonstrates the same orientation of driven gear teeth at the same point on each orbit. To determine that the alignment of the teeth of the driven gear will be correct when the drive gear is to mesh with it, it is necessary to examine the amount of misalignment of the driven gear when the input arm is angularly aligned with the driven gear. For example, in a case with five driven gears and four drive gears, as shown in Figure 3.3, it would be necessary to examine the misalignment of the driven gear at intervals of 90° of input angle position, which corresponds to when a different drive gear will enter into engagement with the same driven gear. That is, every 90° of input will result in five engagements of different driven gears at every 18°, thus every 90° the same driven gear will be engaged. Figure 5.2 shows such a case, and demonstrates zero misalignment of the driven gear when the arm is at 0° and 90°, 18° and 108°, 72° and 126° etc. (positions where the input arm would align with a particular driven gear), which shows that the driving and driven gears will mesh properly.

Case 2: For each orbit of the drive gear about the reference gear, the drive and driven gears will rotate about their axes an angular amount that is divisible evenly by the angular spacing between their teeth. This will cause the same orientation of drive and driven gear teeth to exist, relative to each other, at the same point on each orbit, as shown by the exact overlapping of the plot shown in Figure 5.3 for sequential orbits. However, the alignment of the teeth will not be correct at every point where a drive and driven gear must mesh (Equation 5.2). Figure 5.3 shows a plot of the misalignment of the teeth of the driven gear relative to the drive gear under case two at each angle of the input arm for three revolutions of the input arm (again the plot only shows the range from 0 to 108 degrees of input arm position for ease of examination). The difference between this case and case one is that, while the same orientation of drive and driven gear teeth will exist relative to each other at the same point on each consecutive orbit, the actual alignment of the teeth will be incorrect for proper meshing when the drive gear orbits past the driven gear (Equation 5.2). For example, in a case with five driven gears and four drive gears, as shown in Figure 3.3, it would be necessary to examine the misalignment of the driven gear at intervals of 0° and 90° of input arm angle, which corresponds to when a sequential drive gears will enter into engagement with the same driven gear. Figure 5.3 is representative of such a case, and demonstrates a misalignment of the driven gear when the arm is at 0° and 90°, 18° and 108°, 72° and 126°, etc. (a position where the input arm would align with a particular driven gear), which shows that the driving and driven gears will not mesh properly, and also that they will have the same amount of misalignment at the same point on each orbit. It is also important to note that the amount or pitch misalignment at 0° is the same as that at 90° (it is also the same at 18° and 108°, 72° and 126° etc.). Because the amount of misalignment at every 90° interval on each orbit is constant, a one time correction for the misalignment would ensure correct meshing for all subsequent orbits.

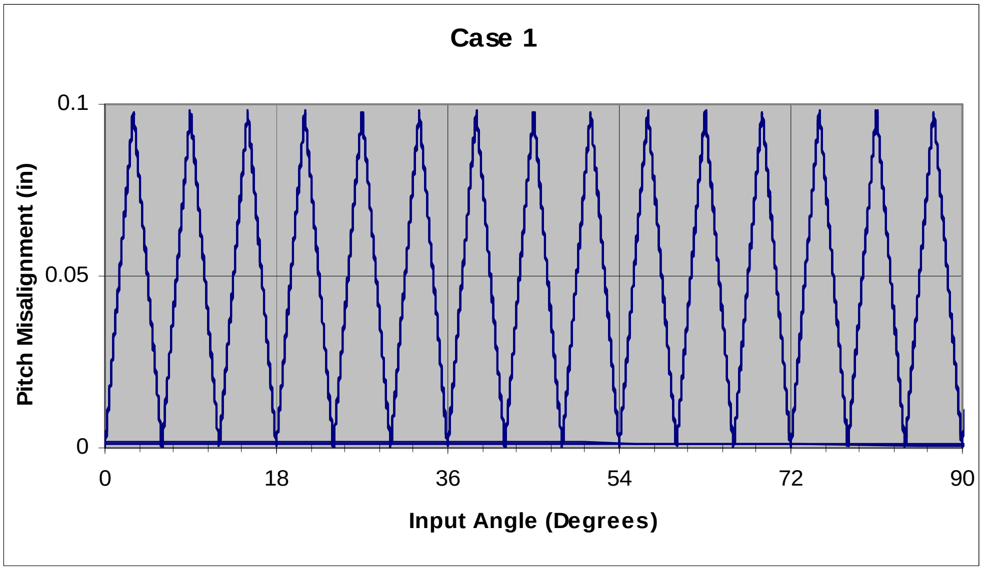


Figure 5.2: Plot of the Pitch Misalignment of the Driven Gear under Case 1 for Three Revolutions of the Input (Vertical Grid Indicates Points of Engagement)

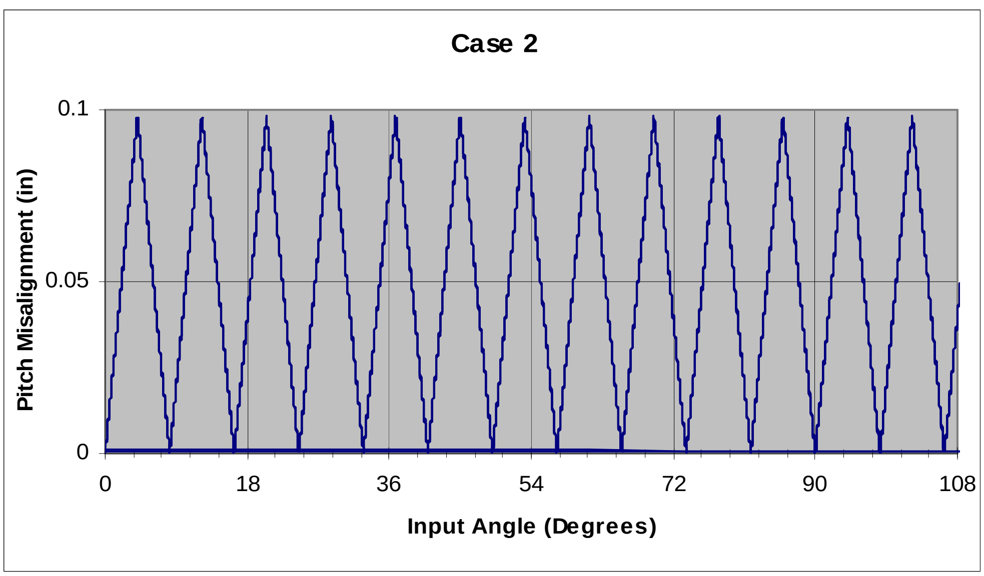


Figure 5.3: Plot of the Pitch Misalignment of the Driven Gear under Case 2 for Three Revolutions of the Input (Vertical Grid Indicates Points of Engagement)

Case 3: For each orbit of the drive gear about the reference gear, the drive and driven gears will rotate about their axes an angular amount that is not divisible evenly by the angular spacing between their teeth, but the amount of non-divisible rotation will remain constant for each revolution. Figure 5.4 shows a plot of the misalignment of the teeth of the driven gear relative to the drive gear under case 3 at each angle of the input arm for three revolutions of the input arm (again the plot only shows the range from 0 to 144 degrees of input arm position for ease of examination). In a case with five driven gears and four drive gears (see Figure 3.3), it would be necessary to examine the misalignment of the driven gear at intervals of 90° of input arm rotation, which corresponds to when a different drive gear will enter into engagement with the same driven gear. Figure 5.4 is representative of such a case, and demonstrates a misalignment of the driven gear when the arm is at 0° and 90°, 18° and 108°, 72° and 126°, etc. (a position where the input arm would align with a driven gear), which shows that the driving and driven gears will not mesh properly. The plot also shows that the amount of misalignment of the driven gear on the first revolution of the input arm is different from the amount of misalignment on the second revolution, as well as on the third revolution. Because the amount of misalignment at the same point on each orbit is not the same, a one time correction of the alignment will not ensure correct meshing for all subsequent orbits.

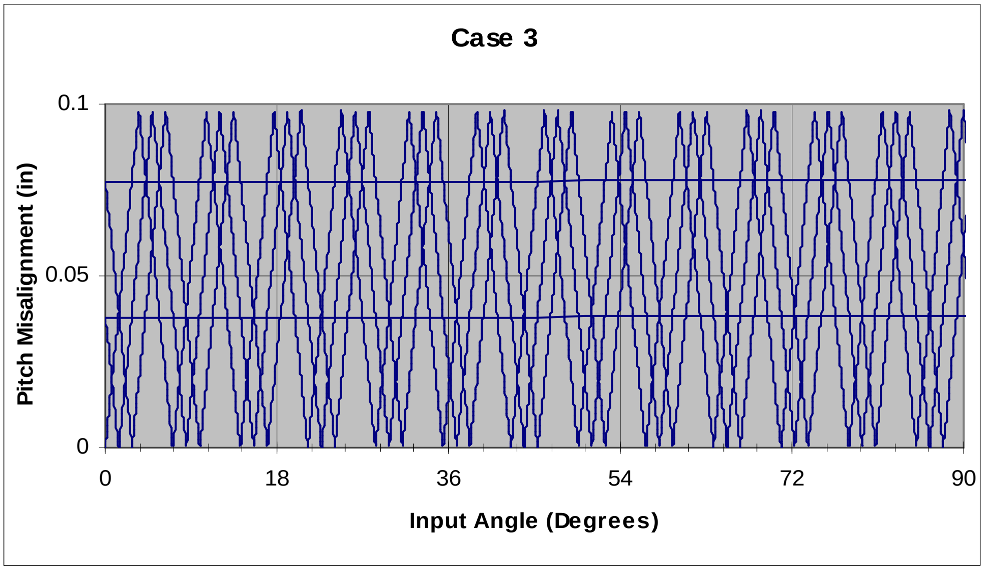


Figure 5.4: Plot of the Pitch Misalignment of the Driven Gear under Case 3 for Three Revolutions of the Input (Vertical Grid Indicates Points of Engagement)

Case three also occurs for certain instances when the drive and driven gears will rotate about their axes an angular amount that is divisible evenly by the angular spacing between their teeth for each orbit of the drive gear about the reference gear. This will cause the same orientation of drive and driven gear teeth to exist, relative to each other, at the same point on each orbit, as shown by the exact overlapping of the plot shown in Figure 5.5 for sequential orbits; however, the alignment of the teeth will not be correct at every point where a different drive meshes with the same driven gear (the amount of pitch misalignment is not the same at 0° and 90°). Figure 5.5 shows a plot of the misalignment of the teeth of the driven gear relative to the drive gear under this latter case at each angle of the input arm for three revolutions of the input arm (again the plot only shows the range from 0 to 144 degrees of input arm position for ease of examination). Again, note that the amount of pitch misalignment at 0° is not the same as that at 90°. Because the amount of misalignment every 90° on each orbit is not constant, a continuous correction of the misalignment would be required every orbit to ensure correct meshing.

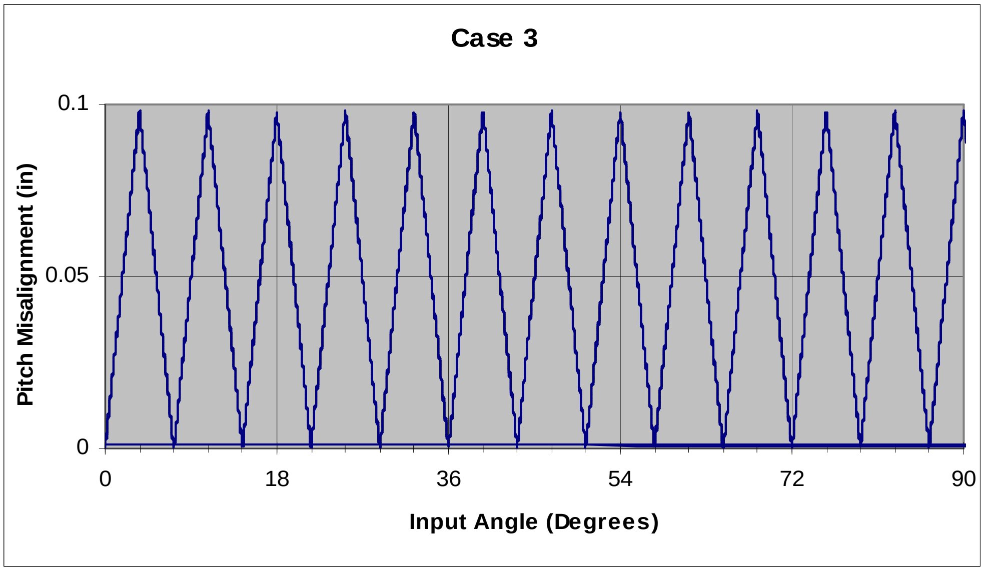


Figure 5.5: Plot of the Pitch Misalignment of the Driven Gear under Case 3 for Three Revolutions of the Input (Vertical Grid Indicates Points of Engagement)

While this chapter has only included the meshing analysis of embodiment 1, it is representative of the meshing characteristics of embodiments 2 and 3. An identical analysis was conducted on embodiments 2 and 3, and it was concluded that the cases presented are reflective of all three embodiments, and thus the meshing problems in embodiment 1 occur in similar form in embodiments 2 and 3.

CHAPTER 6 CHARACTERISTICS OF A SOLUTION

The previous chapters have described the nature of the non-integer tooth problem, and have detailed its occurrence in the new proposed embodiment, as well as three published embodiments belonging to the Positive Engagement family. This understanding of the non-integer tooth problem is the necessary foundation for understanding and generating a solution to the problem, not only for a particular embodiment, but also for the family in general.

Chapter 5 showed that the non-integer tooth problem is generally manifest as a misalignment of the drive and driven portions of the transmission when they should engage. This suggests two possible courses of action for approaching the generation of a solution to the non-integer tooth problem, as shown in Figure 6.1. The first method of addressing the problem is to generate a method of correcting the misalignment. This method would probably be specific to a particular embodiment, as the problem is manifested in unique ways for each possible embodiment. The second method of addressing the non-integer tooth problem is generate an embodiment which does not have the problem. This chapter will explore both possibilities for a solution, and provide, as possible, functional specifications for each case.

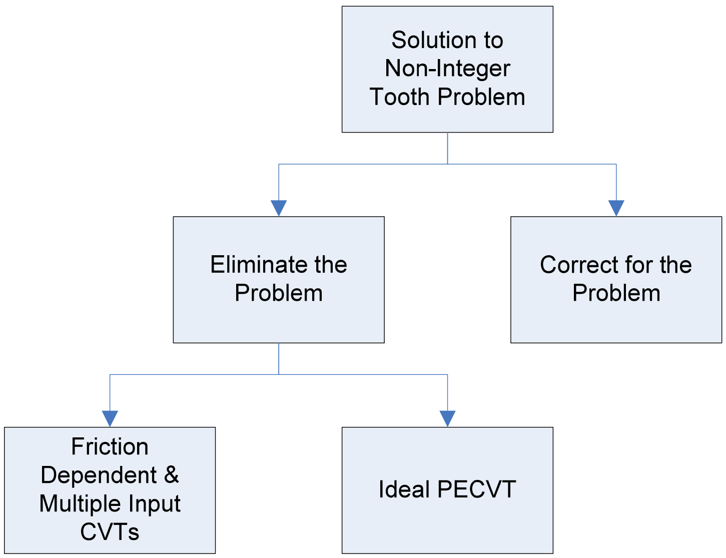


Figure 6.1: Approaches to Solving the Non-Integer Tooth Problem

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A Solution by Correction for the Problem

This section pursues the generation of functional specifications for a solution to the non-integer tooth problem through incorporating a correction. These functional specifications will describe what a solution must do to be a valid solution, not how it will do it. As stated, the method of correction should be unique to a particular embodiment. Therefore, this section will focus on the occurrence of the non-integer tooth problem in the new proposed embodiment, whose meshing was analyzed in Chapter 5.

Because of the complex nature of the occurrence of the non-integer tooth problem in the new proposed embodiment, a visual representation of the problem will clarify its occurrence and the nature of the corrections necessary to overcome the problem. Figure 6.2 shows an embodiment similar to that represented in Figure 5.1, having three drive gears and five driven gears. The virtual circle is represented as a chain having a pitch equal to that of the circular pitch of the gears, and having an integer number of links, or teeth, as described in the three cases presented previously. The representation of the virtual circle as a chain is solely done for visual purposes, aiding in showing the correct alignment positions of the drive and driven gears.

Figure 6.3 shows the top gear pair from Figure 6.2. The alignment of the drive gear relative to the chain (the virtual circle) is such that the teeth of the drive gear align with the pins of the chain. The alignment of the driven gear relative to the chain (the virtual circle) is such that the driven gear teeth align with the spaces in the chain, as a sprocket meshing with the chain would align. This orientation is the proper orientation of the drive and driven gears, and is necessary to ensure proper meshing. This same orientation of the drive and driven gears, relative to the chain representing the virtual circle, must occur for each of the drive and driven gears at any of their possible positions.

The image shown in Figure 6.4 shows the bottom left set of gears from Figure 6.2. It shows that both the drive and driven gears shown are in the correct orientation relative to the chain (the virtual circle), as described in the previous paragraph, to ensure their proper meshing.

In order to now demonstrate the occurrence of the non-integer tooth problem, Figure 6.5 shows the new proposed embodiment, similar to that shown in Figure 6.2, except that in this case the chain has a non-integer number of links, representing a virtual circle with a non-integer number of teeth. The misalignment of the links, shown in Figure 6.6 (upper gear pair from Figure 6.5), demonstrates that the chain has a non-integer number of links, and therefore the virtual circle has a non-integer number of teeth. Figure 6.6 also shows that the drive and driven gear of the upper gear pair are aligned correctly with the chain (the virtual circle). This alignment is purposeful, as it aids in manifesting the misalignment of the other drive and driven gears in the embodiment.

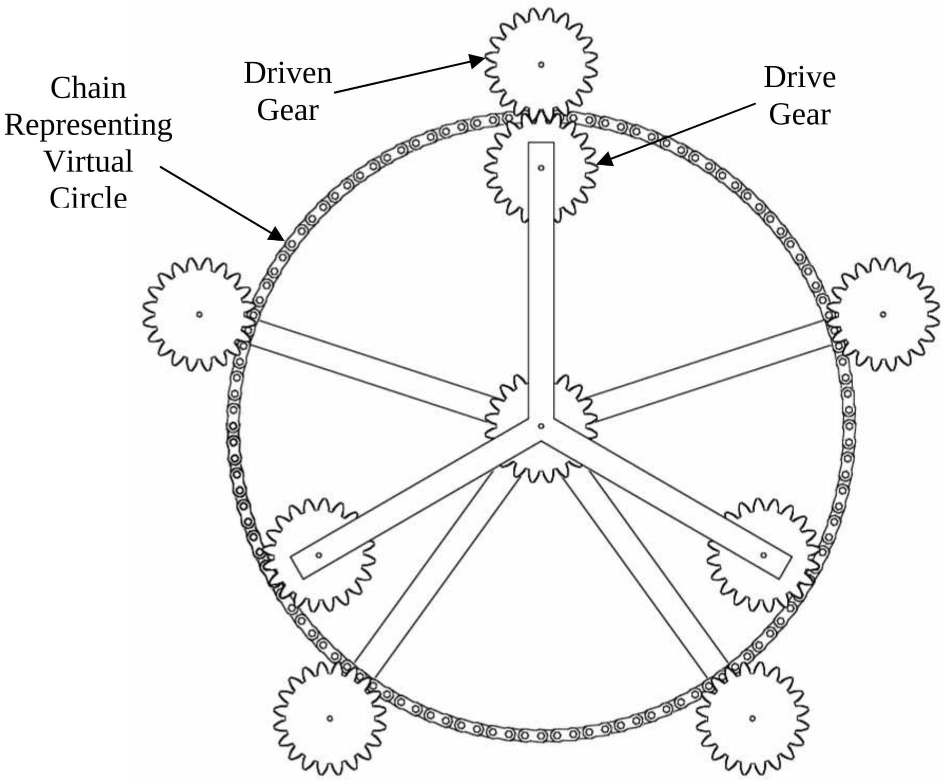

Figure 6.2: Visual Representation of New Proposed Embodiment

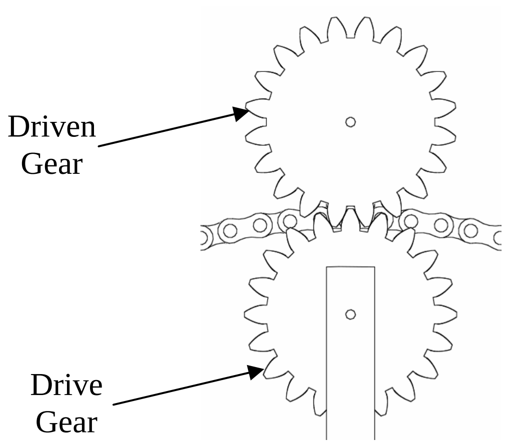

Figure 6.3: Top Gear Pair from Figure 6.2

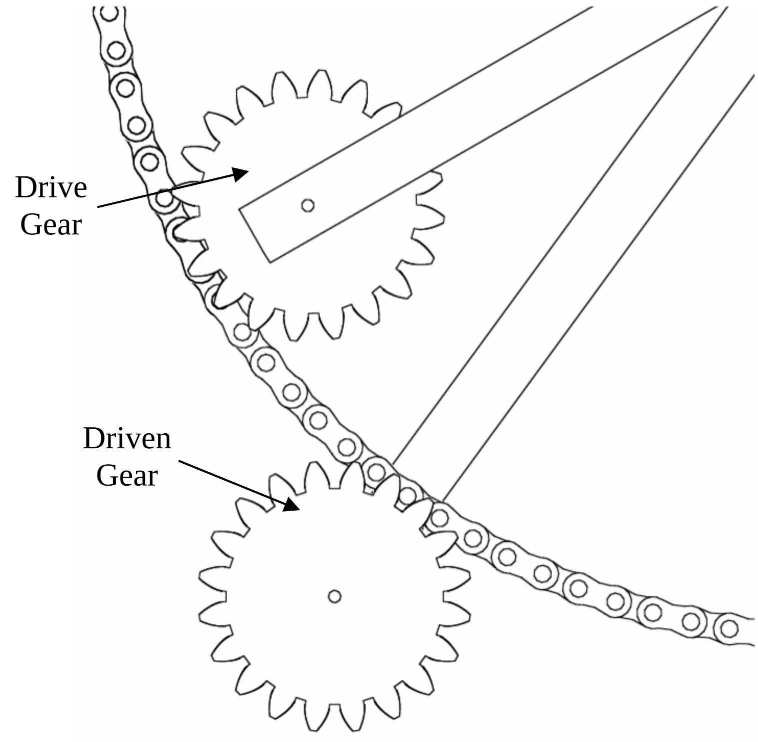

Figure 6.4: Bottom Left Gears from Figure 6.2

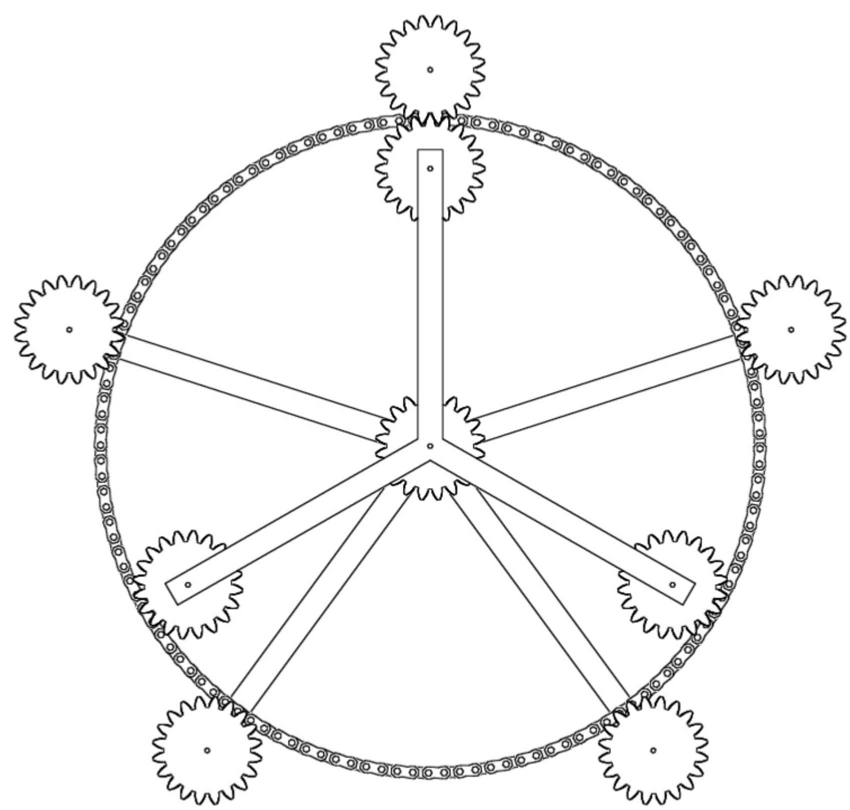

Figure 6.5: New Proposed Embodiment at a Ratio Expressing the Non-Integer Tooth Problem

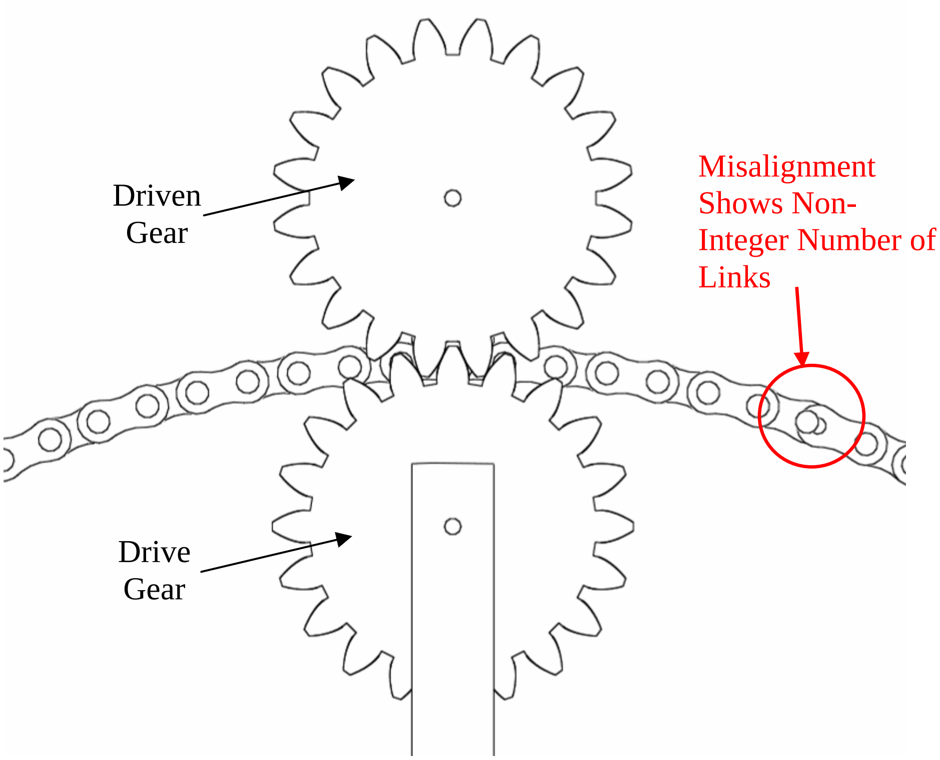

Figure 6.6: Non-Integer Link Portion of Chain from Figure 6.5

The misalignment of the bottom left drive and driven gears from Figure 6.5 is shown in Figure 6.7. It can be seen that the drive gear, which is intended to align with the pins of the chain, is slightly misaligned. The driven gear, which is intended to mesh with the spaces in the chain, is also slightly misaligned. This indicates that when the drive gear is intended to mesh with the driven gear, they will be misaligned.

While the misalignment shown in Figure 6.7 is very small, Figure 6.8 shows that the top left driven gear from Figure 6.5, which is intended to mesh with the spaces in the chain, is very misaligned. This indicates, when compared to the misalignment shown in Figure 6.7, that the amount of misalignment of the drive and driven gears is not the same for each drive and driven gear.

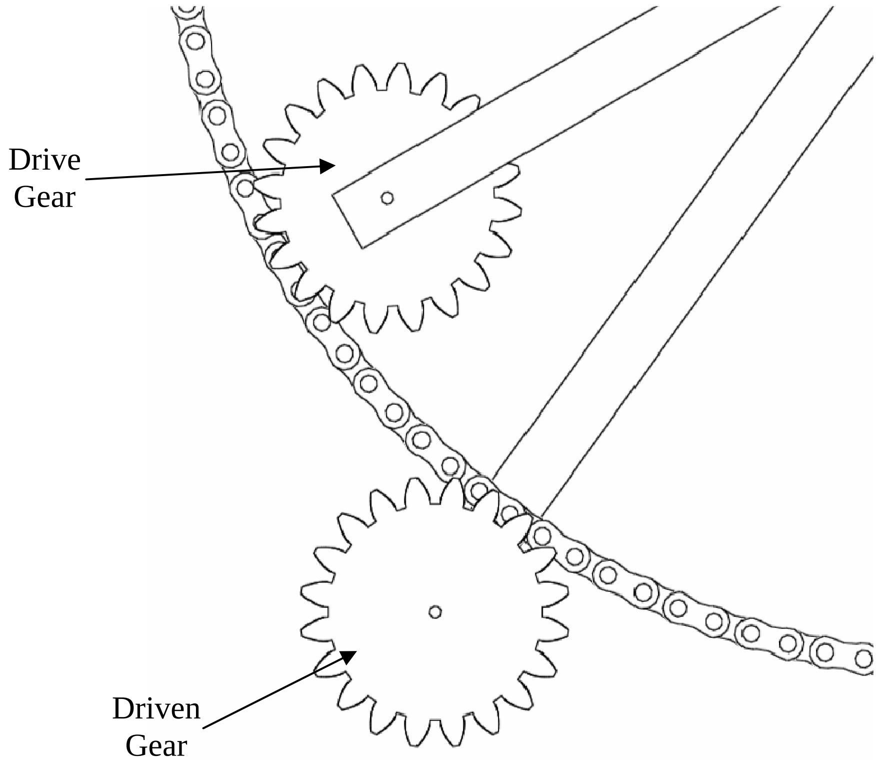

Figure 6.7: Misalignment of Drive and Driven Gears Due to the Non-Integer Tooth Problem

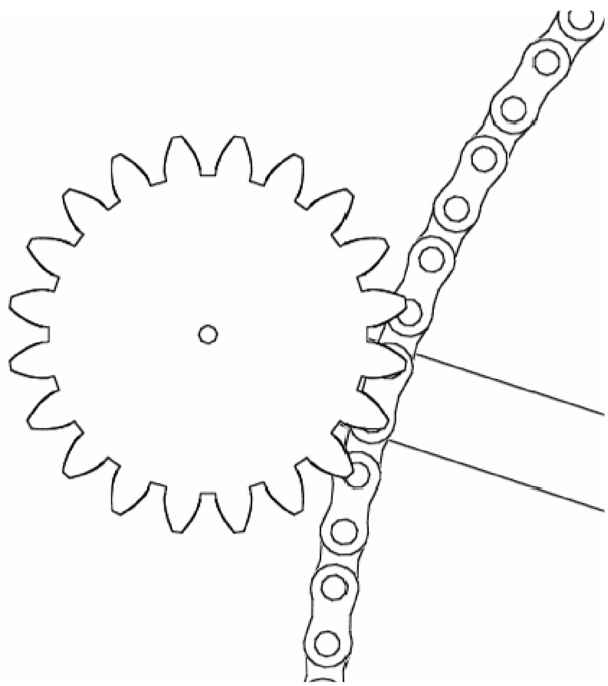

Figure 6.8: Top Left Gear from Figure 6.5

CHARACTERISTICS OF A SOLUTION BY CORRECTION

This graphical representation of the alignment of the drive and driven gears shows that the non-integer tooth problem occurs in the proposed embodiment as an accumulated misalignment of the drive and driven gears. This leads to the conclusion that a solution must involve the reorienting of the drive and/or driven gears to negate the accumulated misalignment before they are required to mesh. This reorientation could include adjustment of the rotation of the driven gears, relative to each other, which would correct for the misalignment, as shown in Figure 6.9. Also, the reorientation could include the translation of the driven gears about the virtual circle, thereby causing the misalignment to be zero at the point and time of meshing with the drive gears (see Figure 6.10). Both of the corrections for misalignment would, of necessity, need to be continuous in nature to accommodate the continuous accumulation of misalignment of the driven gears.

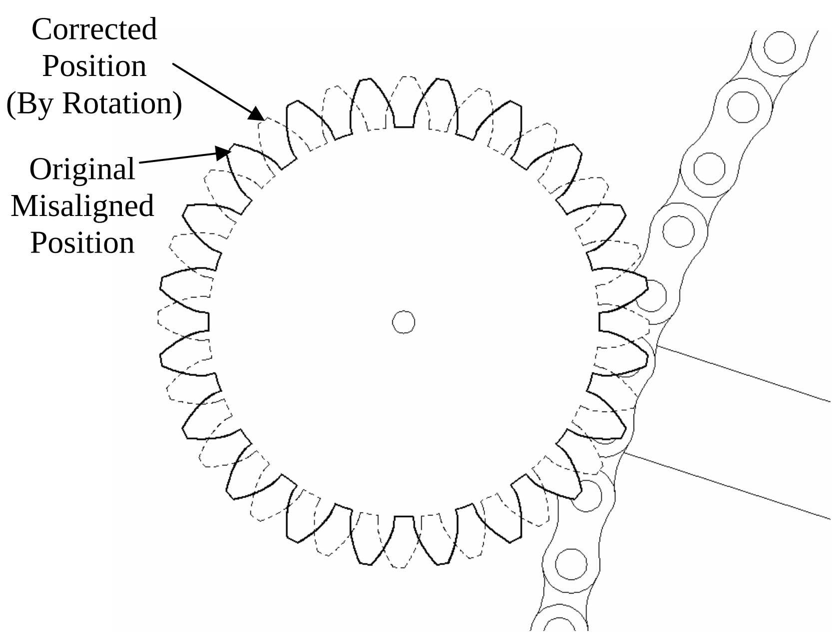

Figure 6.9: Correction of the Driven Gear Misalignment by Rotation

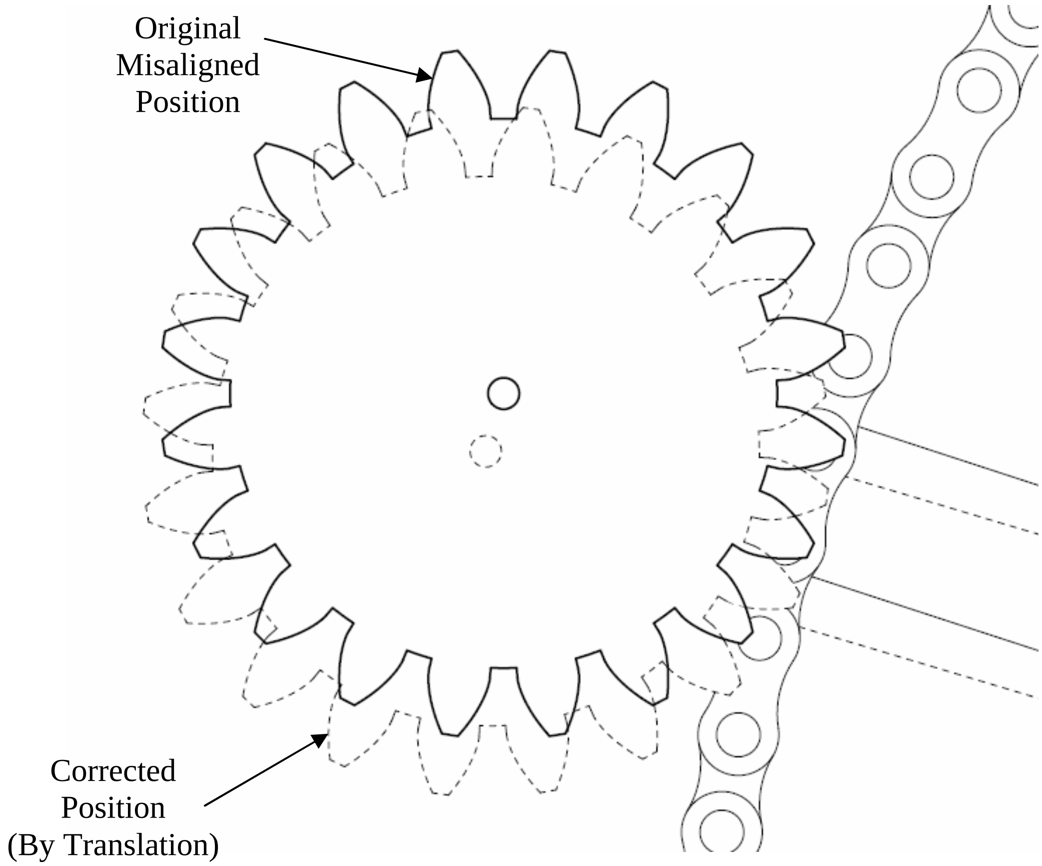

Figure 6.10: Translation Correction

QUANTIFYING THE AMOUNT OF MISALIGNMENT

In order to understand the amount of correction that is necessary in the proposed embodiment, it is important to quantify the amount of misalignment. The following equations allow us to quantify the amount of misalignment at any operating condition of the proposed embodiment.

To quantify the misalignment resulting from the non-integer tooth problem, the following equation must first be satisfied:

$$\frac{S}{X} = Integer \tag{6.1}$$

Where: S = # of Teeth on a Driven Gear

X= # of Driven Gears

Equation 6.1 ensures that the angular spacing of the driven gears is such that their position relative to the orientation of their teeth is correct for quantifying their misalignment. The ensuing equations allow us to quantify the amount of misalignment. The degrees of spacing between mesh points of the input and the output, with respect to the angle of the input arm is:

$$\frac{2 \cdot \pi}{X \cdot Q} = L \tag{6.2}$$

Where: Q = # of Drive Gears

L= Degree Spacing of Mesh Points with Respect to the Input Arm (in Radians)

The angle of the input arm, as a function of its rotational velocity, at any time t, is:

$$\theta_{Arm} = \omega_{in} \cdot t \tag{6.3}$$

Where: θ_{Arm} = Angle of Input Arm

ω_{in} = Angular Velocity of Input Arm (radians/sec)

t = time (sec)

Equations 6.2 and 6.3 tell us that there will be mesh points every time that the following equation is satisfied:

$$\theta_{Arm} = \frac{2 \cdot \pi \cdot n}{X \cdot Q} \tag{6.4}$$

Where: n = An Integer (1, 2, 3, …)

At this point the times when the mesh points will occur can be calculated from the following equation:

$$t = \frac{\theta_{Arm}}{\omega_{in}} \text{ or } t_n = \frac{2 \cdot \pi \cdot I}{X \cdot Q \cdot \omega_{in}} \tag{6.5}$$

Where: t_n = Time of the n^{th} mesh point

I = Maximum Integer that satisfies Equation 6.6

$$I \leq \frac{\omega_{Driven} \cdot t_n \cdot S}{2 \cdot \pi} \tag{6.6}$$

Now, proper alignment occurs when the driven gear has rotated a full tooth width, or a multiple of a tooth width, between the engaging of the input with sequential mesh points. If the following equation is satisfied, then no misalignment occurs:

$$\omega_{Driven} \cdot t_n = \frac{2 \cdot \pi \cdot n}{S} \tag{6.7}$$

Where: ω_{Driven} = The Angular Velocity of the Driven Gear About its Own Axis

If Equation 6.7 is not satisfied, then we must calculate the angle of the driven gear at the time when it should be engaged with the input. If the angular velocity of the driven gear (ω_{Driven}) is not changing with time (the equation for ω_{Driven} is derived in Chapter 3 for the proposed embodiment and its variants), then:

$$\theta_{Driven} = \omega_{Driven} \cdot t \tag{6.8}$$

Where: ω_{Driven} = Angular Velocity of Driven Gear (radians/sec)

θ_{Driven} = The Rotational Angle of the Driven Gear About its Own Axis

If ω_{Driven} is changing with time, then:

$$\theta_{Driven} = \int_{t_1}^{t_2} \omega_{Driven} \cdot t \cdot dt \tag{6.9}$$

To calculate the amount of misalignment when a driven gear is to be engaged with the input, it is necessary to subtract the actual angle of the driven gear at the time of meshing from the correct meshing angle. Therefore, the amount of misalignment (M) is equal to:

$$M = \theta_{Driven} - \left(\frac{2 \cdot \pi \cdot n}{S} \right) \tag{6.10}$$

In terms of the size of the teeth on the driven gear, the misalignment is:

$$M = \left(\omega_{Driven} \cdot t_n \right) - \left(\frac{2 \cdot \pi \cdot n}{S} \right) \tag{6.11}$$

Therefore, the necessary correction (C, in radians), assuming a rotational correction of the misalignment of the driven gear is:

$$C = M \cdot \frac{2 \cdot \pi}{S} \tag{6.12}$$

If the desired correction is instead a translational correction of the driven gear (translating it around the virtual circle), that amount of translational correction is:

$$C = M \cdot \frac{\pi}{P_d} \tag{6.13}$$

Where: P_d = Diametral Pitch of the Driven Gear

From the meshing analysis performed in Chapter 5, it is clear that the amount of misalignment is not constant for each sequential meshing of the input with a particular driven gear. It is possible, however, to quantify the maximum amount of misalignment that can occur. For any ratio in which misalignment will occur (cases 2 and 3 described in Chapter 5), the maximum amount of misalignment is equal to the circular pitch of the driven gear. This assumes that a correction by rotation can only occur by rotating the driven gear in one direction, and that a correction by translation can only occur by translating the driven gear in one direction around the virtual circle. If corrections were allowed in both directions, the maximum amount of misalignment would be equal to half of the circular pitch of the driven gear.

CHARACTERISTICS OF A SOLUTION BY CORRECTION IN THE GENERAL CASE

Applying these observations positive engagement family in general, it is clear that in the general case there is an accumulation of misalignment between the engaged members. This accumulation of misalignment results from the fact of changing the ratio of the transmission. The two methods of changing the transmission ratio of a positive engagement transmission are illustrated by examining the most basic of positive engagement transmissions – a gear pair – as shown in Figure 6.11. The gear pair shown represents one transmission ratio, which is determined by the ratio of the effective diameters of the two gears. To vary the transmission ratio of the transmission shown in Figure 6.11, the diameter of one of the gears must be either increased or decreased, relative to the other. This can be accomplished by adding teeth to or subtracting teeth from one of the gears while maintaining a constant diametral pitch. The difficulty with this method is that if the diameter is to be varied in a continuous manner, teeth will not be added in integer increments. This means that a gear with overlapping teeth will result, meaning that it will not mesh properly at at least one point (See Figure 6.12).

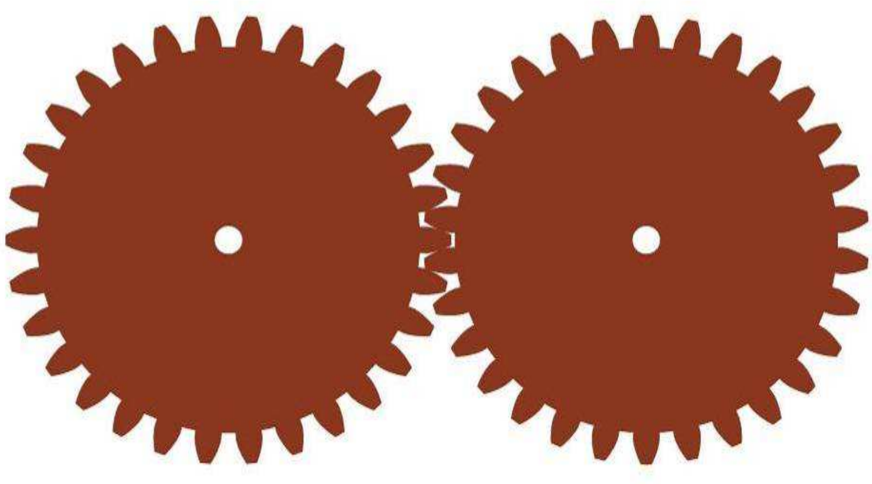

Figure 6.11: Most Basic Positive Engagement Transmission

The alternative is to increase or decrease the diametral pitch of one of the gears while maintaining a constant number of teeth. The diametral pitch can be varied in a continuous manner. This results in a continuously variable diameter, and therefore a continuously variable transmission ratio. Changing the diametral pitch of one of the gears, however, produces a mismatch of diametral pitches, meaning that the gears will not mesh properly.

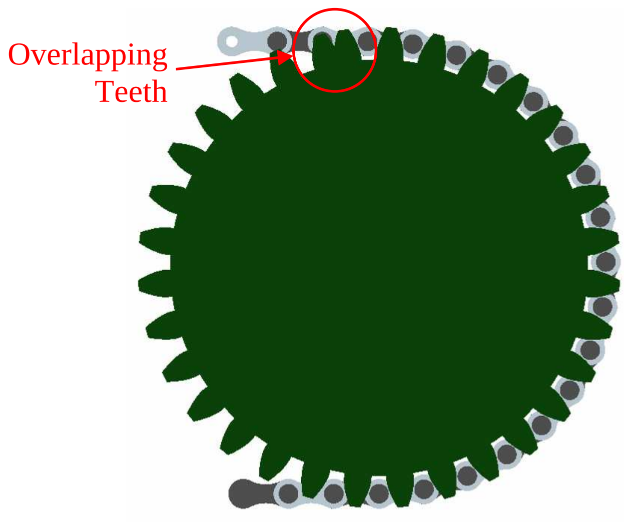

Figure 6.12: Gear with a Non-Integer Number of Teeth

The problem of increasing or decreasing teeth of one of the portions of the transmission in non-integer steps is shown previously in Figure 6.6. In the case presented, the full embodiment of which is shown in Figure 6.2, the set of three drive gears forms a virtual drive gear and each of the driven gears is a separate driven gear. As the size of the virtual drive gear is increased or decreased, it is akin to adding or subtracting teeth from the virtual gear. This is because the pitch of the virtual gear is forced to be constant because it must be equal to the pitch of the drive gears of which it is composed.

To correct for the misalignment resulting from a non-integer number of teeth on the variable size gear (virtual gear) of an embodiment, either a rotational correction of the constant size gear must occur, about its own axis, or it must be translated around the circumference of the variable size gear. Both forms of correction have been previously discussed for the specific case embodied in Figure 6.2. A rotational correction has also been discussed for the Fixed-Pitch CVT, described in Chapter 4.

The problem of unmatched pitches is clearly seen in the Anderson CVT (see Figure 6.13). The cones used in this case have a constant number of teeth, but due to the increasing diameter of the cone from one end to the other (changing diametral pitch), the circular pitch of the teeth is constantly changing as the ratio changes. The chain that meshes with the teeth on the cone has a constant number of teeth and a constant pitch, which means that when the circular pitch of the teeth on the cones is not equal to or evenly divisible by the pitch of the chain, the chain and the teeth on the cones will not mesh properly.

To correct for the misalignment resulting from the mismatch of the pitches of the drive and driven members, the circular pitch of one of the engaged members must be adjusted to match the other. This is accomplished by circumferential movement of the engaging portion of one of the members, as demonstrated by the floating sprocket bars (see Figure 6.14) in the Anderson CVT. It is important to note that the only purpose of a correction is to ensure that the pitches of the engaged members are compatible, which will ensure proper meshing.

Figure 6.13: Anderson CVT

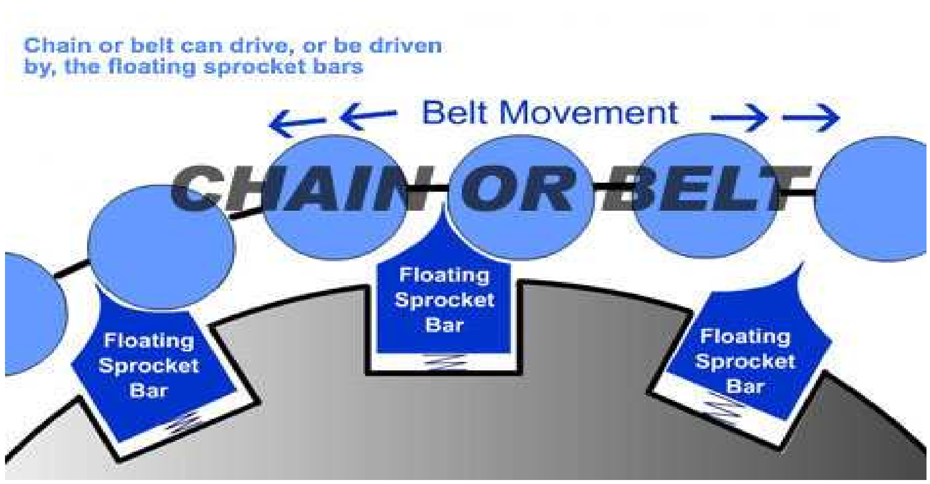

Figure 6.14: Floating Sprocket Bars

A SOLUTION BY ELIMINATION OF THE PROBLEM

An alternative method of addressing the non-integer tooth problem, instead of trying to correct for it, is through its elimination, as is shown in Figure 6.1. The review of literature presented in Chapter 2 has provided a brief introduction to the first method of eliminating the problem, which is to allow for power transmission either though friction or by providing a separate variable input from a device like a hydrostatic unit or electric motor. Relying on friction sacrifices the benefits of positive engagement, but gains the ability to transverse a continuous range of transmission ratios without being concerned with the alignment of the driving and driven portions of the transmission. The use of a secondary variable input is also less desirable because of the inherent inefficiency of hydrostatic drives and electric motors.

The second method of elimination of the problem is to create a PECVT that incorporates all of the benefits of positive engagement without being subject to the non-integer tooth problem. This would be the ideal CVT and would have the following basic characteristics:

- High torque capability
- High RPM ratio range
- High efficiency
- Provide continuous engagement of the input with the output portions of the transmission
- Provide positive engagement of the input and output without reliance on friction
- Generate no misalignment or be insensitive to misalignment between engaging portions of the transmission that would hamper meshing

The review of available literature on the development of modern CVTs reveals no published devices having all of these characteristics. The development of such a device has long been the goal of many transmission designers, but the difficulty of the problem has thus far precluded its invention. All attempts to produce the ideal CVT have attempted to address the non-integer tooth problem through some form of correction, rather than addressing the occurrence of the problem itself.

CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS

The purpose of this research has been to investigate the family of positive engagement, continuously variable transmissions, which have the possibility for higher efficiency and torque capabilities than the friction dependent CVTs that are currently in use. The analysis of the positive engagement CVT family reveals that all published embodiments belonging to this family must overcome a problem called the non-integer tooth problem. This research has described this problem as it exists in three published embodiments. This has been done to show several ways in which this problem can be manifest.

This research has also investigated a conceptual embodiment of a new positive engagement, continuously variable transmission that was proposed for investigation at Brigham Young University. This research has examined both the kinematic and meshing characteristics of the proposed embodiment, as well as two variant embodiments. From the derivation of the kinematic equations governing the motion of the proposed new transmission in its various embodiments, it can be concluded that each of the embodiments would function kinematically, allowing the selection of infinitely variable gear ratios over a finite range.

The meshing analysis of the proposed embodiment, as well as the two variants, however, has shown that a meshing problem exists between the driving and driven portions of the transmission. This problem has been identified as the non-integer tooth problem. The cases in which the non-integer tooth problem occur in the proposed new embodiment have been classified, which aids in understanding the problem, as well as the general characteristics of a possible solution that may be applicable to the entire PECVT family.

While solutions to the non-integer tooth problem have been presented from published embodiments of the positive engagement family, which essentially accomplish the matching of pitches between the driving and driven members, the presented solutions are not optimal solutions. This is because these solutions reduce the efficiency of the device and lessen the ability of the transmission to transmit continuous power without oscillations. An optimal solution would eliminate the effects of the non-integer tooth problem or eliminate the problem altogether. The optimal solution would not rely on overrunning clutches or another method of overcoming the non-integer tooth problem.

Perhaps the most important contribution of this research is the method that it establishes for analyzing embodiments of the positive engagement family. This method establishes an approach by which future embodiments can be analyzed to determine their functionality, as well provides a method by which to compare the operating characteristics of various embodiments. More importantly, the understanding of various positive engagement CVT embodiments that the method provides is the foundation for the generation of a successful solution to the non-integer tooth problem.

RECOMMENDATIONS FOR FUTURE RESEARCH

Because the purpose of this research has been to expose the nature of the non-integer tooth problem in the PECVT family and generate the characteristics of a solution to the problem, no work has been done to transform the proposed characteristics into a viable solution. Therefore, future work should center on using this understanding of the problem, as a guide for finding a solution which meets the presented characteristics. Also, as future embodiments are produced, the analysis methods employed in this research can be used as a guide for determining their meshing characteristics.

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