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An Approach to Concept Development for Compliant Mechanisms Possessing High Coefficients of Restitution

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AN APPROACH TO CONCEPT DEVELOPMENT FOR COMPLIANT MECHANISMS POSSESSING HIGH COEFFICIENTS OF RESTITUTION

by

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ABSTRACT

AN APPROACH TO CONCEPT DEVELOPMENT FOR COMPLIANT MECHANISMS WHICH RETURN ENERGY UNDER IMPACT LOADING

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The design of structures and mechanisms subject to impact loading has historically involved designing in such a way as to minimize damage induced by the impact. This has traditionally been accomplished by absorbing and dissipating the energy of the impact. However, in some applications it is desirable to harness the energy and return it to the impacting object to maximize the coefficient of restitution (COR), resulting in large rebound velocities. The use of traditional rigid-body mechanisms to achieve high-COR mechanisms is limited by issues of friction, durability, poor strain-energy distribution and others. Compliant mechanisms do not posses the same limitations and are well-suited to these types of applications. The principles needed to realize these types of designs are found in existing literature but are confined to very specific applications such as hollow-body golf club heads.

The contribution of this thesis is an approach to the generation and evaluation of compliant mechanism concepts for use in impact applications where a high COR is required. This approach is based loosely on common general concept development processes found in literature. This thesis describes the process including the use of lumped mass or mechanical models, the categorization of strain-energy storage, the use of both closed-form and finite-element static models and the use of dynamic finite-element models to determine if a configuration is eligible to be used in a final design process. This thesis also contributes a case study in the development of configurations for metalwood golf club driver heads.

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CHAPTER 1 INTRODUCTION

1.1 Motivation

When an object experiences a change in velocity due to contact with another object, there are resultant forces on both objects. Newton's second and third laws describe this phenomenon. The force acting on the objects depends on the mass of both objects, the velocity changes of the objects, and the amount of time over which the contact and the changes in velocity occur. When the time duration of the contact is short, and/or the changes in velocity are large, the forces involved between the two objects become very large.

These large forces present a problem when the colliding objects are structures or mechanisms. The forces can lead to damage severe enough to compromise the integrity of the structure or the functionality of the mechanism. Regardless of whether an object's intended use involves impact loading or not, designers must still often consider impact loading and its consequences on their product. For this reason, impact loading is often an important factor in the design of new products.

In most cases, an object is designed to absorb and dissipate the energy so that damage to the object is minimized. Sometimes this is accomplished through adding otherwise useless mass in order to aid in the absorption of the energy. This increases the inertia of the impacted object which reduces the change in acceleration experienced. This brute-force method is illustrated by the armor added to military vehicles. Another approach is to add an energy-absorbing mechanism to an existing structure or system. This approach can be seen in the development of low-speed crash bumpers on modern automobiles.

When a load is periodic in nature, a common practice is to design a structure or mechanism so that the low natural frequencies of the structure do not coincide with the frequency of the applied load, thereby avoiding resonance. This is commonly seen in seismic engineering of buildings or in rotating machinery.

It may be desirable to return the energy from the impact to one or both of the objects. In these cases, the energy from an impact needs to be carefully controlled and directed in order to maximize the efficiency of the impact.

A good measure of the efficiency of an impact is the coefficient of restitution (COR). This factor is a ratio of the post- and pre-impact relative velocities. For a perfectly elastic collision, the COR is equal to one while in a perfectly plastic collision, the COR is equal to zero.

One way to accomplish energy return in an impact is to design a mechanism that transforms the kinetic energy into potential energy. A mechanical system designed to transfer or transform motion, force, or energy is defined as a mechanism [Howell, 2001].

The energy can then be transferred back to the impacting object in the form of kinetic energy.

Mechanisms traditionally have rigid bodies connected by revolute or sliding joints to control motion, and torsional or linear springs to store energy. The use of rigid-body mechanisms to return energy has several disadvantages. Friction in joints results in energy losses which lower efficiency. In addition to lower efficiencies, without very tightly controlled tolerances in the joints, secondary impacts occur there, producing higher stresses and possibly reducing reliability and longevity. Traditional linear and torsional springs with the desired stiffness characteristics may fail to meet other functional criteria such as mass or size requirements. Rigid-body mechanisms may also be at a disadvantage in these types of applications because of poor strain energy distribution. In a rigid-body mechanism, most of the mass has little strain energy stored in it so there is more mass which adds to inertia and may reduce the COR.

General considerations for high COR are well represented throughout literature. These general considerations include mass, stiffness, natural frequency and material behaviors. However, these general considerations are limited to very few, narrow applications and the research doesn't contain a general approach for evaluating concepts. Because of this, a new approach needs to be developed to aid in the concept development for compliant mechanisms possessing high coefficients of restitution.

1.2 Discussion

Given the limitations of rigid-body mechanisms described above, it is desirable to develop an improved approach to the development of mechanisms to return energy to an impacting object. One approach makes use of compliant mechanisms. In order to understand the advantages of this approach, some background discussion on impact loading and compliant mechanisms is provided below.

1.2.1 Impact Loading

Forces are categorized as impact or static only by the time duration of the force with respect to the natural frequency of the system to which the force is applied. When the time duration of the load (defined as the time it takes the load to rise from zero to its peak value) is half the period of the natural frequency of the system or less, it is usually considered an impact load. When the duration of the load is more than three times the natural frequency of the system, it is considered a static load. In between those two extremes is a situation in which the forces can be classified as either impact or static [Norton, 1998].

Before an impact occurs, either one or both of the objects in the impact have a certain amount of kinetic energy and momentum due to its velocity. A very small amount of the kinetic energy is converted into heat and sound during the collision, but the vast majority of it is either converted into strain energy in the objects themselves, into kinetic energy in the form of vibrations, or is transferred between objects and results in post-impact velocities. The design of the objects and the nature of the collision are what determine how the energy gets distributed between strain energy, vibrational energy, post-impact kinetic energy and wasted energy.

The impact of a golf club on a golf ball is a good illustration of how COR is a measurement of impact efficiency. In reality, the club approaches the stationary golf ball with some initial velocity. However, the impact can just as accurately be modeled as a stationary golf club being approached by the ball with an initial velocity. In this model, the ball strikes the face of the club. The primary purpose of the club is to maximize the COR of the impact, meaning the club absorbs a maximum amount of energy from the impact and returns that energy to the ball in the form of velocity.

1.2.2 Compliant Mechanisms

A compliant mechanism is defined as a mechanism that gains all or some of its motion from deflection of flexible members [Howell, 2001]. This differs from a rigid-body mechanism in that rigid-body mechanisms gain all or some of their motion from the rotation or translation of rigid members about kinematic joints, including pins, sliding joints, etc. Many advances have been made in the last decade in the design and analysis of compliant mechanisms. With these advances, the opportunities to implement compliant mechanisms in place of rigid-body mechanisms have grown.

Compliant mechanisms traditionally offer several advantages over rigid-body mechanisms. The elimination of kinematic joints also eliminates wear, particle generation, and friction in joints. Other advantages include the ability to tightly control the motion of

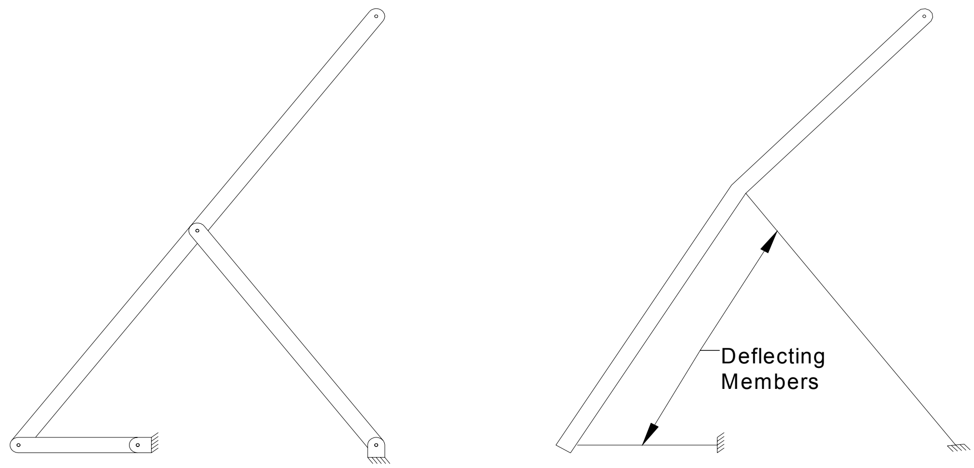


Figure 1.1 A straight-line mechanism and its compliant equivalent.

the mechanism and the reduced cost of assembly and often manufacturing. Compliant mechanisms are also often more reliable since there are fewer parts.

Several of these advantages would be conducive to the design of mechanisms for high-COR impact applications. Because friction and its associated energy losses are eliminated, more energy can be controlled and returned. A compliant mechanism has a continuous geometry, so the problems associated with secondary impact would be eliminated. Compliant mechanisms are often more durable because of the reduction or elimination of wear and particle generation. Stiffnesses can also be more tightly controlled because it is a function only of geometry and material properties and does not include friction.

1.3 Objective

The characteristics of compliant mechanisms make them an ideal fit for use in high-COR impact applications. The objective of this thesis is to identify and define an approach for generating and evaluating compliant mechanism concepts for use in high-COR applications. The approach is defined by modifying an existing concept development process and adapting it by taking the general principles relating to high-COR design found scattered throughout the existing literature, organizing and clarifying them, and applying them.

This approach is illustrated through a case study in the evaluation of concepts for a golf club head which makes use of compliant mechanisms to achieve a high coefficient of restitution. The case study and the research focus on generating and evaluating a number of concepts to determine which are viable and would be suitable for a more detailed final design process.

1.4 Contribution

The main contribution of this thesis is to produce a well-defined approach that can be used by a designer to generate and evaluate a number of compliant mechanism concepts for use in an impact application in which a high coefficient of restitution is desirable. This process can be applied to a much wider variety of applications than the procedures currently found in the literature. This is illustrated through the case study involving a golf club head, highlighting factors considered to be important in impact loading during the evaluation phase of the case study. It also contributes to the understanding of behavior of compliant mechanisms experiencing impact loading.

1.5 Thesis Outline

Chapter 2 contains the results of a review of literature of previous research on impact loading and compliant mechanisms, as well as specific energy-return mechanisms. Chapter 2 also includes a brief discussion and literature review concerning advancements made in golf club technology. Chapter 3 outlines the research approach and introduces the process involved in the evaluation of energy-return compliant mechanism concepts under impact loading. Chapter 4 is a discussion of lumped-mass models and their use in identifying more specific specifications. Chapter 5 describes the process of classifying methods of strain energy storage to generate some general compliant mechanism concepts. Chapter 6 is a discussion of the use of static analysis of compliant mechanisms in order to produce more specific configurations and then use the results as a the first criteria for evaluating concepts. Chapter 7 discusses the process of taking a configuration obtained from a static analysis, further evaluating and refining it through finite element dynamic simulations to determine if it is fit for further development. Chapter 8 consists of the case study of the golf club. Chapter 9 contains the conclusions and recommendations obtained from this thesis.

CHAPTER 2 LITERATURE REVIEW

In considering existing literature applicable to this thesis, two major categories were reviewed. These two categories are compliant mechanisms and impact. With respect to impact, a small amount of basic background as well as current literature were considered. A section concerning the history and current state of golf club research is also included here in order to provide context for the case study.

2.1 Compliant Mechanism Background

The idea of using deflecting members to gain motion and energy storage as opposed to rigid members connected through kinematic joints is nothing new. Nature abounds with a variety of ways of using this principle. Mankind has also used compliance in catapults and bows which have existed for thousands of years. However, many new applications of compliant technology required more advanced materials in order to be viable.

Many of the advantages of compliant mechanisms are mentioned in Chapter 1. In addition to these advantages, compliant mechanisms also possess some unique challenges that needed to be solved before they could be truly useful as replacements for many rigid-body mechanisms. Some of these challenges included finding new ways to analyze large deflections, finding ways to relate compliant mechanism kinematics to rigid-body kinematics and others.

In order for compliant mechanisms to be useful as replacements for many rigid-body mechanisms, the ability to analyze deflections beyond the linear range was needed. Many models used to analyze deflections in beams, such as those taught in strengths of materials or machine design courses make use of simplifying assumptions which limit their usefulness to small deflections. While these assumptions may be perfectly valid and justified in many applications, some compliant mechanisms experience large deflections which undermine the linear models' accuracy.

To account for large, nonlinear deflections, new models must be used. Some of these include the use of elliptic integrals or numerical methods. One additional method that has great usefulness is the pseudo-rigid-body model [Howell, 2001; Howell and Midha, 1995]. The premise is that a beam which derives its motion from bending can be modeled as a rigid beam with a torsional spring or springs and pin joint(s) at calculated positions. The positions of the pin joints and the torsional springs are calculated depending upon the end and loading conditions (standard cantilever beam, fixed-guided segment, small-length flexural pivot, fixed-fixed beam with different loading conditions). The torsional spring constants are calculated as a function of the bending moment of inertia and the material properties. The pseudo-rigid-body model also provides methods for calculating stresses. The pseudo-rigid-body model is limited in some respects because models for all compliant mechanisms have not been developed.

The pseudo-rigid-body model is usually applied for static loads. However, research has also shown that it can be useful in predicting the first modal frequency of a compliant mechanism [Lyon and Erickson, 1999]. Because the COR is related to the first modal frequency, this may be an important factor if the impacted mechanism is experiencing large deflections.

2.2 Impact Background

Impact has been a topic of scientific study and research for centuries. Galileo was the first to develop the initial concepts used to analyze rigid-body impact. Newton furthered the knowledge of impact and also introduced the idea of the coefficient of restitution [Goldsmith, 1960].

The most basic (and most predominant) approach to analyzing impact problems is called stereomechanics [Goldsmith 1960]. This method of analysis makes use of the impulse momentum as well as the conservation of momentum relationships shown in equations (2.1) and (2.2), respectively.

$$\Delta m\vec{v} = \int_{0}^{t} \vec{F} dt \qquad \textbf{(2.1)}$$

$$\sum_{i} m_i \vec{v_i} = \text{constant} \tag{2.2}$$

Stereomechanics also makes use of the conservation of energy relationship shown in equation (2.3).

$$\frac{1}{2}\sum_{i} m_i v_i^2 = \text{constant} \tag{2.3}$$

When limited only to these relationships, all energy losses in the impact are considered negligible. In order to account for energy losses during an impact, the coefficient of restitution must be introduced. The coefficient of restitution is defined as the ratio of the relative velocities before and after impact. For one-dimensional impacts, equation (2.4) represents the coefficient of restitution.

$$e = \frac{v'_1 - v'_2}{v_2 - v_1} \tag{2.4}$$

Where the numerator is the relative velocity after impact and the denominator is the relative velocity before the impact.

While the coefficient of restitution can be used to model general energy losses in a system, it is a difficult variable to predict using stereomechanics and must often be found by experiment. Stereomechanics also makes no claim to be able to account specifically for the lost energy. It is not known how much of the energy is lost to internal damping and plastic strain of the material or other possible sources of energy loss. If the sources of energy loss are not understood, it becomes very difficult to design with the objective of minimizing those losses. Stereomechanics also treats the impacting objects as point masses, thus no information is available concerning deflections or stresses involved in the impact.

In order to improve the analysis of impacts, vibration theory is included. Vibrations in impact were first studied by Bernoulli, Navier, and Poisson [Goldsmith, 1960]. The study of vibrations is the study of the transformation of kinetic energy to potential energy and vise-versa. In impact, this corresponds to the energy from the motion of the impacting objects being transformed to potential energy in the form of strain energy and then returning some portion of that energy to a kinetic form. Vibration theory allows for better understanding of the sources of energy loss as well as the forces involved. Deflections and stresses can also then be better understood.

Vibration theory may include the use of lumped mass or mechanical models. A perfectly elastic material can be modeled as a series of masses connected by linear or non-linear springs. Some materials require the addition of dashpot elements in order to model internal damping and visco-elastic behaviors. These include the Kelvin-Voigt, Maxwell, and standard linear solids [Goldsmith, 1960].

Part of the reason why vibration theory is important in impact analysis can be seen from the example of a simply supported beam which is impacted transversely by a mass moving at a given velocity. Using conservation of momentum and conservation of energy, the impacting mass would be modeled as a single point mass, and the entire mass of the beam would also be modeled as a single point mass. In reality, only a portion of the mass of a beam in bending should be included in its inertia because the entire beam is not in motion. Along with other advantages, vibration theory can help determine how much of that mass should actually be included for inertial considerations.

2.3 Current Impact Research

Research on impact is very much alive and well. For the purposes of this thesis, current literature and research on impact has been limited to three categories. The first of these categories is material considerations. Much of the existing literature is related to the way specific materials deal with impact and how to improve materials behaviors under impact loading. The second category included here is the design of mechanisms to increase the coefficient of restitution after impact. The third category relates to golf research.

2.3.1 Material Considerations

Technology in the twentieth century made extraordinary progress in the development of new materials. Many of the technologies which society appreciates today required new materials to be developed before they could be realized. Composite materials are a specific example. The research cited under material considerations are mostly cited for reference only, and are not related directly to this thesis.

As new materials are developed, the material properties need to be understood in order to predict how the materials will respond to different applications. This is the case with impact applications. Most impact analyses are carried out with respect to energy-absorbing structures. Thus the materials in those structures become greatly scrutinized.

There has been a great deal of research done on reinforced concrete [Miyamoto and King, 1996] and composite materials [Liu and Swaddiwudhipong, 1997] under impact loading because of the fact that they are widely used in structures which may experience impact loading.

Materials are also constantly being modified so they exhibit specific properties. Much current research on composite materials focuses on making them more resistant to impact damage if they are to be used in a specific impact application. For this thesis, material considerations are limited. Basic mechanical properties (density, strength, modulus, etc...) are the main considerations. Some of these properties are strain-rate dependent.

2.3.2 Coefficient of Restitution

Most of the current research addressing the coefficient of restitution is very application-specific and deals with sports equipment such as tennis rackets and golf clubs. Coefficient of restitution has been discussed in golf literature for quite some time [Cochran and Stobbs, 1968]. But most of the early references were merely definitions. It wasn't until the mid 1990's that the research shifted to determining ways to design clubs specifically to increase the COR [Science and Golf; Michal and Novak, 2001]. This research is specifically about how to make the current hollow-body design more efficient. While the applications and the scopes of the individual papers vary, some general principles can be obtained from them.

One of the clear principles is the fact that to increase the COR in a collision, both objects involved in the impact must be considered during the design. Brody [1995] notes that because tennis balls are designed in such a way that they will absorb energy when they deform, the strings of the tennis racket can be designed and set to deform more than the ball, thus increasing COR. Yamaguchi and Iwatsubo [1999] determine that high COR's are obtained when the mechanical impedance of the ball *and* the clubface are matched. Lumped-mass models of *both* the ball and the club are used by Cochran [1999] to simulate impact.

Several of the key design parameters also become apparent throughout the literature as well. The parameter which is given the most attention is the stiffness of both objects. Because the existing literature focuses on the current or traditional design of metalwood heads, much of the discussion focuses on making the club face more flexible by making the club face thinner [Johnson and Hubbell, 1999]. In order to make the face thinner, more exotic materials with higher stress limits must be used. One additional way to increase the flexibility is to use materials with lower Young's modulus values. The ratio of the yield strength to the modulus is a key variable in all compliant mechanisms because it indicates the amount of elastic energy storage that can be stored [Howell, 2001; Michal and Novak, 2001].

Other research mentions the idea of the natural frequencies associated with the club and the ball. The impedance matching that was mentioned earlier is related to natural frequencies. Some discusses the use of finite element analysis to determine the natural frequencies of golf clubs [Hocknell et all, 1998]. It is clear that mass is an important design consideration, not only because natural frequencies are a function of both stiffness and mass, but because the conservation of momentum plays and important part in impact analysis.

While the existing literature is helpful in identifying some key parameters, it is limited by the fact that the scope of the existing literature which focuses on design for high COR is very narrow. It focuses on tennis rackets or golf clubs. Even then, it focuses only on the traditional design of those objects.

2.4 Golf Club Literature

The technology of golf has advanced through an evolutionary process. From the replacement of hickory shafts with the latest composite materials to the replacement of persimmon hardwood driver heads with advanced metallic glass, material advances have allowed for the most significant advances in golf technology [Shira and Froes, 1997]. It has been pointed out that advances made in golf clubs have been paralleled by advances in tennis rackets [Davis, 1997].

The invention of the hollow-body metal wood has not only effected the COR, but probably had even a more significant impact on the rotational stability by increasing the polar moment of inertia of the club head [Winfield and Tan, 1996; Davis, 1997].

Much has also been written about the concept of "feel" in a golf impact and whether or not feel can be engineered. Interestingly, feel is most often associate with the sound made during a club/ball impact [Varoto and McConnell, 1995; Hocknell et all, 1998]. However, feel is also related to the vibrations transmitted through the shaft and felt by the golfer [Varoto and McConnell, 1995; Ekstrom, 1996].

2.5 Summary

The existing literature contains those principles and design characteristics that are essential in a high COR design. However, these principles are applied to only a very narrow band of technology (traditional hollow-body golf club designs) and nothing exists which takes the principles in the existing literature, collects them and applies them much to a new, comprehensive process for concept development.

CHAPTER 3 RESEARCH APPROACH

3.1 Introduction

This chapter outlines the process used accomplish the objective of this thesis, namely to create an approach used to generate and evaluate compliant mechanism concepts to be used to obtain maximum energy return under impact loading.

A key portion of any product development process is concept generation and selection. Ulrich and Eppinger [2000] have named this portion of the process "concept development". According to their model, concept development includes the identification of customer needs and translation of those needs into functional specifications. The next step is the generation and selection of concepts according to those specifications. In this thesis an additional step has been added after concept generation and selection. That step is a refinement of the chosen configuration. After this is accomplished, the configuration is tested in order to determine if it should be forwarded into a more final design process.

The process described above forms the basis for the research contained in this thesis. These steps were modified to fit a more specific process of developing compliant mechanism concepts that would be suitable for final design for use in an impact application in which a high coefficient of restitution is required. This chapter outlines each step in the process and describes how they were adapted to high-COR impact applications.

3.2 Functional Specifications

Understanding the desired specifications of a product is vital to developing one that is worthwhile and useful. These specifications become the standards by which concepts are judged worthy of further development. For this thesis, the process of defining specifications is divided into two parts.

Traditionally, customer needs and specifications are developed through a process of talking with users of the product and then translating their comments into more technical metrics which can be quantified. This "customer input" process remains an integral part of any concept generation and evaluation process. It is no different for compliant mechanisms which produce high COR's under impact loading. Some general information must be gleaned through simply asking questions. What types of impacts can be expected? What objects will be impacting the mechanism? What are the speeds involved? What has been done in the past? What other constraints should be considered like size and mass? What is the desired COR? Many of the specifications can be determined through this process. Because the customer input process is general to most product development, it will not be discussed in further detail in this thesis.

However, some of the more detailed specifications will not be known unless some of the more basic physical phenomena are understood. There are a variety of tools that can be used to aid in the understanding of impacted systems. The tool used in this research is lumped element or mechanical models.

Lumped element models are models in which the bodies involved in an impact are broken down into equivalent lumped mass (point mass) elements and massless springs. This includes both the impacting and impacted object. Any energy losses are modeled as damper or dashpot elements. The most simplified lumped element model would be two point masses connected by a spring.

In order for the lumped element models to be accurate, they must be able to model all of the appropriate types of material behavior, from purely elastic to plastic deformation to visco-elastic behavior. This is because the design of a compliant mechanism for maximum energy return depends largely upon the behavior of the materials involved in the impact. The models for these complex materials are usually obtained through extensive testing.

The use of lumped element models allows for a better understanding of what is happing in impacts. Their analysis also allows for the mass and stiffnesses values to be obtained which maximize the coefficient of restitution in a given impact. These mass and stiffness values are additional specifications which can be used in the development of compliant mechanism configurations. The use of lumped element models is discussed in Chapter 4.

3.3 Concept and Configuration Generation

After the set of specifications is set up, the next step of the process is to generate a wide variety of concepts to meet those specifications. As with the previous step, this step is divided into two parts, which are specific to compliant mechanisms in high-COR applications.

The first part of this process consists of an exploration and categorization of strain-energy storage possibilities. This is similar to a brainstorming process in that the goal is to identify as many strain-energy storage types as possible. The two primary objectives of this phase of the process are to identify as many types of strain-energy storage as possible and then to identify what key variables are necessary in order to identify specific configurations within the general concepts. This process of strain-energy categorization and concept generation is discussed in-depth in Chapter 5.

At this point, a distinction is made between concepts and configurations within this thesis. The categories of strain-energy storage are made up of general concepts like fixed-fixed or fixed-free cantilever beams, but the concepts do not include detailed information like the number or geometry of beams. That specific information is associated with configurations. One way to think about it is that a CAD model could not be created for a concept but it could for a configuration.

After a variety of concepts have been generated and the key variables for each concept have been identified, the next step is to use models involving those key variables to identify possible configurations. The static, closed-form models for force-deflection-stress behavior would be used first. Some of these models lend themselves easily to design, while others may require an iterative approach. Regardless, the aim of this portion of the concept development process is to produce configurations from which a selection can be made for further development. The process of generating more detailed configurations is covered in Chapter 6.

3.4 Configuration Selection

At some point, a decision must be reached concerning which configuration or configurations should be developed further. In this approach, that decision is made after a number of configurations are generated using the static, closed-form models. These configurations are then examined within the context of the functional specifications and other considerations. This stage in the process is not necessarily elementary. Many factors must be considered, and it is unlikely that one configuration will stand out substantially above the rest. The more likely solution is that there will be some configurations and concepts that do not appear viable and some others that appear comparable to each other. Those in the first group will be eliminated and the decision will have to be made from the remaining configurations. Because the configuration selection is so applications-specific, no universal rules apply to the decision. It is left to the designer's judgement.The configuration or configurations that are chosen will then be refined through further static and eventually dynamic analysis.

3.5 Configuration Refinement

After a configuration or configurations are chosen for further development, additional tools are used in order to validate the closed-form solutions and further develop the chosen configurations. These additional tools include and static finite-element analysis and static prototypes. The configurations are analyzed in a finite-element environment to determine if there is agreement between the closed-form solutions and the finite-element analysis. The finite-element analysis can also be used in an iterative process used to improve the performance of the configurations. Because this is an iterative process, it is less efficient to use finite-element analysis rather than the closed-form models as the primary method used to generate the configurations.

Static prototypes can be used mostly to validate both the closed-form and finite-element static models. These prototypes are not generally of the actual configurations because they may be too expensive to build and are still in the early stages of development. Rather, they are similar to the configurations, but are constructed with cheaper materials and maybe on a smaller scale in order to be less expensive. The prototypes can then be tested under static conditions to validate the other static models.

This process of configuration refinement is included in Chapter 6, which includes all static analysis in the thesis. At the end of this stage, a configuration or configurations exist (in model form only) that can then be tested to determine if they do behave as they have been predicted too.

3.6 Configuration Testing

Once a configuration or configurations have been selected and refined, dynamic analysis and testing is the next logical step. Most good finite-element software packages include the ability to model impacts. Using dynamic finite-element software, a configuration or configurations are still further refined while also determining its suitability. If the configurations prove to be viable, the final result from the entire concept development process would be a refined geometry that could then be the starting point in a more detailed, intensive design process. Because physical prototypes can be very expensive and the their development is actually part of a final design step, physical prototype dynamic testing is not included as part of this research. The testing of the selected configurations using dynamic finite-element analysis is discussed more in Chapter 7.

3.7 Case Study: Metalwood Golf Club Head

This thesis contains a case study on the generation and evaluation of concepts for a new compliant energy-return mechanism for use in metalwood driver golf club heads, an application where maximizing energy-return is of the highest priority. An example of the entire process outlined above is given. This case study is contained in Chapter 8.

CHAPTER 4 LUMPED ELEMENT MODELS

In order to have a better knowledge of the required characteristics of any compliant mechanism concept, the dynamics of the impact must be understood. One method of effective dynamic analysis to evaluate a large number of concepts is to use lumped element models. This chapter discusses the elements that make up lumped element models, a method for analyzing lumped element models, and the use of lumped element models results to establish concept guidelines. Examples of methods for determining proper lumped element models are found in literature. [Ujihashi, 1994]

4.1 Background

To more fully comprehend the usefulness and application of lumped element models, a brief background on their parts is included here. The purpose of lumped element models is to be able to break down a more complex system into a simpler form in order to get a better understanding of its behavior. The elements that make up lumped element models include point masses, massless springs, and massless dampers or dashpots.

4.1.1 Point Masses

In lumped element models, all of the mass in a system is broken into discrete blocks. For strictly linear motion, these blocks have zero volume and have no shape. When the rotation of an object needs to be included in the dynamic analysis, size and shape are significant and are included in the model. The higher the number of blocks, the more accurate the model, but the more difficult it is to analyze as well. The higher the number of blocks, the closer the model actually approximates a continuous model which would have an infinite number of blocks.

One of the prime considerations for determining the appropriate number of blocks in a model is the number of natural frequencies to be modeled. With one lumped element and a massless spring, the first modal frequency is somewhat predicted, but the second is not even close. Adding an additional mass and spring adds more accuracy to the first modal frequency, and more closely begins to approximate the second. Because the COR of an impact is primarily concerned with the first modal frequency so the lumped element model is often made up of just one or two masses.

4.1.2 Massless Springs

Connections between point masses are most commonly made with massless springs. These elements have the ability to exert forces between the masses, but are modeled as not having any inertia of their own. Spring elements can account for the behavior of an actual spring in traditional mechanisms, or they may model the material characteristics associated with flexibility in compliant mechanisms, including the overall force-deflection characteristics and the localized contact deformation. The modeling of the localized contact stiffness is not as crucial when one of the objects is much more flexible than the other.

$$F = k(x_1 - x_2) \tag{4.1}$$

$$F = k(x_1 - x_2)^{\alpha} \tag{4.2}$$

The behavior of the springs in the lumped element models is either linear or non-linear as shown in equation (4.1) and equation (4.2) where **F** is the force, ***k*** is the spring stiffness, $\mathbf{x_1}$ and $\mathbf{x_2}$ are the displacements of the respective ends of the springs and α is the nonlinear exponent. In many models, both types of springs are included. The choice of spring behavior and stiffness to use in a lumped element model is dictated by the actual behavior of the mechanism being modeled. Physical testing is often required in order to determine what characteristics should be modeled in any of the lumped elements. Because the energy imparted to a spring element is conserved within the system, they are not able to model any sort of energy losses. Additional elements are needed.

4.1.3 Dashpot or Damper Elements

Whether modeling friction losses in traditional mechanisms or the internal damping which exists to some extent in all materials experiencing deflection, some element must be able to model energy losses in a system. This element is the dashpot or damper element.

$$F = c(\dot{x}_1 - \dot{x}_2) \tag{4.3}$$

$$F = c(\dot{x}_1 - \dot{x}_2)^{\alpha} \tag{4.4}$$

These elements are also used as connections between mass elements, either in parallel, in series with a spring element or sometimes without any spring element at all. As shown in equation (4.3) and equation (4.4), the force generated by a dashpot is dependent only upon the relative velocity experienced by both ends of the dashpot and can be linear or nonlinear. Dashpots, like masses, are only significant in dynamic situations and play no role in any type of static loading. In addition, the energy imparted to a dashpot element is dissipated and none of it is returned to the system.

4.2 Establishing Concept Guidelines

The results of lumped element model analysis can be used to define more specific design specifications. By allowing some of the model parameters to be variable, the target values for those variables can be established. The use of lumped element models to determine additional design guidelines involves formulating the proper lumped element models for both objects involved in the impact, the modeling of the dynamic behavior and the analysis of the results.

4.2.1 Formulating Lumped Element Models

The process of modeling a system by the use of lumped elements can range from a rigorous and thorough testing program to an initial estimation. One approach is shown by Ujihashi [1994] in the development of a lumped element model of a golf ball.

In this approach, golf balls were fired by an air cannon at a steel target that functioned as a load cell. The impact was filmed by high speed camera. Understanding the load characteristics and the impact and rebound velocities allowed for a lumped element model to be constructed. The predicted behavior of the model was tested against the behavior of an actual golf ball and showed good results.

In contrast to the experimental method described above, the lumped element model for a golf club has traditionally been developed simply by dividing the entire mass into two point masses with a linear spring between them.

4.2.2 Dynamic Modeling

Once an appropriate lumped element model is created, the next step is to analyze that model to be able to predict behavior of the equivalent mechanism. The world of dynamic systems is the world of differential equations. In analyzing dynamic systems, the first step is to obtain the proper differential equations, or the equations of motion, and then to use appropriate tools to understand the dynamics of the model.

The ways in which equations of motion can be obtained for a particular system are numerous and vary widely. The equations of motion for this chapter are discussed within the context of a golf club/ball model which is related to the case study later in the thesis. The lumped element model is shown in Figure 4.1.

In the figure, the block labeled $\mathbf{M_f}$ is the mass of the face portion of the club. This does not only include some portion of the actual impact surface mass of the club, but in the

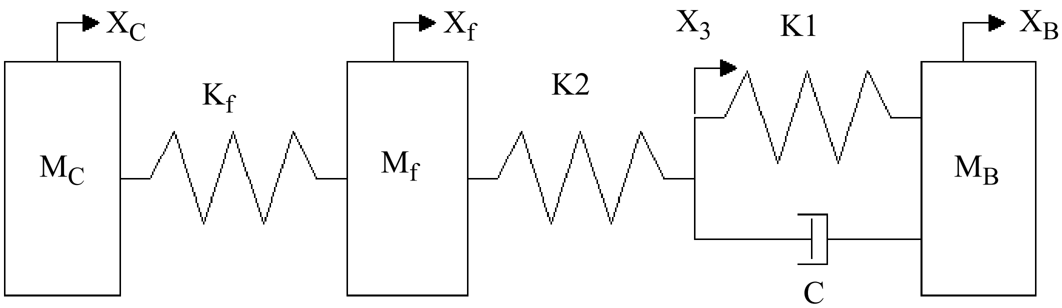


Figure 4.1 Lumped element model of golf club/ball model including Ujihashi ball model. $\mathbf{M_c}$, $\mathbf{K_f}$, and $\mathbf{M_f}$ are all part of the club model itself. $\mathbf{K_2}$, $\mathbf{K_1}$, **C** and $\mathbf{M_b}$ are all part of the ball model. $\mathbf{X_c}$, $\mathbf{X_f}$, $\mathbf{X_3}$, and $\mathbf{X_b}$ indicate the degrees of freedom and the arbitrary positive directions.

case of configurations with a mechanism behind the face, $\mathbf{M_f}$ includes some portion of the mass of that mechanism as well. The block labeled $\mathbf{M_C}$ is the remaining mass of the club. $\mathbf{M_B}$ is the mass of the ball itself. All elements to the right of $\mathbf{M_f}$ are parts of a golf ball model found in literature. [Ujihashi, 1994].

The equations of motion can be obtained through a variety of different methods from free-body diagrams to Lagrange's method. It is assumed the reader can use any of these methods to develop the equations of motion for themselves. For the system shown in Figure 4.1, there are four degrees of freedom so the four equations of motion are:

$$K_f(X_f - X_C) = M_C\ddot{X}_C \tag{4.5}$$

$$K_f(X_C - X_f) + K_2(X_3 - X_f) = M_f\ddot{X}_f \tag{4.6}$$

$$K_2(X_f - X_3) - K_1(X_3 - X_B) - C(\dot{X}_3 - \dot{X}_B) = 0 \tag{4.7}$$

$$K_1(X_3 - X_B) + C(\dot{X}_3 - \dot{X}_B) = M_B\ddot{X}_B \tag{4.8}$$

In equation (4.7), the sum of the forces is equal to zero because it is a point that has a degree of freedom, but has no mass. These equations of motion can also be expressed in state-space form.

$$\frac{\mathrm{d}X_C}{\mathrm{d}t} = \dot{X}_C \tag{4.9}$$

$$\frac{\mathrm{d}\dot{X}_C}{\mathrm{d}t} = \ddot{X}_C = \frac{K_f}{M_C}(X_f - X_C) \tag{4.10}$$

$$\frac{\mathrm{d}X_f}{\mathrm{d}t} = \dot{X}_f \tag{4.11}$$

$$\frac{\mathrm{d}\dot{X}_f}{\mathrm{d}t} = \ddot{X}_f = \frac{K_f(X_C - X_f) + K_2(X_3 - X_f)}{M_f} \tag{4.12}$$

$$\frac{\mathrm{d}X_3}{\mathrm{d}t} = \dot{X}_3 = \frac{K_2(X_f - X_3) - K_1(X_3 - X_B)}{C} + \dot{X}_B \tag{4.13}$$

$$\frac{\mathrm{d}X_B}{\mathrm{d}t} = \dot{X}_B \tag{4.14}$$

$$\frac{\mathrm{d}\dot{X}_B}{\mathrm{d}t} = \ddot{X}_B = \frac{K_1(X_3 - X_B) + C(\dot{X}_3 - \dot{X}_B)}{M_B} \tag{4.15}$$

Both forms of the equations of motion can be analyzed in different ways. One of the most effective ways to analyze the second form of the equations is to use a computer program with a runge-kutta simulation to predict the behavior. One tool that may be very useful depending upon the application is non-dimensionalization. Instead of varying individual variables, variable ratios can be used. In the golf club/ball model shown in figure 4.1, the key ratios to be tested would be the mass of the face $\mathbf{M_f}$ over the mass of the ball

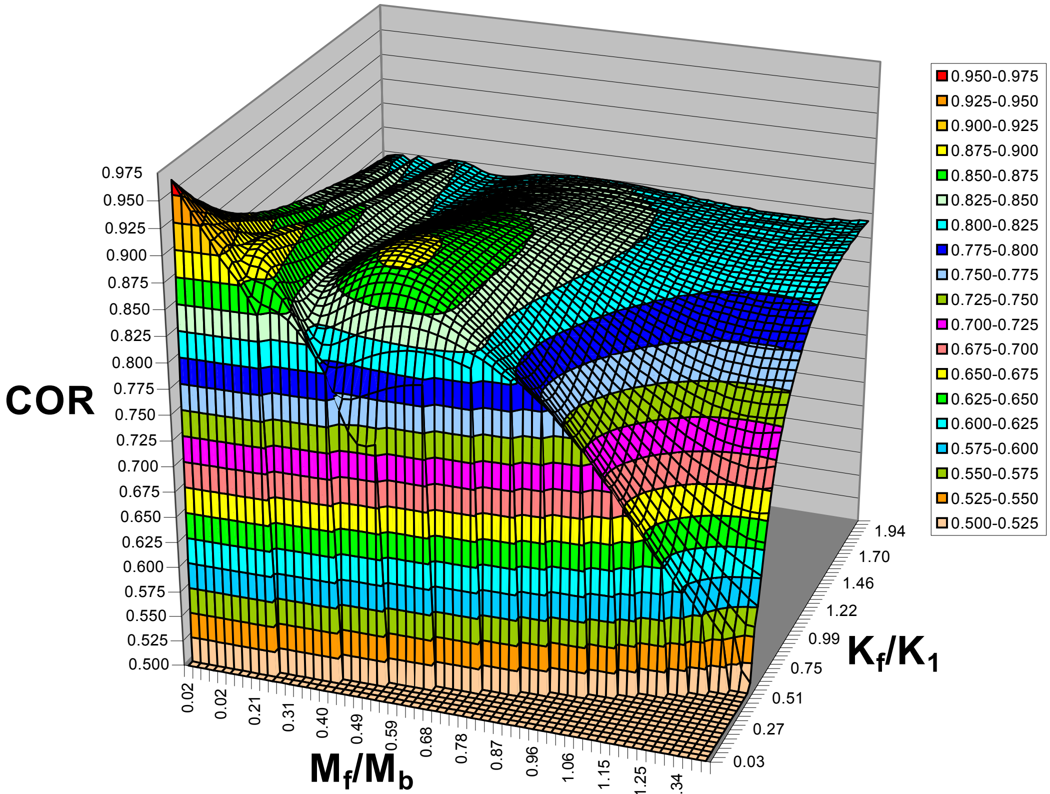


Figure 4.2 COR results for Ujihashi (1994) lumped element ball model.

$\mathbf{M_b}$ and the face stiffness $\mathbf{K_f}$ over $\mathbf{K_1}$ in the ball. The results were plotted as a 3D surface and are shown in Figure 4.2.

There are several interesting trends visible in this data. The first is that the COR generally increases as the mass and stiffness ratios decreases. At high stiffness ratios, the sensitivity to changes in the mass ratio decreases. At low stiffness ratios, the system becomes very sensitive to the mass ratio and almost appears unstable. These trends show mostly what would be expected from this system. Higher mass ratios result in higher inertia for the face and higher deflections in the ball, thus wasting energy in the internal damping. High stiffness ratios reduce the amount of energy that can be stored in the club head.

Low stiffness ratios result in a large amount of post-impact vibration in the club head, thus reducing the amount of energy returned to the ball.

It is from the results of the lumped element simulation that additional design guidelines can be established. From figure 4.2, it is clear there are two areas on the graph which represent fairly high COR's. The highest coefficients are located at mass and stiffness ratios of almost zero. It is very unlikely that feasible designs can be realized in that area. The other region of high COR is between mass ratios of approximately 0.35 to 0.7 and a stiffness ratio range of approximately 0.3 to 1.35. These two ranges give guidelines for the further development of concepts and provide additional criteria for evaluation and selection.

4.2.3 Impact Forces

An additional piece of information that may be gleaned from lumped element models is the peak force to be used in the static analysis. While the stiffness and the mass may be evaluated independently of the force, the stresses cannot be, and so the forces must be included in the analysis. The results of the lumped element model may be used as one approach to identify those forces. The velocity profile of one of the objects may be used to calculate the accelerations and then the forces experienced by that object. In the case of the ball/club model in Figure 4.1, the velocity changes of the ball mass are used to calculate accelerations and then forces. The peak force computed by this method is approximately 9000 Newtons.

Experimental evidence is a more precise method of determining the peak force. Instrumented equipment would not only expose the peak force, but also disclose the force profile over the area of impact, which cannot be determined from the lumped element model data.

To choose between these two methods to determine an equivalent static force to be used in the static analysis, many factors must be taken into account. Using the largest values results in the most conservative concept from a stress standpoint, but this overdesign may result in poorer performance than could otherwise be obtained. In some cases, the cost of an extensive testing program or other similar methods to determine the impact force may be prohibitive. In these cases, the lumped element model may be the only option. This method may or may not include some safety factor.

4.2.4 Concept Guideline Summary

Before the evaluation process can really begin, the results and conclusions from the lumped element model need to be summarized. This summary would include mass and stiffness target values for high COR concepts. In the context of the golf club and Ujihashi ball model, if the minimum COR is 0.850, the optimum coefficients are found when the mass and the stiffness are both minimized. However, there are other combinations of stiffness and mass which offer promising results. The range of those combinations is summarized in Table 4.1. The results from this lumped element analysis, combined with the results from another model will be used in the case study in Chapter 8.

4.3 Summary

In this chapter, the concept of lumped element or mechanical models was introduced. It is shown how these models can help predict the performance of a dynamic system. These models are useful in determining some of the specifications needed in order to maximize the coefficient of restitution of an impact. Much of this chapter has been explained within the context of a lumped element model of a golf club and golf ball system. Once the constraints are known, then they can be used to evaluate and further develop actual concepts under static conditions. This static analysis is the next step in the research and is explained in Chapter 6.

TABLE 4.1 Concept guidelines from the Ujihashi ball model for 0.850 COR.

Category	Value
Maximum Stiffness Value	6.5×10^6 N/m
Minimum Stiffness Value	1.5×10^6 N/m
Maximum Mass Value	29 g
Minimum Mass Value	15 g

CHAPTER 5 STRAIN-ENERGY STORAGE

This chapter provides an organization to the generation process for high-COR mechanism concepts. It looks first at existing energy-return mechanisms and the elements they use to store strain-energy, including any existing compliant systems. It then looks at categories of strain-energy storage in compliant mechanisms. General concepts are generated within those categories. Several possible configurations are shown in the figures in this chapter. The goal of this phase in the process is to identify as many types of energy storage as possible and to determine the key variables necessary in order to produce realizable configurations.

5.1 Existing Energy-Return Mechanisms

There are many mechanisms which are designed to transform strain-energy into kinetic energy under non-impact conditions. They are used in everything from firing mechanisms in firearms to mouse traps to ball-point pens. Because the strain-energy in these mechanisms is not imparted due to an impact, there are many differences from an impact application, but there are enough similarities that further discussion is useful.

5.1.1 Storage Elements

Whether the strain-energy is imparted by impact or by any other way, it must be stored in something. Springs are the most common strain-energy storage element in rigid-body mechanisms.

Springs come in many different configurations, but the most identifiable would be the coil spring. Coil springs are in everything from small ball point pens to automobile suspensions. Coil springs store the majority of their strain-energy in the form of torsional strain. Coil springs have many advantages. Their behavior is well understood and easily predicted. They are manufactured to fit the linear spring model quite well. They are readily available and relatively inexpensive. Coil springs only allow for linear motion, which may be an advantage or disadvantage depending upon the application.

Torsional springs are also useful, especially when rotational motion is required. These springs usually follow a linear model, with rotation instead of linear motion being the variable. These springs are often used in door hinges and automobile suspension.

Leaf springs are another category of traditional spring. They are commonly used in bows for firing arrows and vehicle suspensions, where they are usually coupled with a damper in order to become an energy-*absorbing* application. The stiffness of traditional leaf springs is increased by adding additional leaves and stacking them on top of one another. This allows for increasing the stiffness without increasing the stress in the beams because relative sliding motion between the leaves is permitted.

Elastomers are also sometimes used as energy storage elements. Elastomers allow for low stiffnesses where other springs may not be effective.

These traditional energy storage elements all have disadvantages when it comes to impact applications. Coil and torsional springs commonly require more complex mechanisms in order to provide the correct motion or function. And often, as the desired stiffness is increased, the mass of the spring may also greatly increase. This presents a problem in some energy-return applications, where the inertia and therefore the mass must be tightly controlled. Leaf springs, especially when stacked, cause additional problems because the sliding motion between plates produces friction, a non-conservative force which decreases the efficiency of the collision. Elastomers dissipate too much energy due to their internal damping characteristics. It is also difficult to obtain very high stiffnesses in these materials.

5.1.2 Existing Compliant Configurations

There are a small number of compliant systems which are designed for energy-return under impact loading. Two of these are tennis rackets and hollow metalwood golf heads.

Tennis Rackets make use of strings in the head to provide the energy storage for the impact with the tennis ball. Tennis balls are designed in such a way as to absorb energy when undergoing strain [Brody, 1995]. Because of this, the deflection of the ball needs to be minimized in order to increase the COR. The energy is mostly stored in tension in the strings.

Golf clubs have become more and more compliant in recent years. As discussed earlier, because golf clubs have traditionally had a certain appearance and have been constructed a certain way, increase in COR has traditionally been accomplished by making the face thinner and larger.

There are limitations with the products above because both have developed in very narrow evolutionary processes. Because of stress constraints, most golf club designs have already reached a point where further progress is largely limited by materials. Golf club faces can only get so thin before the stresses become too high.

5.2 Classification of Strain-Energy Storage

Different ways to store strain-energy can be organized and classified in order to gain a better understanding of the general characteristics of each category and to assist in the generation of concepts within the categories. This classification is based on the type of loading and geometry of any mechanism.

5.2.1 Axial Loading

Axial loading can be further broken down into the two categories of tension and compression. Axial loading offers the advantage of using the entire cross-section of material to store the energy. However, this can also be a disadvantage because a specimen loaded in pure tension is limited to lower stress levels than a specimen in bending in order to prevent yielding to the point of failure. A specimen loaded in compression is limited by the critical load to avoid buckling when the beams are long and slender.

The geometry of axially loaded beams is mostly dependant upon the cross-sectional area. The actual shape of the cross-section is not as critical except as it affects the area moment of inertia so as to oppose or to aid buckling.

5.2.2 Bending Loading

Bending loading is similar to axial loading in that the predominant stresses imposed in a material are either tensile or compressive. The stress distribution in bending versus axial loading is different in that in bending there is a neutral axis at which the stress is zero, and the stress increases linearly with distance away from that neutral axis.

This category is categorized mainly by types of applied load and also the end or support conditions for the configuration. Loads can be applied to beams as point or distributed loads and the behavior of the beams is significantly different. In addition, strain-energy can be imparted to the beam through applied moments. However, it is with end conditions that the behavior of bending beams varies the most. At each end of a beam, the end condition can range from free to fully constrained in all degrees of freedom. Several end condition combinations are evaluated in this thesis. A brief explanation of each of the combinations follows.

The first combination considered is that of a simply supported beam. With these conditions, both ends are supported by a joint that allows for rotation, but no translation in the normal to the longitudinal axis of the beam. Translation parallel to the longitudinal axis is usually allowed by one of the end constraints. For a force directed normal to the longitudinal axis of the beam, simply supported beams have no reaction moments, only reaction forces in the direction opposite to the applied force.

A fixed-free cantilever beam has the next pair of end conditions. On one end of the beam, the beam is fixed rigidly, allowing no translation or rotation. The other end of the beam is left free, allowing both translation and rotation. These end conditions constrain the reaction forces and moment to the fixed end of the beam.

A fixed-fixed cantilever beam is fixed and allows no translation or rotation at either end. This produces very stiff configurations but also raises the stresses. Both reaction forces and moments are present at both ends.

A fixed-guided cantilever beam is similar to the other types of cantilever beams already discussed, but with the distinction that at one end, it allows translation but not rotation. In other words, the fixed end has both a reaction moment and forces, but the guided end has only a reaction moment. This type of beam is very common in compliant mechanisms.

5.2.3 Torsion Loading

Torsion is an additional method of obtaining deflection and storing strain-energy. Applied moments are often used to impart torsion strain-energy into an object. However, when the motion of an impacting object is in one direction (as is usually the case), the loading condition usually becomes combined loading. This usually comes about because the way to take a uni-directional force and create a moment is to apply it to some moment

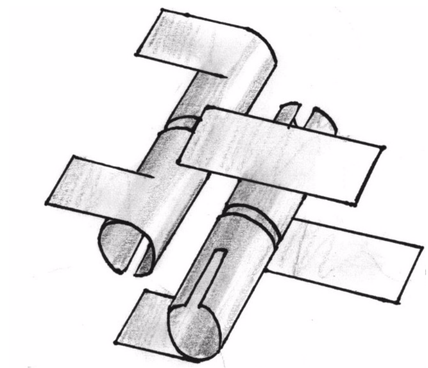


Figure 5.1 Split-tube flexure configuration.

arm. In addition to the moment created, a reaction force must also occur to offset the unidirectional force. This combined loading creates different stress distributions than pure torsion and may also place some limits on geometry which are discussed in Chapter 8.

There are several existing mechanisms which make use of torsion. One of these is the split-tube flexure shown in Figure 5.1. The use of hollow tubes allows for a higher average strain-energy storage rate. The split in the tube allows for greater deflections. When those options aren't available due to high loads and thus stresses, a more traditional torsion bar can be used. This is usually just a solid bar as opposed to one that is hollow.

5.2.4 Initially-Curved Beams

There are some mechanisms that do not fall completely into one of these categories because they may be a combination of two or more loading conditions. One of these additional categories that is of interest in this thesis is initially-curved beams. These beams can have identical end conditions to cantilever beams and are loaded either transversely or pseudo-axially.

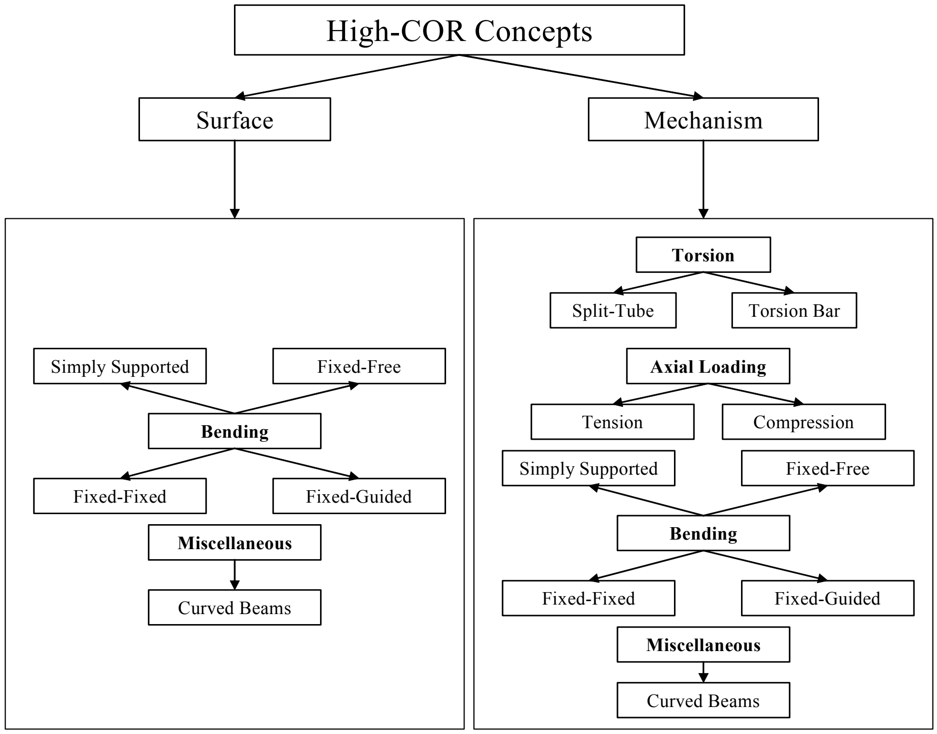


Figure 5.2 Strain-energy storage organizational chart.

Figure 5.2 is an organizational chart which shows the relationship between the types of loading and geometries.

5.3 Concept Generation

In addition to strain-energy storage categories, concepts are further categorized by the overall type of design. In any impact loading application, there is some surface on which the impact occurs. Any strain-energy storage in that surface itself is usually through bending, although the boundary conditions may vary. In this thesis, some concepts derive all of their deflection from this impacted surface and are called "surface" concepts. How-

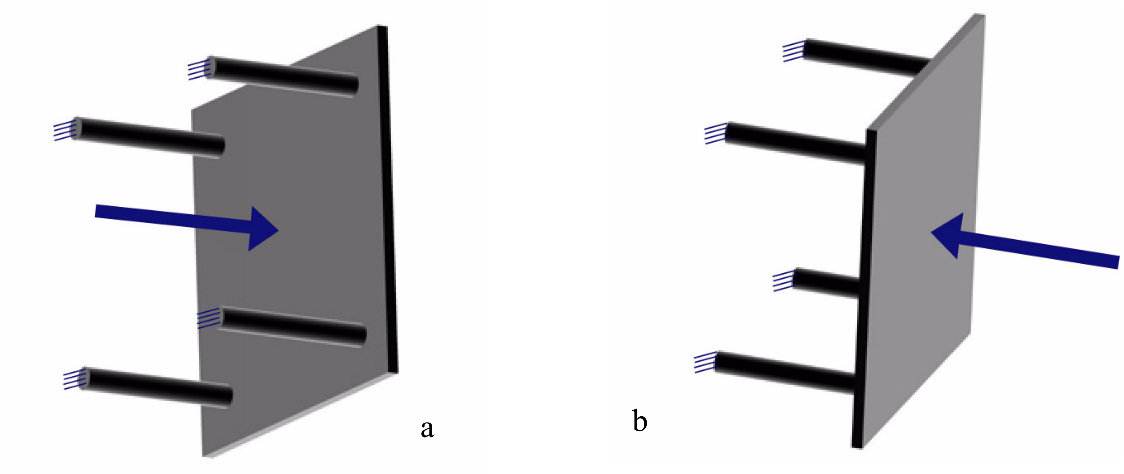
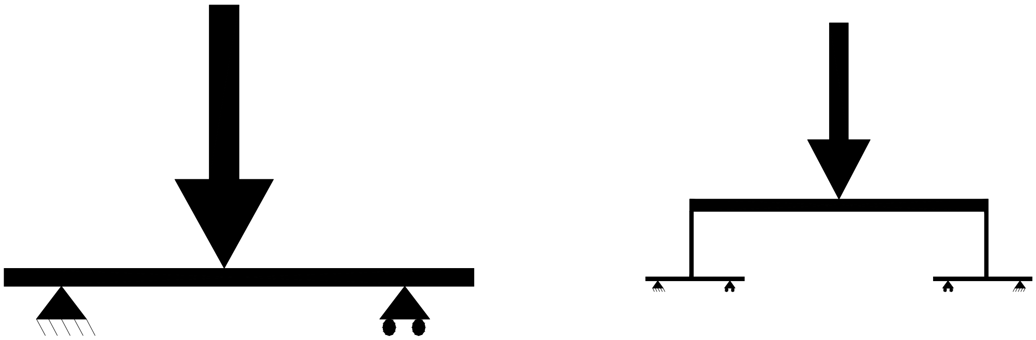


Figure 5.3 Possible tension (a) and compression (b) configurations.

ever, most of the concepts discussed here derive their motion from a combination of surface deflection as well as the deflection of some mechanism fastened to that surface and are called "mechanism" concepts.

After strain-energy storage categories have been identified and the distinction between surface and mechanism categories has been made, the next process is to generate concepts within each category. This is a creative process, and no concept that is generated is evaluated for its performance at this stage in the process. The main objective of this process is to identify the key variables that must be determined in order to produce a configuration. Each category is considered and the key variables are recorded. Some possible configurations are shown in the figures. The arrows indicate both the impacted face and also the force acting upon the face.



a b

Figure 5.4 Simply supported beam surface (a) and mechanism (b) examples.

5.3.1 Axial Concepts

Concepts in this category are made up of some surface subject to impact which would be then fastened to columns either in front or behind it. Depending on which side the columns are in relation to the surface would determine whether or not the columns would be in tension or compression. The cross-sectional geometry of the beams would not be as important as the total cross-sectional area and the length. Two examples are shown in Figure 5.3. The key variables to be identified in these concepts are number of beams **n**, length **L** of beams as well as the cross-sectional area, which can most easily be defined by a radius **r**.

5.3.2 Bending Concepts

The behavior of all bending concepts is dependant upon the end or support conditions of the beams. In a general sense, their behavior is also dictated by the same variables, regardless of end conditions. These variables are the number and geometry of the beams in the mechanism.

Any concept using simply supported beams, whether on the impacted surface itself or in a mechanism fastened to it, would be subject to the motion of at least one of the pin joints on each beam in addition to those variable mentioned above. Both surface and mechanism configurations are shown below. The first makes use of a simply supported beam or beams on the impacted surface itself, while the second is a mechanism concept and would consist of a simply supported beam or beams connected to the impacted surface. The key variables for both of these concepts are the number of beams **n**, the width **w**, height **h**, and length of the beams **L**. These variables are explored in the static analysis section in Chapter 6. These two concepts are seen in Figure 5.4.

The generation of concepts in all the other bending concepts are similar to that of simply supported beams in that the variables of number and geometry of the beams themselves are dealt with later in Chapter 6 on static analysis. However, some general examples of these concepts are shown in Figure 5.5.

5.3.3 Torsion Concept Generation

All torsion concepts generated fall under the mechanism category. Two general concepts were generated in this category. One makes use of split-tube flexures and the other uses torsion bars. The key variables for torsion concepts are made up of the geometry of the torsion members themselves, which includes at least one radius, and the length.

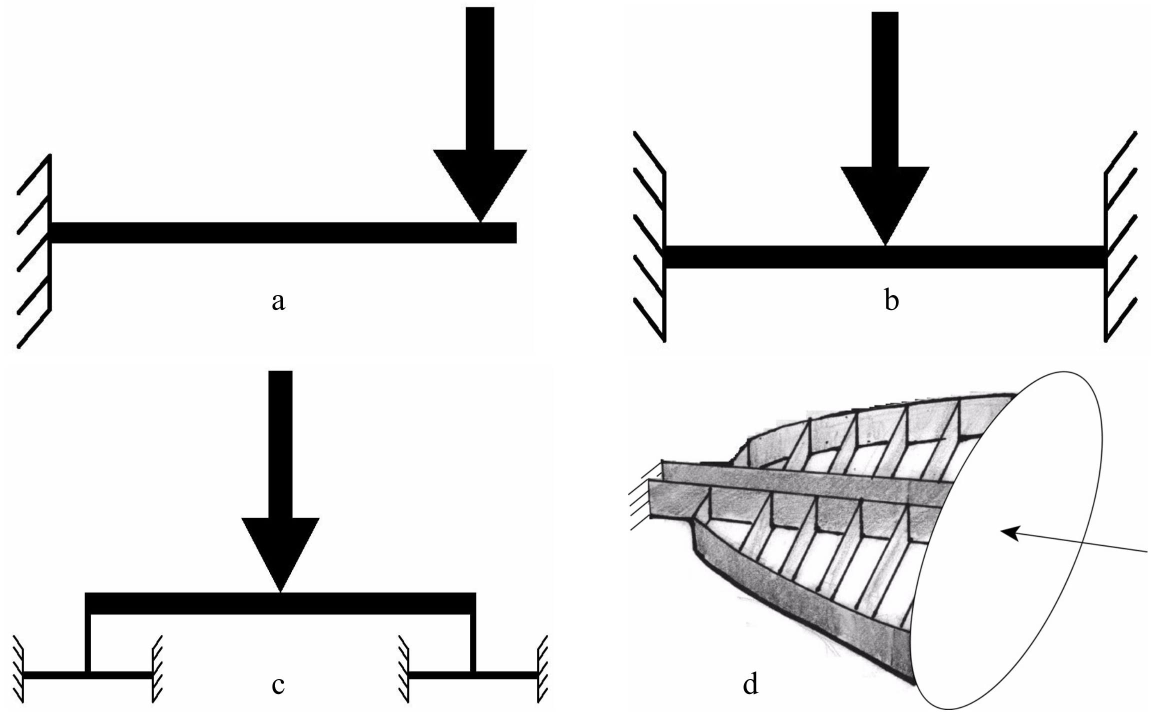


Figure 5.5 Several bending concepts: (a) Fixed-free surface, (b) Fixed-fixed surface, (c) Fixed-fixed mechanism, (d) Fixed-guided mechanism.

The key variables also include the geometry of the moment arm. One possible torsion configuration makes use of either split-tube flexures or torsion bars attached directly to the impacting surface, allowing greater rotation at its boundaries (as opposed to fixed end conditions). The second possible configuration uses either torsion bars or split-tube flexures in a mechanism connected to the impacted surface through a link that would allow the impacted surface more of a linear motion. These configurations are shown in Figure 5.6. As with the bending concepts, key variables that control behavior, such the number and geometry of beams are analyzed in Chapter 6.

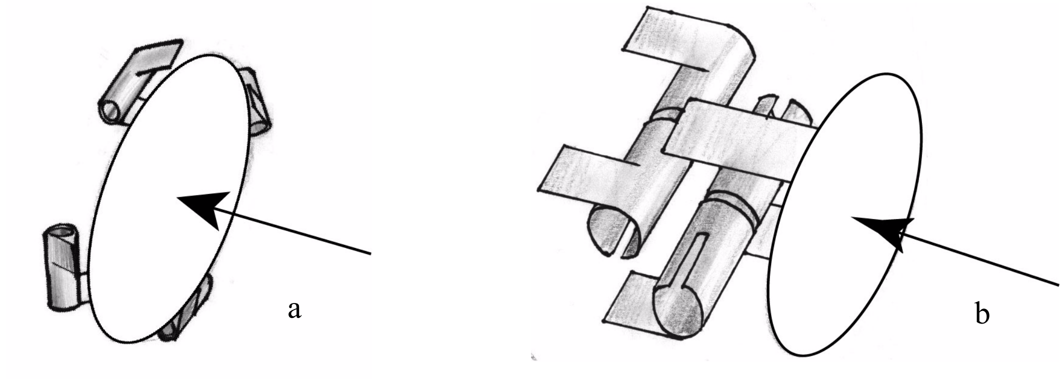


Figure 5.6 Torsion bar (a) and split-tube flexure (b) configurations.

5.3.4 Miscellaneous Concept Generation

As mentioned earlier, initially-curved beams are additional concepts of interest. There are two different loading conditions that have been considered for initially-curved beams. These conditions are shown in Figure 5.7. They are called pseudo-axial loading and transverse loading. These initially-curved beams are interesting in that some of the strain-energy is stored in bending modes and some of it is stored in axial modes. As in some of the previous concepts, the key variables associated with the initially-curved beam concepts are left until the static analysis portion of the thesis and both surface and mechanism concepts exist for initially-curved beams

An additional concept that does not fall neatly under any of the existing categories is that of straight-line mechanism. These mechanisms have existed in traditional mechanisms for quite some time and compliant versions have been developed. The idea is that a certain point on one of the links follows a straight line over a certain portion of the motion of the mechanism. This attribute may be useful in trying to attain linear motion of an

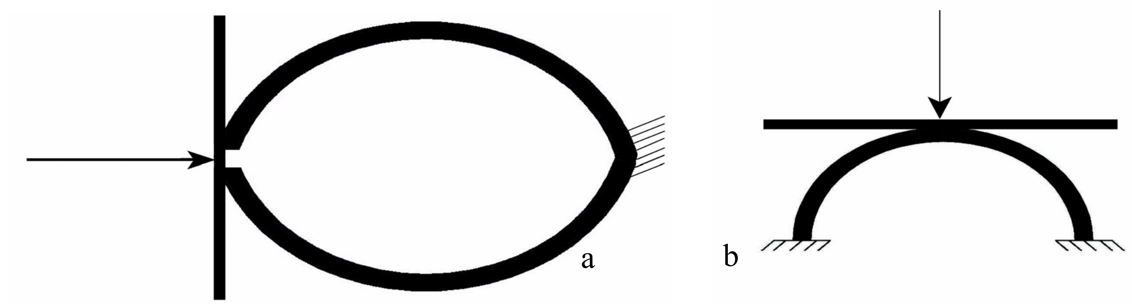


Figure 5.7 Pseudo-axially loaded (a) and transversely loaded (b) initially curved beams.

impacted surface. The difficulty in classifying this mechanism is that the compliant members do not fall into one of the traditional bending categories. However, the deflecting members in a straight-line mechanism can adequately be modeled as fixed-guided cantilever beams. A straight-line mechanism is shown in figure 1.1.

5.4 Summary

This chapter has first identified some of the traditional ways to return energy in a mechanism, including some existing compliant mechanisms. It has then identified ways of categorizing compliant mechanisms for the purpose of organizing any existing mechanisms and generating new mechanism concepts. General concepts were discussed and identified in those categories and several possible configurations were shown.

CHAPTER 6 STATIC ANALYSIS

With the guidelines established by the results from the simulations of the lumped mass models and the general concepts from the strain-energy storage categorization, the next stage in the process can be used to evaluate the viability of different concepts and also to create the first stage of configurations. This chapter explains the process and principles that can be used to evaluate general concepts by creating specific configurations with closed-form static models. After one of the configurations is chosen, static finite-element models are used to refine the model and validate the closed-form models with static prototypes.

6.1 Material Considerations

In addition to the stiffness and mass constraints, material properties play a key role in the performance of any compliant mechanism. As with any design process, the material selection is based both upon performance and cost as well as other factors. If performance is of a higher priority than low cost, the list of possible materials will be large at the beginning of the process. However, if the cost is of a high priority, or if the application has traditionally made use of certain materials, then the list may be considerably smaller. When considering materials to be used in the static analysis, there are three main properties that are essential in the static analysis. These are the density, the strength, and the modulus.

Because the COR is dependant upon mass, density plays an important role in the design considerations. Mechanisms experiencing impact loading often experience very high loads. These loads may produce high stresses. For this reason, the yield and ultimate strength levels of a given material are also crucial elements in the design process. Compliant mechanisms rely upon the deflection of members in order to gain their motion. The modulus of a material is closely related to its flexibility, thus it is also an important property to consider.

At this point a distinction should be made between stiffness and strength. Objects that are very stiff are often assumed to be very strong, while flexible objects may be perceived as weak. This is not necessarily true. An object's strength is related solely to its material properties, while its flexibility is related to geometry as well as material properties. For this reason, stiffness and strength are somewhat independent of one another.

6.2 Closed Form Solutions for Force, Deflection, and Stress Behavior

Closed form solutions for force, deflection and stress characteristics offer several advantages in the initial stages over other techniques such as numerical solutions. First, closed form solutions are the simplest methods of those available. Second, they lend themselves more readily to design approaches because design variables can be solved for in the

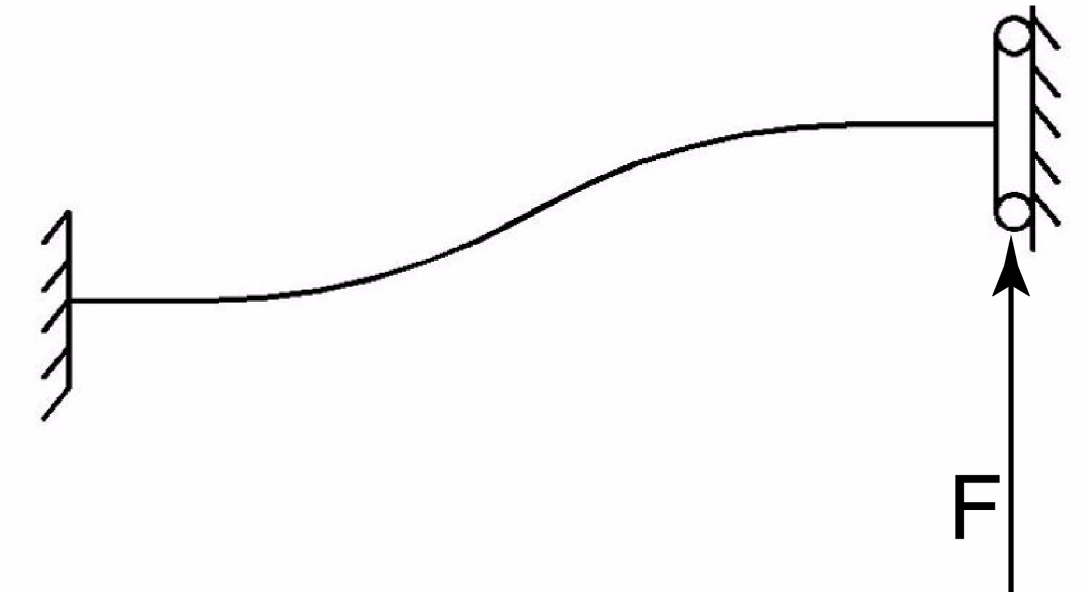


Figure 6.1 Fixed-guided cantilever beam.

closed form solutions. Other methods are very useful as a way to validate the closed-form solutions. For these reasons, closed form solutions are used in the first steps of design.

6.2.1 Linear Beam Theory

Most closed form solutions are closed because they make use of some simplifying assumptions. These assumptions limit the use of the solution to small deflections or else the solution's accuracy is compromised. Nevertheless, because of their simplicity, linear beam theory is a good starting point for static analysis. A very wide variety of sources exist for finding closed form solutions for force, deflection, and stress analysis. As an example, the development of a solution for a fixed-guided cantilever beam shown in Figure 6.1 is included here.

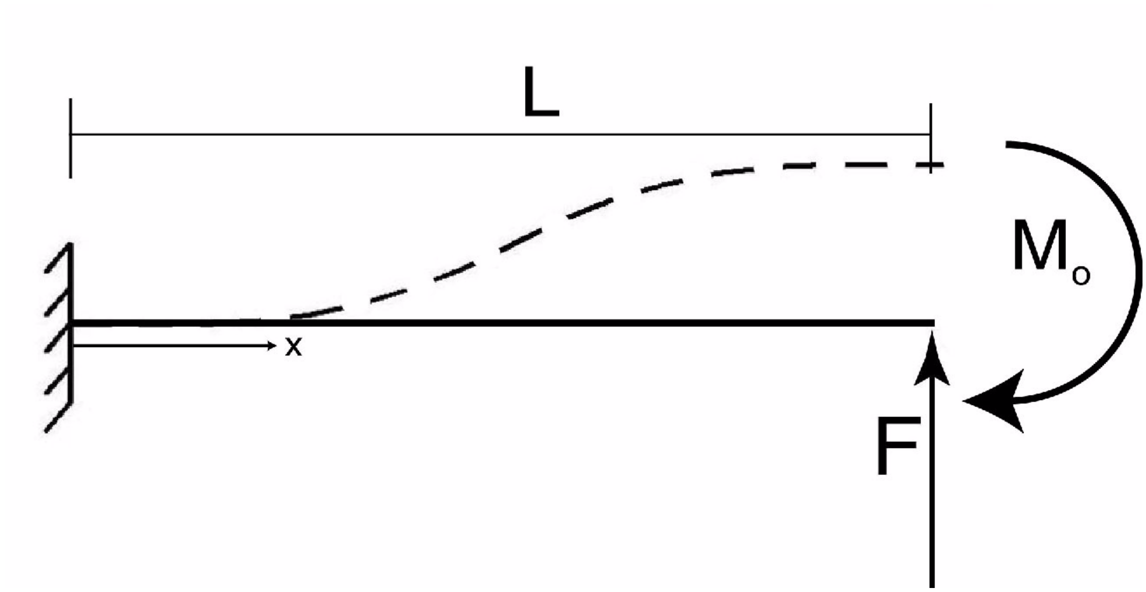


Figure 6.2 Fixed-guided cantilever beam with end condition equivalent loads on right side.

The first step is to obtain an expression for the deflection of this type of beam for a given load. It may be obtained from the expressions for an applied load and an applied moment together because the system is statically indeterminant. Figure 6.2 shows the same beam with the equivalent loading conditions on the right side. The expression for the deflection can be obtained from Howell [2001; Howell and Midha, 1995], and is shown in equation (6.1).

$$y(x) = \frac{F}{2EI}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) \tag{6.1}$$

Where **E** is the modulus and **I** is the area moment of inertia. The maximum deflection is then given when **x = L**.

$$y_{max} = \frac{FL^3}{12EI} \tag{6.2}$$

The angle of the beam as a function of location is given in equation (6.3).

$$\theta = \frac{F}{2EI}(x^2 - Lx) \quad \textbf{(6.3)}$$

The maximum angle of the beam occurs when x = L/2.

$$\theta_{max} = \frac{FL^2}{4EI} \quad \textbf{(6.4)}$$

$$M_0 = \frac{FL}{2} \text{ at } x=0,L \quad \textbf{(6.5)}$$

The expression for stress is found in equation (6.6).

$$\sigma = \frac{Mc}{I} \quad \textbf{(6.6)}$$

where **M** is the moment at any area along the beam, **c** is the distance from the neutral axis (centroid of the cross section of the beam) to the point at which the stress is being measured. To acquire the maximum stress, $\mathbf{M = M_0}$ and **c** is located at the maximum distance away from the neutral axis.

To use these expressions in a design approach, the design variables must be isolated. Keeping in mind that each of these equations are for a single beam, there are four basic design variables in a fixed-guided design: **n** (number of beams), **L**, **I**, and **E**. Depending upon the cross-sectional geometry chosen, **I** could consist of up to two variables.

Algebraic manipulation of the equations above yield some interesting results. Taking equation (6.2), and restating it for a number of fixed-guided beams in parallel, produces equation (6.7).

$$y_{max} = \frac{FL^3}{12nEI} \tag{6.7}$$

Dividing equation (6.7) through by F and inverting, an expression for the overall stiffness is obtained.

$$K = \frac{12nEI}{L^3} \tag{6.8}$$

Assume a rectangular cross-section so $I=bh^3/12$.

$$K = \frac{nEbh^3}{L^3} \tag{6.9}$$

Examining the stress equation (6.6) and the moment equation (6.5), equation (6.10) is developed.

$$\sigma = \frac{3FL}{bh^2} \tag{6.10}$$

Use equation (6.9) and equation (6.10) to solve for variables **h** and **b** and then substitute into mass equation (6.11) where ρ is the density of the material to obtain equation (6.12).

$$m = Lbh\rho \tag{6.11}$$

$$m = \frac{9\rho E F^2}{K\sigma^2} \qquad \textbf{(6.12)}$$

The variables in this equation can be sorted in to several general categories. The first category is that of material variables. A given material choice determines ρ, **E,** and σ (the maximum allowable stress). The other variables in the equation are determined either in the lumped mass model analysis (target stiffness **K**, and possibly **F**) or in an experiment (**F**). Some of the trends shown by this equation are worth noting and may be counter-intuitive. First of all, one way to decrease the mass is to increase the target stiffness. Another way is to allow greater stresses. This equation also shows one of the key material property ratios used in all compliant mechanisms. For a low mass, a high S_y/E ratio, where S_y is the yield strength of a given material, is desirable in addition to a low density. In applications in which the largest amount of elastic strain energy is to be stored, the key ratio is actually Sy^2/E.

While it is interesting to note what variables are *in* equation (6.12), it is just as worthwhile to observe what variables are *not* in it. Notice that the mass of the system is independent of the number and length of the beams. However, the actual geometry of the system (**b** and **h**) is not independent of these variables. In order to have practical configurations, those variables must be considered.

The mass calculated in equation (6.12) should not be assumed to be the total mass of a mechanism because it doesn't include any of the mass of the impacted surface, nor should all of that mass be assumed to be included in $\mathbf{M_f}$ in the lumped mass model. In reality, only some portion of the calculated mass is included. Depending on the geometry, that portion may or may not be simple to determine.

The numerical coefficient in equation (6.12) also of interest. When a similar analysis is applied to bending concepts with different end conditions, the equation has an identical form with the exception of the coefficient. This coefficient is also known as the *specific volume efficiency* [SAE, 1982]. For a cantilever concept loaded by a point load with any of the following end-condition combinations: fixed-fixed, fixed-free, fixed-guided and simply supported, the specific volume efficiency is one/ninth. In essence, all of these configurations are equal when it comes to mass, stiffness and stress considerations. Differentiating between them will have to be based upon other criteria, such as size, ease of manufacture and other considerations.

When the same process is applied to axially-loaded beams, the specific volume efficiency obtained is one. This implies that an axially-loaded beam can store nine times more energy per unit mass than a bending concept. There is one problem in this direct comparison between bending concepts and axial concepts using specific volume efficiency. That problem involves the maximum allowable stress. In a bending concept with a ductile material, the maximum allowable stress can be significantly above the yield stress because of the shape of the stress distribution. Only the outer fibers of a beam would actually experience that full stress. In an axially-loaded concept, the entire cross section of the beam would experience that stress, so the maximum allowable stress in equation (6.12) would be limited to a value below the yield strength of the material.

At first glance, it may seem that the static analysis of the bending concepts has yielded little information that would be helpful in determining which would be the most viable configuration for a given application. The original proposed criteria were stiffness, mass and stress. Table 6.1 shows a comparison of all bending and axial concepts. Note that all bending concepts are equal when it comes to specific volume efficiency, while the geometry of the beams involved is different for all cases.

The specific volume efficiency is quite simple to find for axial and bending loading conditions whose models are linear. The models for other categories of energy storage (torsion, initially-curved beams, etc.) do not lend themselves to being evaluated in a simi-

TABLE 6.1 Specific volume efficiency comparison of bending and axial concepts.

	Mass Equation	Specific Volume Efficiency	Height Equation	Width Equation	Radius Equation
Simply-Supported	$m=\frac{9F^2E\rho}{K_{opt}\sigma_{allow}^2}$	1/9	$h=\frac{K_{opt}L^2\sigma_{allow}}{6EF}$	$w=\frac{54F^3E^2}{nK_{opt}^2L^3\sigma_{allow}^2}$	NA
Fixed-Free	$m=\frac{9F^2E\rho}{K_{opt}\sigma_{allow}^2}$	1/9	$h=\frac{2K_{opt}L^2\sigma_{allow}}{3EF}$	$w=\frac{27F^3E^2}{2nK_{opt}^2L^3\sigma_{allow}^2}$	NA
Fixed-Fixed	$m=\frac{9F^2E\rho}{K_{opt}\sigma_{allow}^2}$	1/9	$h=\frac{K_{opt}L^2\sigma_{allow}}{12EF}$	$w=\frac{108F^3E^2}{nK_{opt}^2L^3\sigma_{allow}^2}$	NA
Fixed-Guided	$m=\frac{9F^2E\rho}{K_{opt}\sigma_{allow}^2}$	1/9	$h=\frac{K_{opt}L^2\sigma_{allow}}{3EF}$	$w=\frac{27F^3E^2}{nK_{opt}^2L^3\sigma_{allow}^2}$	NA
Tension	$m=\frac{9F^2E\rho}{K_{opt}\sigma_{allow}^2}$	1	Length Equation	$L=\frac{\pi nEr^2}{K_{opt}}$	$r=\sqrt{\frac{F}{\pi n\sigma_{allow}}}$

lar manner. These categories must be evaluated based upon their individual equations for force, deflection, and stress relationships.

Until now, the only models that have been considered have been those which are valid for small deflections and thus can make use of linear beam theory. When the deflections are beyond the linear realm, new tools must be used. These tools include elliptic-integrals, numerical (finite-element) methods, and the pseudo-rigid-body model. Only the latter two are discussed within this thesis.

6.2.2 Pseudo-Rigid-Body Model

The pseudo-rigid-body model was developed to allow for the closed-form analysis of beams experiencing large, non-linear deflections and was introduced in Chapter 2. The best resource on the pseudo-rigid-body model is by Howell [2001; Howell and Midha, 1995]. The analysis of a fixed-guided cantilever beam is shown here for purposes of example.

The pseudo-rigid-body model for a fixed-guided beam is shown in Figure 6.3. In this model, **l** is the length of the beam, γ is the characteristic radius factor and is equal to 0.8517 for a fixed-guided beam and Θ is the angle of the pseudo-rigid member. While this model allows for the analysis of larger deflections than the linear model, it is limited in that Θ is restricted to angles below 64.3 degrees.

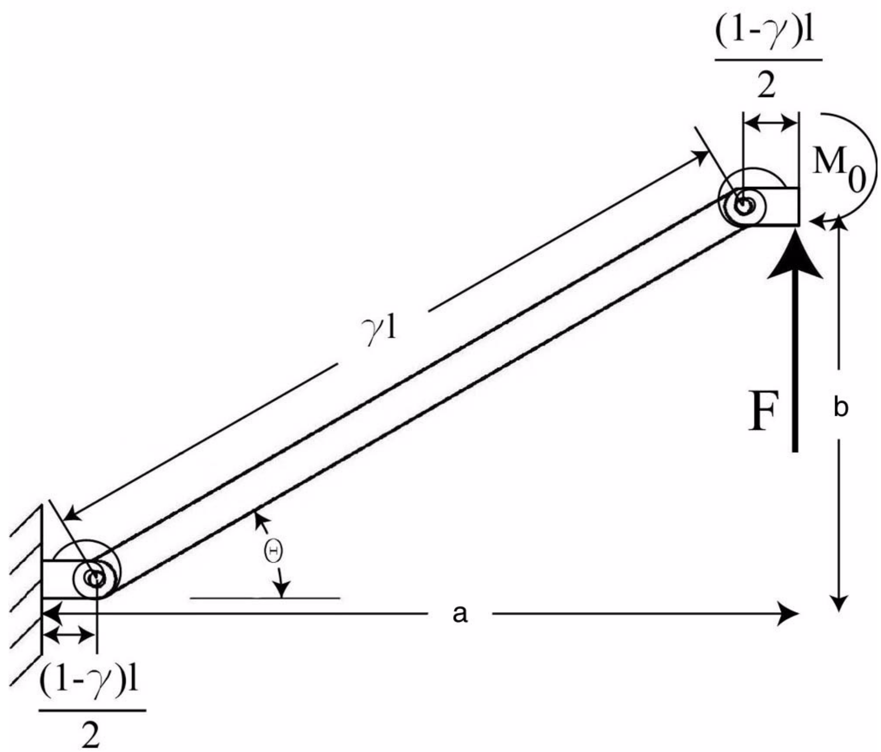


Figure 6.3 Fixed-guided cantilever beam pseudo-rigid-body model.

The link lengths and pin joint locations account for the motion of the guided end of a fixed-guided beam, but it is the torsional springs at the pin joints that account for the force-deflection characteristics. The torsional spring constant is found in equation (6.13).

$$K = 2\gamma K_{\Theta} E\frac{I}{l} \tag{6.13}$$

where $\mathbf{K}_{\Theta}$ is the stiffness coefficient and is equal to 2.67 for fixed-guided beams. **E** is the material modulus. Note that the units on the torsional spring constant **K** are torque/radians. Equation (6.14) shows the maximum moment in the beam. It is located at either end of the beam. Equations for **a** and **b** are also below.

$$M_0 = \frac{Fa}{2} \tag{6.14}$$

$$a = l - \gamma l(1 - \cos\Theta) \tag{6.15}$$

$$b = \gamma l \sin\Theta \tag{6.16}$$

The manipulation of the closed-form solutions for the pseudo-rigid body model is shown in equations (6.17) through (6.21).

$$T = 2K\Theta \tag{6.17}$$

where T is the torque applied and is equal to

$$T = F\gamma l \cos\Theta \tag{6.18}$$

Setting equation (6.17) and equation (6.18) equal to each other and solving for **F** gives equation (6.19).

$$F = \frac{2K\Theta}{\gamma l \cos\Theta} \tag{6.19}$$

Substituting equation (6.13) into equation (6.19) yields equation (6.20).

$$F = \frac{4K_{\Theta}EI\Theta}{l^2 \cos\Theta} \tag{6.20}$$

Solving equation (6.19) for the area moment of inertia **I**.

$$I = \frac{Fl^2 \cos\Theta}{4K_{\Theta}E\Theta} \tag{6.21}$$

The next step is to look at which variables are known and which are unknown and which are known in equation (6.21). **F** is known from the lumped mass model or from experimental data and $\mathbf{K}_{\Theta}$ is known from the pseudo-rigid-body model. **E**, **l**, Θ and **I** are all unknown at this point in the analysis process.

Θ is the first of the unknown variables to be eliminated. Because the desired stiffness is known from the lumped mass results, the deflection **b** is also known. This allows Θ to be solved from equation (6.16). Because equation (6.21) is non-linear, it is not simple to do this algebraically, but the numerical value can be inserted.

The next two unknowns must be chosen arbitrarily. **E** can be eliminated simply by choosing one or more materials to evaluate. The length **l** must also be chosen. The experience of the user of the equations must be enough to dictate reasonable a reasonable choice. Because of these two design choices, the process to find the best configuration is most likely an iterative one.

Once all of the other unknown variables are eliminated from the equation, **I** can be solved for. Because **I** is also made up of two independent variables, **w** (width) and **h** (height) of the beam for a rectangular beam are both unknown. The use of the stress equation can be of use to solve for these two variables. The equation for stress is given as equation (6.21).

$$\sigma = \frac{Fac}{2I} \quad \textbf{(6.22)}$$

where **c** = **h/2**. Substituting equation (6.15) into equation (6.22) and solving for **h** yields

$$h = \frac{4\sigma I}{F(l - \gamma l(1 - \cos\Theta))} \quad \textbf{(6.23)}$$

where σ is the maximum allowable stress based upon material property and the amount of allowable plastic deformation. Once **h** is known, the equation for **w** can be obtained through the algebraic manipulation of the equation for the area moment of inertia.

$$w = \frac{12I}{h^3} \tag{6.24}$$

Once **w** and **h** are known for a given **l**, then the mass and the stiffness are known for a particular configuration for fixed-guided cantilever beam. As mentioned before, this process may be iterative because choosing new **l** and materials makes a difference in the final geometry. However, it should be noted that changing **l** has been shown to have very little, if any effect upon the mass of the system. This is identical to the results shown in the linear analysis. Changing **l** does impact the geometry (**w** and **h**) considerably. Therefore, it is not safe to assume that the choice of **l** is inconsequential.

Models similar to that shown above for the fixed-guided segment are available for a variety of beams with different end conditions. The analysis process for those conditions are similar to that shown for the fixed-guided beam.

Depending upon the amount of deflection in a given concept, the results of the linear model and the pseudo-rigid-body model may be very similar. A comparison of the results for both models is addressed in the case study in Chapter 8.

Simplified non-linear analysis methods for some of the other strain energy storage categories listed in Chapter 4 such as torsion do not exist. In those cases, more advanced non-linear analysis such as finite-element methods must be used if the accuracy of the linear models is in question.

Regardless of the methods used, the general strain energy storage concepts must be analyzed thoroughly so that a reasonable comparison can be made in order to determine which categories should be pursued beyond this initial stage.

Once the most viable configurations are chosen, then those configurations are pursued further within the closed-form solutions until initial geometries exists. These initial configurations contain enough information to make solid CAD models and FEA geometries to be analyzed.

6.3 Static Model Validation

Before further development continues, the results of the closed form static models should be validated. The use of finite-element analysis and possibly actual physical prototype testing are the best tools to accomplish this.

6.3.1 Finite-Element Analysis

Finite-element analysis is not well suited to initial design because the iterative process can be so lengthy. However, it is more effective after an initial geometry already exists so there is a starting point. It may also be the only option as well because it may be cost prohibitive to produce and test physical prototypes. Because the static, closed-form models are for the mechanisms behind the impacted surface, the first generation of finite-element testing would only be for the force-deflection-stress characteristics of those mechanisms and the impacted surface itself would be modeled as rigid. The force-deflection-stress results could then be compared with those expected results from the closed-form solution and possibly with those from any physical prototype testing.

6.3.2 Physical Prototypes

Physical prototyping and testing is one of the best tools to validate both the closed-form results and finite-element analysis. However, prototypes and testing can be very expensive. One thing to consider is that the physical prototype used to validate closed-form models need not be even close to the actual production prototype expected in the later stages of design development. Readily available materials which are easy to obtain and work with are used rather than more expensive and exotic materials that would be used in a final design. The stiffness and mass targets do not have to match those obtained in the lumped mass model results. If the closed form and finite-element results indicate that the deflection is in the non-linear range, then the prototype that is constructed must be tested in the non-linear range as well. An example of these types of static model validation is shown in the case study in Chapter 8.

When any of these results disagree with one another, appropriate steps must be taken to find out the cause. If the physical testing was carefully performed, the most confidence should be placed in it. Some changes may be needed in either the closed-form or finite-element analyses in order to bring them into conformance with each other and with the physical prototypes results.

6.4 Configuration Refinement

Once the different models are validated and some initial geometry exists, the impacted surface is also considered. If the boundary conditions of the impacted surface are very simple, some closed-form solutions may exist, but more likely this analysis will require finite-element analysis.

Beginning with the initial geometries generated in the previous steps, the entire configuration can be modeled in a finite-element software package. The static force applied to the model may be obtained from the lumped mass models or from an experimental testing program as discussed earlier.

The process of evaluating and refining the geometry is an iterative one. Most finite-element software allows for some level of automation which makes the process faster. The goal is to approach the appropriate mass and stiffness targets which were determined in the lumped mass model results while remaining under the appropriate stress levels. If some portion of the deflection is obtained from the impacted surface itself, there will be some uncertainty about what portion of that its mass and what portion of the mass of the mechanism behind the surface should be included in that which is compared with the target value.

Due to this ambiguity, the matching of the mass of the concept with the mass target value obtained from the lumped mass model is not likely to be completed at this stage. This static design may have mass values that are close to the target values, but optimizing the design from a mass standpoint cannot be done using static analysis. However, because the impacted surface is included in the analysis, an impact force profile can be used so the accuracy of the model can be improved. The questions which revolve around the mass at this point have to be handled in the next step of evaluation. The next step is dynamic finite-element analysis.

6.5 Summary

In this chapter, the method of utilizing the results from the lumped mass model analysis was introduced and explained. First, material properties are considered and several material candidates are chosen. Closed-form models, both linear and nonlinear, were introduced and were illustrated through the example of a fixed-guided cantilever beam. The closed-form solutions for the mechanism without the impacted surface can be confirmed through both finite-element and physical prototype testing. After an initial geometry for the mechanism exists, the impacted surface can be included and further refinement of the overall geometry can be achieved. This refined geometry is then used in finite-element dynamic analysis. This dynamic analysis is discussed in Chapter 7.

CHAPTER 7 DYNAMIC ANALYSIS

The next step in the generation and evaluation process is to use dynamic modeling tools to further refine the configuration and to determine whether or not the configuration is truly viable. The static analysis to this point has allowed for the evaluation of mostly the stiffness of a given configuration. The mass of an object plays no role in its static force-deflection characteristics. Further evaluation is needed to determine whether or not the mass values are acceptable, and most importantly, whether the COR is acceptable. This further evaluation is performed in dynamic finite-element modeling of the impact.

7.1 Introduction

At this point in the evaluation process, an initial configuration geometry exists which is primarily a result of the static analysis. This geometry has been shown to have the appropriate static stiffness value, but the mass value is somewhat vague due to the fact that not all of the mass should be included in the inertial face mass $\mathbf{m_f}$.

The dynamic analysis provides several results that are key to finishing the evaluation of any geometry. The first data provided is some idea about how sensitive the performance of the configuration is to changes in the inertial face mass. Dynamic analysis also allows for the lumped mass model results to be compared to dynamic simulation data and recognize how effective the lumped mass model is for a particular configuration. Finally, the dynamic simulation is the final authority on whether or not a concept is considered viable. If the concept is viable, it would then be promoted into a more specific and expensive final design process.

7.2 Dynamic Finite-Element Analysis

Most finite-element software packages allow for a dynamic analysis in which an actual impact can be modeled and simulated. This is the method used to evaluate concepts in a dynamic context for this research. In any impact, there are at least two objects involved. Finite-element models must be developed for all objects involved in order for the simulation to be valid. In this chapter, the impacting and impacted objects refer to the object striking the designed mechanism and the mechanism itself respectively.

7.2.1 Impacting Object

Building a model for the impacting object can be a difficult and intensive proposal. As would be expected, the more simple the impacting object, the more simple the finite-element model. More complex models are necessary for more complex objects.

The process of developing finite-element models is very similar to the process for developing the lumped mass model as discussed in Chapter 4. However, it is likely to be more involved because of the complexity of the finite-element models as compared with lumped-mass models. One of challenges most likely to be encountered is a material that is not found in the standard material libraries of the finite-element software or that doesn't follow common material assumptions. In these cases, new materials must be modeled within the software. This process alone can be very time-consuming. The geometry of the impacting object may also be complex, which also adds to the difficulty of creating a model. Creating a model will probably involve a comprehensive experimental program to determine if the model for the impacting object is appropriate and accurate.

However, this is a one-time process because once a proper model is created for the impacting object, that model can be used in the evaluation of any concept that experiences impact with that object. Libraries can be built which contain models for all of the impacting objects which can be used to evaluate concepts.

7.2.2 Impacted Object

The geometry and material behavior for the impacted object should be fairly well defined at this point in the evaluation process. If finite-element analysis was used in the static analysis portion, a finite-element model will already exist. This model may be slightly modified in order to increase the speed of the analysis, such as using shell elements rather than solids, but the geometry and material model should remain the same.

This model is then be used to model impact with the impacting object model developed earlier.

7.3 Testing Program

There are two portions to actually analyzing the concept configuration. The first part of the test is to actually test the original geometry as created in the static analysis step. The next stage is to set up a simple optimization system in order to understand sensitivities and find better designs close to the original.

7.3.1 Original Model Test

When the original geometry is tested, there are three major results that must be considered. The first is the coefficient of restitution. Because this is the object of this process, it must be the prime consideration. However, it is not the only consideration. Stress levels must also be evaluated in order to determine if the impacted object can withstand the impact in its original configuration. In addition to these two results, the deflection of the concept is also of interest. While this data is not of primary importance, it is a way to check the predicted behavior from the static analysis with the behavior shown by the dynamic analysis.

It should not be surprising if there are discrepancies with the predicted COR, stress and deflection values of the static analysis and the dynamic analysis. This divergence is due to a number of factors. The COR may be different because of some timing issues such as if the impacting object is beginning to rebound out of phase with the impacted surface on the impacted object or the equivalent mass may deviate from the target value more than initially thought. The deflection may be different because the stress wave speed may be fast enough that not all of the maximum deflection occurs at the same time as it would in a static deflection. This divergence does not necessarily indicate faulty static or dynamic models. However, close attention must be paid to the results of both analyses to watch for indicators that would suggest errors in either.

7.3.2 Simple Optimization and Configuration Refinement

In addition to analyzing the original model, a program to test variations of that design should also be used. The first step in this program is to choose which variables should be altered. With any concept, there are a given number of those variables. However, it is not reasonable to choose to vary all of those variables. This would be nearly equivalent to starting with just a basic concept and performing all of the concept evaluation within the dynamic simulation context while skipping the static analysis. It would be very time consuming and inefficient. It is best therefore to choose two or three variables to change. The decision regarding which variables to choose would depend on the configuration itself and would be up to the discretion of the designer.

The range over which to modify the variables is another decision that must be reached. While this is also at the discretion of the evaluator, it is recommended that some original envelope be set with a maximum and a minimum. As mentioned earlier, most finite-element software allows for some degree of automation and so a series of these tests can be run using that automation to test over the range of the envelope. The results can be expressed in some sort of matrix form. Depending upon the number of variables chosen and the number of steps within the envelope, the matrix could be very large. When the data within the matrix is evaluated, trends can be seen which may necessitate the modifying of some of the ranges over which the variables are altered. For instance, the COR may increase as a certain variable is increased as well, and reach a maximum at the maximum variable value. It would be prudent to increase the maximum value allowed for that variable to see if that trend continues. In this process, both COR and stress, as well as any other design constraints must be considered.

7.4 Summary

The dynamic analysis process defined in this chapter represents the final step in the evaluation of a high-COR compliant mechanism concept. Once it is completed, enough information is available to make an informed decision regarding whether or not a concept should be thrown out, set aside for the moment, or promoted into a detailed and final design process. Chapter 8 will be a case study involving metalwood golf club heads and will illustrate the entire process from start to finish.

CHAPTER 8 CASE STUDY: GOLF CLUB HEAD

In order to illustrate the concept development process and tools defined in this thesis, a case study generating and evaluating new concepts for use in a metalwood driver golf club head is presented in this chapter. First, a brief introduction to the case study is given and then the steps of the process are presented and explained.

8.1 Introduction

Golf drivers have developed through a long evolutionary process which has produced a highly optimized configuration. The overall look of the design is very similar to actual wood drivers from fifty years ago, although now they are made using very advanced metals or even composite materials. The evolution of golf technology is discussed in both Chapters 1 and 2.

Because of the manner in which golf technology has advanced, unconventional configurations have not been thoroughly examined as viable possibilities. As a result, the Utah Center of Excellence for Compliant Mechanism Development and Commercialization at Brigham Young University was approached by TaylorMade Golf Company about generating and evaluating non-traditional configurations for a golf driver possessing a high coefficient of restitution. The functional specifications are first explained, then the evaluation process developed in this thesis is used to determine a configuration that could be used in a final design process.

8.2 Functional Specifications

In this case study, the functional specifications had largely been defined before the project began. The main objective of this project was to generate and evaluate a configuration to determine if it could produce a higher COR than traditional configurations. Additional considerations are described below. These were included to help ensure that the final configuration from this research would be a viable starting point for input into a final design process.

8.2.1 Coefficient of Restitution

Coefficient of restitution is of prime importance in this research and so it is the primary functional specification. The lower limit for the configurations in this case study was set at 0.850. There are some implicit assumptions in that specification. Whether or not a configuration met this standard would be determined by simulating the United States Golf Association test for COR within a finite-element program. This test uses approximately 150 feet/second as the closing speed between the club and the ball.

8.2.2 Overall Mass

The majority of driver heads today have an approximate mass of 200 grams. There are advantages and disadvantages to increasing the mass of a club head. For a given velocity, higher mass makes for higher momentum which can then increase the COR by itself. However, the speed of the head at impact is dictated by a player's ability to swing the club. More massive heads are more difficult to accelerate and may lead to slower club head speeds at impact. For this reason and others, the total mass of any configuration was set to be approximately 200 grams.

8.2.3 Durability

The impact of a golf ball with a golf club is very violent, with the ball often experiencing an acceleration of 30,000 to 50,000 g's. This leads to very high forces experienced by both objects. A practical driver must be able to withstand a large number of these impacts before it fails and even before its performance is reduced. The actual number of impacts is a value set by the individual manufacturers. Because many of the factors that influence durability are further refined in a final design process and validated using physical prototypes, the concepts that are developed here comply with the durability requirement only through ensuring that proper stress levels are maintained during the impact. These proper stress levels are defined by both standard material properties as well as engineering experience with the chosen materials.

Several other considerations were used to aid in the selection of the chosen configuration. While the value of the required polar moment of inertia was not set, it was considered from a qualitative standpoint. Other considerations such as overall size of the concept were also included in a similar fashion.

8.3 Lumped Element Models for Golf Clubs and Balls

Lumped element models are used in order to identify the desired characteristics for a given concept configuration such as mass, stiffness and others. As mentioned in Chapter 4, the models may be developed from a rigorous experimental program or from simple assumptions. The models used in this case study are a combination of both types. Both of the ball models used here are found in literature and were developed with experimental data.

8.3.1 Ujihashi Ball Model

The ball model by Ujihashi [1994] was introduced in Chapter 5 in order to illustrate the process of using lumped element models. The model consists of a single linear spring in series with an additional linear spring and linear damper in parallel with respect to each other. It is shown in the right side of Figure 4.1 on page 32.

8.3.2 Johnson and Leiberman Ball Model

In addition to the Ujihashi model, Cochran [1999] utilizes the Johnson and Leiberman ball model which was developed through experimental research. This model consists of a non-linear spring in parallel with another nonlinear spring and linear damper which are in series with one another. The model is shown in figure 8.1. The values for this model

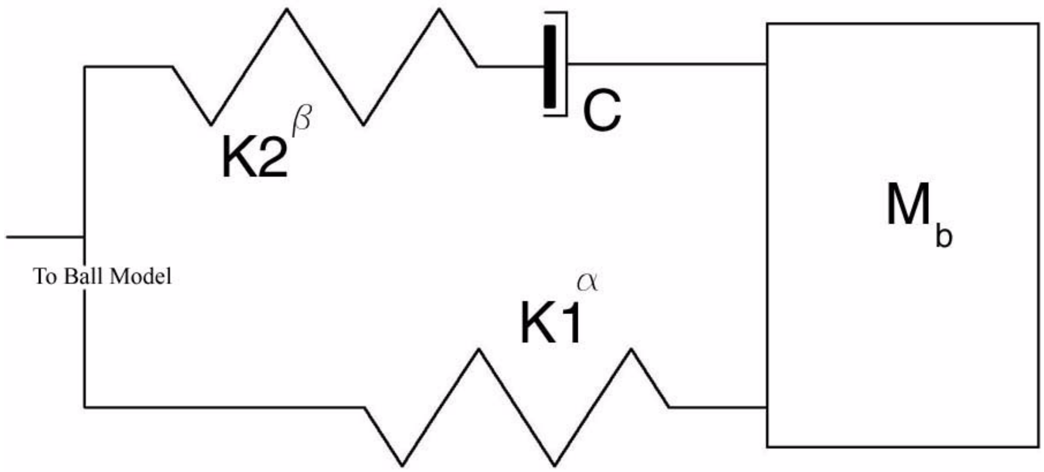


Figure 8.1 The Johnson and Leiberman ball model.

are: **K1** = 15.9 x 10^6 N/m, α = 1.5, **K2** = 47.6 x 10^6 N/m, and β = 1.64. The mass values $\mathbf{M_b}$ are the same for both models at 45.2 grams.

Because the mass of golf balls is fixed by United States Golf Association rules, the use of non-dimensionalization is not as helpful. Because of this, the analysis of lumped mass models in this case study is performed without non-dimensionalization.

8.3.3 Club Model

The model for the club was created using simplifying assumptions. This reduced the complexity of the model and allowed results to be obtained more quickly and in an easier form than for a more complex model. This model consists of two variable masses, one which represents the mass of the face itself and one for the remainder of the club mass. A massless spring of variable stiffness joined the two together. In order to be in line with the overall mass specification, the sum of the two masses is limited to approximately 200 grams.

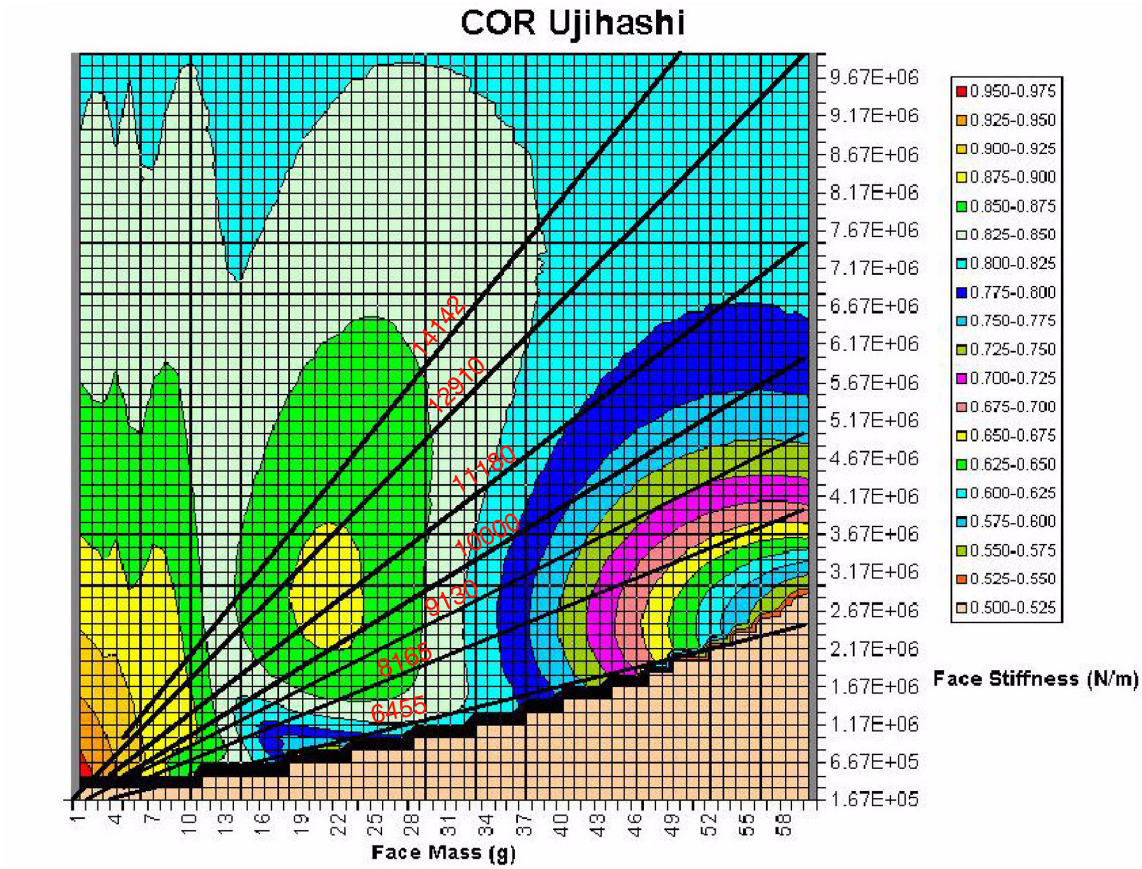


Figure 8.2 Lumped element simulation results from use of Ujihashi ball model.

8.3.4 Lumped Element Analysis

Both models were analyzed using the same method, a Runge-Kutta simulation program in MATLAB. The simulations used a range of face stiffnesses $\mathbf{K_f}$ and face masses $\mathbf{M_f}$ in order to determine ranges for those variables that yielded acceptable performance. The results utilizing the Ujihashi model are shown in figure 8.2. This is a from a plan view to show the variable ranges more clearly that an orthogonal view. The radial lines represent different frequencies of the club face. They serve to indicate that club face frequency is not the only key to a good design. The results for both models are similar, but not exactly the same. Table 8.1 shows the summarized results for the lumped element models. The variable ranges shown are produced by combining the results from both models. The fact that some of the stiffness and mass values from the models are also not realistic was considered as well. Using these values, the process of static analysis and further concept refinement was begun.

8.4 Concept Generation

The concepts generated in this case study follow the pattern shown in Chapter 5. The general concepts are categorized by the form of strain-energy storage used. The general categories with figures of possible configurations are given below. Most of the concepts below are mechanism concepts which make use of deflection in the face as well as gaining deflection from some mechanism behind the face. However, some surface concepts are also included.

8.4.1 Axial Concepts

Because more deflection is gained from tension rather than compression in axial loading, tension concepts were considered here. The concept makes use of a face which would be attached to a variable number of beams. These beams would be attached to the

TABLE 8.1 Lumped element model variable ranges for stiffness and mass.

Variable	Maximum	Minimum
Face Stiffness (N/m)	2×10^6	6×10^6
Face Mass (g)	20	35

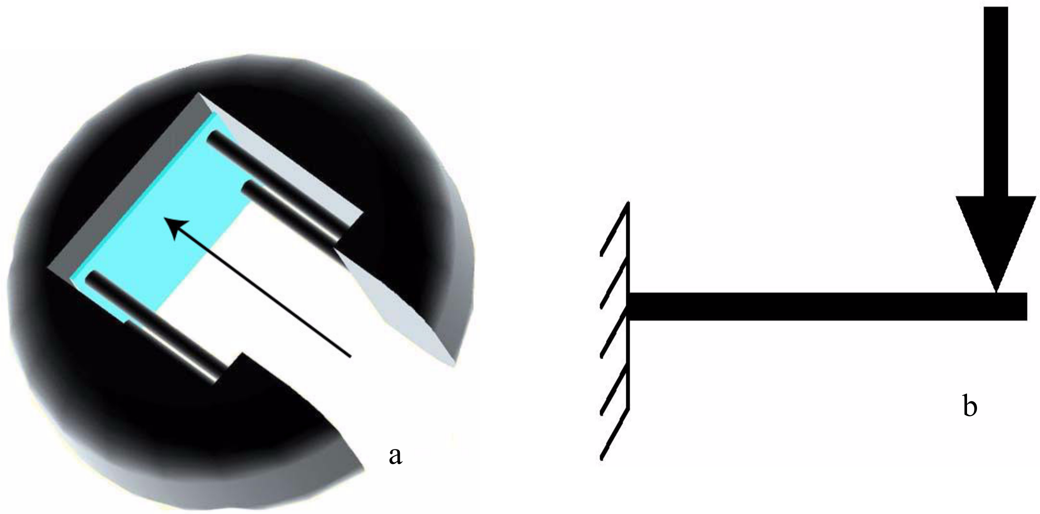


Figure 8.3 An (a) axial tension concept and a (b) fixed-free concept. The arrows indicate the direction of the direction of the impact force on the face.

remainder of the club body. For simplicity, the beams would be cylindrical. This reduces the number of variables that define the geometry of the beam to two, length **L** and radius **r.** The number of beams **n** is another variable that must be investigated.

8.4.2 Bending Concepts

Bending concepts are organized by the end conditions. The end conditions which are represented here are simply supported beams, fixed-free cantilever beams, fixed-fixed cantilever beams and fixed-guided cantilever beams.

8.4.2.1 Simply Supported Beams

Simply supported beams were determined to not be viable concepts because the end conditions do not provide sufficient support to retain the face when not being impacted without the use of pin joints, which are not desirable for a variety of reasons including friction, secondary impacts and others mentioned in Chapter 1.

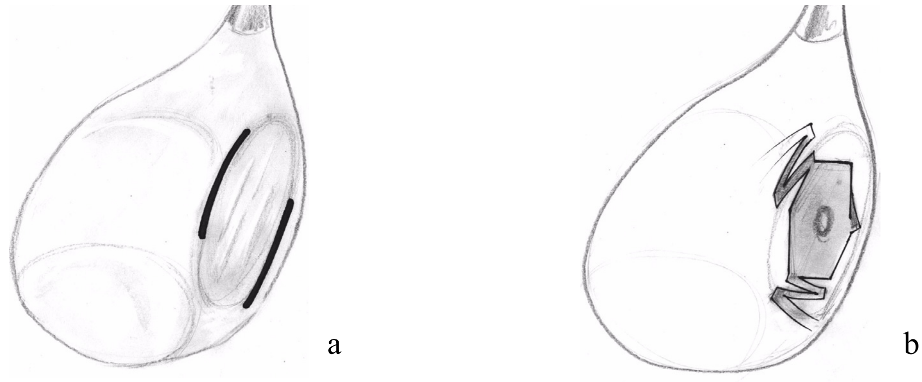


Figure 8.4 A fixed-fixed (a) and fixed-guided (b) concept. The fixed-guided concept uses an ortho-planar spring. The hexagon is the center platform and the legs are the beams to the sides.

8.4.2.2 Fixed-Free Cantilever Concepts

In order for one end of the beam to actually be free, the beam must constitute the impacted face. This then becomes a surface concept. A rectangular cross-section is chosen for the beam. There are three variables that must be chosen, width of the beam **w**, height of the beam **h**, and length of the beam **L**. The number of beams is one in this case if the beam is to act as the face itself. One general concept of this category is shown in figure 8.3b on the right with an arrow indicating the direction of the force on the face.

8.4.2.3 Fixed-Fixed Cantilever Concepts

Both surface and mechanism concepts are feasible using a cantilever beam which is fixed on both ends. A rectangular cross-section is chosen once again. The variables to be determined are length **L**, width **w**, height **h** and the number of beams **n**. The number of beams is not of concern when it is a surface concept because the face would be the only beam. An example of a surface configuration is shown in figure 8.4.

8.4.2.4 Fixed-Guided Cantilever Concepts

Fixed-guided cantilever beams lend themselves more readily to mechanism concepts. The variables to be determined in this concept are essentially the same as all other bending concepts. Figure 8.4 shows a fixed-guided concept that makes use of an ortho-planar spring technology developed at Brigham Young University. [Parise et al., 2001].

The pursuit of four key variables summarizes the further refinement of bending concepts. Those variables are the number of beams, and the length, width and height of those beams. Those variables will be found in the static analysis portion.

8.4.3 Torsion Concepts

Several configurations of torsion bars were generated and evaluated. One key question that needed to be answered was whether or not the torsion cylinders should be hollow or solid. In addition to that, the number of torsion bars $\mathbf{n}$, the length $\mathbf{L}$, radii $\mathbf{r_1}$ (and possibly $\mathbf{r_2}$) and the geometry of the moment arms are all variables that need to be defined in order to have a practical configuration to evaluate. A possible torsion concept is shown in figure 8.5.

8.4.4 Initially-Curved Beams

This concept makes use of initially curved beams attached on the back of the face. The key variables which need to be determined are radius $\mathbf{r}$, width $\mathbf{w}$, height $\mathbf{h}$ and the

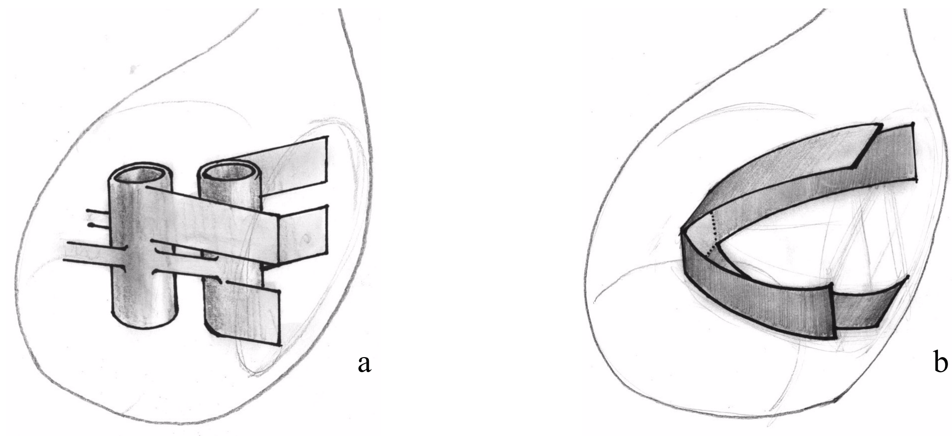


Figure 8.5 A torsion (a) and an initially-curved beam (b) concept.

number of beams **n**. One potential initially-curved beam configuration is shown in figure 8.5.

As mentioned in Chapter 5, the purpose of the initial concept generation is to identify feasible general concepts and identify the key variables in each concept that must be defined in order to determine its behavior. These key variables are summarized in table 8.2. The actual values for those variables were determined in the static analysis portion of the process. Instead of generating a large number of concepts and performing a static anal-

TABLE 8.2 Summary of key design variables.

Concept	Key Variables
Tension Concepts	**n, L, r**
Fixed-Free Cantilever Concepts	**w, h, L**
Other Bending Concepts	**n, w, h, L**
Torsion Concepts	**n, L, r_1, r_2, Moment Arm Geometry**
Initially-Curved Beam Concepts	**n, r, w, h**

ysis on each one, the static design analysis will be implemented on each general concept. Using the results from the lumped element models which are shown in the next section, several feasible configurations for each general concept can be created.

8.5 Static Analysis

After identifying the significant variables in the concept generation stage and the target values from the analysis of the lumped element models, the generation of actual configurations using static analysis was begun. In this section, the potential materials are introduced and discussed, and then the analyses of each main concept category identified in the concept generation section are summarized. These summaries include discussion of the key trends found in the closed-form solutions. A single configuration was chosen based on these closed-form solutions and was then modeled in a finite-element environment for further development and evaluation. All mechanism configurations which were analyzed do not include the effects of the face itself. That will not be included until the finite-element portion of the process.

TABLE 8.3 Recommended materials and properties.

Material	Modulus E (GPa)	Density (g/cm^3)	Allowable Stress (MPa)
Titanium 6-4	116	4.85	2000
465 Stainless Steel	210	7.8	2600
Aluminum	72	2.75	1100

8.5.1 Materials

Three materials were recommended by TaylorMade Golf. These three materials, along with their respective properties are shown in table 8.3. While the density and modulus numbers are very similar to those found in the literature, the allowable stress figures are all about twice the yield strength found in the literature. This number is based on engineering experience from TaylorMade and includes the implicit assumption that this stress level is only allowed in bending because the stress distribution still leaves a large portion of the material below the yield point. However, when the material is stressed in ways other than pure bending, this number needs to be modified. In the closed-form solution, all three materials were considered.

8.5.2 Axial Concepts

The key variables identified earlier in this chapter for this type of concept are: **n** (number of beams), **L** (length of beams), and **r** (radius of beams). The algebraic manipulation of the equations modeling axial loading shows that the mass of this concept is independent of the number of beams. It is intuitive that this would be the case. The stress in a axially-loaded member is only dependent upon the load and the area. This basically means that the length and overall area are fixed for a given stiffness and allowable stress. Axial concepts cannot use the allowable stress number given in table 8.3, because the entire cross section will experience the same stress. The allowable stress must be reduced to some point below the yield point. In this case study, maximum allowable stress was limited to 0.9 times the yield stress.

8.5.3 Bending Concepts

The closed-form models for bending concepts, both linear and non-linear, were manipulated so as to be able to solve for the mass of the concept and the cross-sectional geometry of rectangular beams used in the concept. To accomplish this, several variables were required. Some of these were known, such as a target stiffness and the static load. It is these known variables that influence the mass of the concept. In addition, for a given stiffness and static load, the mass was constant for all bending concepts. Other variables were arbitrary, including the number and length of beams. These variables only influence the geometry, and that influence varies depending upon the end conditions of the beam. However, their values were not inconsequential. They had to be chosen so that the geometry was practical. The end condition combinations that were considered were simply supported, fixed-fixed, fixed-free, and fixed-guided. For the fixed-free and the fixed-guided end conditions, both linear and non-linear pseudo-rigid-body models were analyzed.

8.5.3.1 Fixed-Fixed Cantilever Beams

As with the simply supported beam concept, only the linear model was used to analyze the fixed-fixed concept. The equations for height and width are given in equations (8.1) and (8.2).

$$h = \frac{1}{12}\frac{KL^2\sigma}{EF} \tag{8.1}$$

$$w = 108\frac{F^3E^2}{K^2L^3\sigma^2 n} \tag{8.2}$$

8.5.3.2 Fixed-Free Cantilever Beams

In the case of fixed-free cantilever beams, both the linear and pseudo-rigid body model were used to analyze the system. First, consider the linear closed-form solution in which the equations for height and width are given as equations (8.3) and (8.4), respectively.

$$h = \frac{2}{3}\frac{KL^2\sigma}{EF} \tag{8.3}$$

$$w = \frac{27}{2}\frac{F^3E^2}{K^2L^3\sigma^2 n} \tag{8.4}$$

With the two sets of equations, a comparison can be made about the height and width equations. In order to provide the same stiffness and stress characteristics, a fixed-free cantilever beam would be eight times as high, but about one-eighth as wide as a fixed-fixed beam of the same length and material.

The nonlinear solution shows very similar trends. It appears that the pseudo-rigid-body model predicts a mass about 4 to 4.5% higher than the linear model depending on the stiffness. The PRBM also predicts a beam height about 5% higher and a beam width about 8% lower than the linear model. It does not appear that these differences are due to the deflection being outside of the linear range because they vary little over the stiffness range. These differences are slight and if more concept refinement is desired, additional methods such as finite-element models and physical prototypes may resolve the variance. Because the results are similar, linear models should be used where possible in order to minimize the complexity.

8.5.3.3 Fixed-Guided Cantilever Beams

The analysis for the fixed-guided concept is much the same as the fixed-free concept. The mass predicted by the linear model for a fixed-guided concept is the same as the mass predicted by the linear models for any other bending concept. The equations for the height and width are given in equations (8.5) and (8.6).

$$h = \frac{1}{3}\frac{KL^2\sigma}{EF} \tag{8.5}$$

$$w = 27\frac{F^3E^2}{K^2L^3\sigma^2 n} \tag{8.6}$$

The variances between the PRBM and the linear model are nearly identical to those found for a fixed-free cantilever beam.

8.5.3.4 Bending Summary

In order to compare the various configurations in the general bending category, each end-condition combination must be considered in context. For instance, the fixed free cantilever concept was determined to be a surface concept, meaning that the cantilever beam would actually be the impacted surface. This places some constraints on the concept right away. A typical golf face is approximately 100 millimeters long (horizontal axis of face) and 50 millimeters tall (vertical axis of face). Therefore, the fixed-free concept should be evaluated near these parameters. Other end-condition combinations may not be quite so constrained, but it is still important to try and develop configurations with reasonable geometry. With this is mind, each end-condition combination was evaluated and table 8.4 shows a selection of the concepts generated. However, all bending concepts shown in the table make use of the maximum allowable stress for the chosen material, use the same target stiffness (6 x 10^6 N/m), and use a static load approximated as 13 kN. All length dimensions are in meters and mass dimensions are in grams.

Table 8.4 also highlights the fact that while the masses are equal, there are significant differences in the cross-sectional geometry of the beams for different end-condition combinations. Because of this, other considerations such as those mentioned earlier were contemplated.

8.5.4 Torsion Concepts

The analysis of torsion concepts was considerably more difficult than the analysis of the bending concepts. One of the major challenges in implementing a torsion concept was determining the method used to apply a moment to the torsion bar. For this case study, it was decided to apply that moment through a moment arm with a point load at the end opposite from the torsion bar. This was probably the most simple method. However, this method still applied a combined load to any torsion bar because the bar also had to react to the point load. This combined loading created a situation in which the torsion bar was twisting and bending. The torsion bar must have a certain geometry in order to provide the desired torsion behavior. But that same geometry also influenced how much the bar bent under the applied load. In addition to these factors, the moment arm itself was considered as part of the inertial mass. It also had some deformation and thus deflection. All of these factors were considered in the analysis.

For this case study, a spreadsheet was set up that allowed for all of the factors mentioned above to be contemplated. It was first thought that hollow torsion bars would provide better specific volume efficiency due to the stress profile that is present in torsion. Since the highest stresses are on the outside fibers of the material, it was believed that if the mass was also concentrated away from the center of the bar, it would be beneficial. However, it was quickly discovered that a hollow structure would not provide the needed stiffness and still be below acceptable stress levels. So solid torsion bars were then considered. Several additional trends were also discovered. The number of torsion bars would not be able to exceed two. If three or more torsion bars were used, the majority of the deflection would actually come from bending of the bars rather than torsion, and then a bending concept might as well be used.

The moment arm itself was also a cause for concern. If the majority of the deflection was to come from torsion, a very stiff moment arm needed to be used. In this case, because this is a force rather than displacement load, the stiffer moment arm would be heavier and thus add to $\mathbf{m_f}$, which is also undesirable.

At that point in the case study, there were other concepts that looked more viable as well as easier to refine. The decision was made to eliminate torsion concepts from further consideration.

8.5.5 Initially-Curved Beams

One miscellaneous category that warranted further investigation and concept refinement was initially-curved beams. As mentioned in Chapter 5, the beams considered in this case study could be loaded in two ways. The first loading condition considered was labeled pseudo-axial loading. This type of loading is shown in the left-hand side of figure 5.7. The other type of loading is known as transverse loading and is shown in the right-hand side of the same figure.

Linear, closed-form models exist for both of these configurations and can be found in Roark's Formulas for Stress and Strain [Young, 1989]. Both models predicted similar behavior, with the transverse loading predicting slightly better performance. Transverse loading was chosen for further development. While the models are technically closed-form, they are quite large and complex, and so the analysis was not as simple as for the bending concepts.

The model for a transversely loaded initially curved beam showed some trends that were somewhat similar to those found in the bending concepts. The mass stayed relatively constant even as the radius of the beams changed. This was somewhat analogous to changing the length of the bending concepts. The mass did increase slightly as the radius was increased. The cross-sectional geometry was still highly sensitive to changes in radius. Overall, the masses of the initially-curved beams were considerably higher than the masses of the bending concepts. However, there were some other advantages that will be discussed in the next section.

The summary of this stage in the generation of configurations is shown in table 8.4. Titanium showed itself to be the superior of the three materials, so all configurations in the table are modeled with titanium. All configurations have the a stiffness of 6 x 10^6 N/m, which is the high end of the stiffness envelope and all configurations have predicted stresses below the set limit. All lengths are in units of meters, and masses in units of grams.

8.6 Configuration Selection

Almost all of the configurations that were developed had their own specific advantages and disadvantages. The axial concepts had low masses, but their geometry was somewhat unwieldy and they didn't appear to be able to hold the face in a manner that was acceptable. Each bending concept (fixed-fixed, fixed-free, etc.) yielded configurations that

TABLE 8.4 Several concept configurations in comparison.

Concept/ Variables	Number of beams	Length	Height	Width	Radius	Total Mechanism Mass	Model
Tension	2	0.2793	NA	NA	0.0015	19.6	Linear
Fixed Free	1	0.045	0.0107	0.0152	NA	35.65	Linear
Fixed-Free	1	0.045	0.0103	0.0166	NA	37.24	PRBM
Fixed-Guided	4	0.045	0.0054	0.0076	NA	35.65	Linear
Fixed-Guided	4	0.045	0.0051	0.0083	NA	37.24	PRBM
Fixed-Fixed	4	0.045	0.0013	0.0304	NA	35.65	Linear
Simply Supported	4	0.045	0.0027	0.0152	NA	35.65	Linear
Initially-Curved	4	NA	0.0015	.065	0.045	135.7	Linear

Figure 8.6 Initially-Curved Beam Configuration.

were acceptable from a mass standpoint, but not all of them showed acceptable geometry. For instance, the fixed-free surface concept did not match the size of a normal driver face. The initially-curved beams showed some good geometry aspects, but looked to be on the heavy side.

Keeping in mind all of the design constraints, a decision was made to pursue transversely-loaded initially curved beams for further development. This decision was based on the fact that while the mass appeared high from the initial static analysis, it appeared that the mass could be reduced as the model was refined. A configuration with four initially-curved beams that curved inward from the outside edges of the back of the face was more similar to current designs which attempt to move the majority of the mass away from the center of gravity to produce high polar moments of inertia. This four-beam design also appeared to have good symmetry to be able to handle off-center ball strikes. Input from TaylorMade golf also indicated that this was a good configuration to pursue. As with

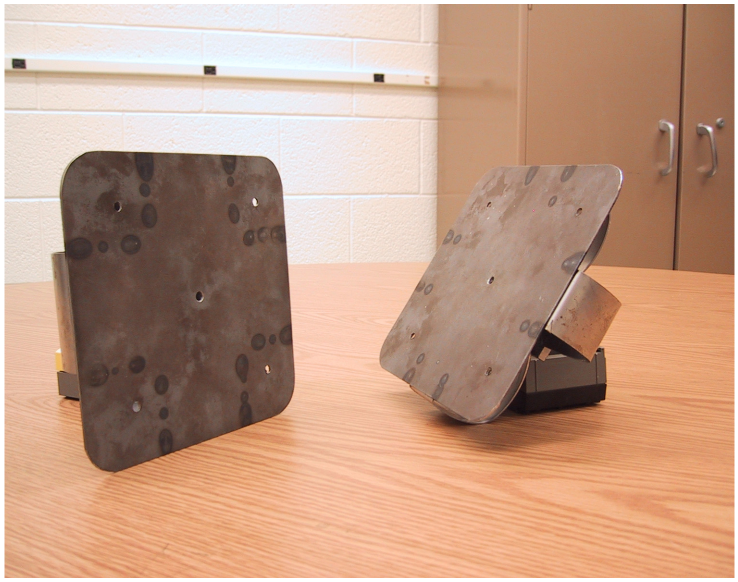


Figure 8.7 Static prototypes for initially-curved beams.

many engineering applications, the advantages and disadvantages create a system that must be balanced to be practical. While the initially-curved beam concept had many advantages, much work had to be done to decrease the mass to a feasible range. The variable values that defined the starting point for further configuration refinement are shown in table 8.4. A drawing of the initial configuration is shown in figure 8.6. At this point in the process, the thickness of the face had not been determined.

In order to validate the finite-element and closed-form analysis, two physical prototypes were constructed. The initial configuration chosen above was to be modeled in Titanium, but the static prototypes were constructed from steel. This was due to cost and facility limitations. It was much easier to model a steel configuration in the closed-form and finite-element worlds than it was to manufacture a titanium model in the real one. The prototypes were similar to the actual initial configuration in that they used the same num-

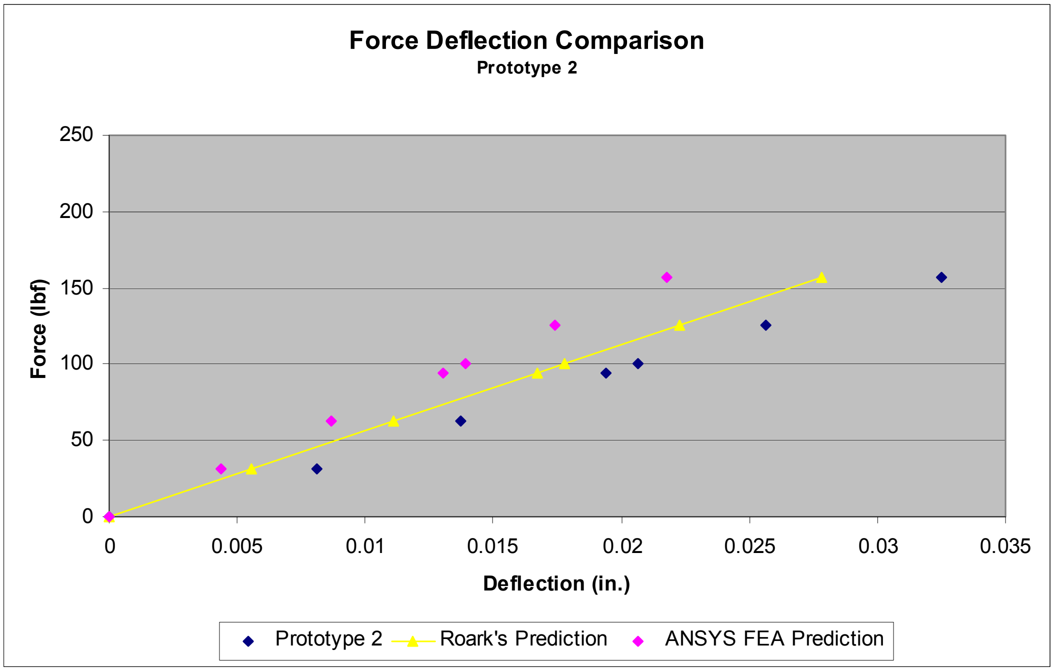


Figure 8.8 Force-deflection validation of closed-form models.

ber of initially curved beams and were approximately the same shape. A photo of the prototypes is shown in figure 8.7.

Force was applied with an air cylinder connected to a pressure regulator and the pressure (force) was measured with a pressure gauge. The displacement was measured with a linear potentiometer. Finite element models of the prototypes were built and analyzed. The comparison between the static testing of the closed-form and finite-element models and one of the prototypes is shown in figure 8.8. The figure indicated a fairly good correlation between all three models. This allowed for the process then to move forward with confidence in the static models, both closed-form and finite-element.

8.7 Configuration Refinement

As mentioned previously, the closed-form solutions did not account for the impacted face itself. The finite-element analysis was used to account for the face. Before any configuration could be refined, a decision had to be made concerning the amount of deflection allowed in the face and in the curved beams. The guidelines for this decision came from mass and stress considerations. The initial decision was to make the face as light as possible while still being able to withstand the impact. This meant that the face had to be thin. The first configuration modeled after the closed-form validation test was the curved beam configuration shown in table 8.4 with a face of thickness 0.003 meters. TaylorMade Golf Company supplied the load profile that was applied to the model. The results showed that this configuration was too flexible and the stresses were too high in the face.

The fact that the closed-form models and the finite-element models began to diverge more as larger deflections were derived from the face was not a surprise. The loads acting on the curved beams were not as simple as those modeled in closed-form. However, those closed-form models had provided a good starting point.

A new configuration was then created with slightly thicker curved beams and also a slightly thicker face. The results were still not within the target range for stiffness or stress. As a result, another configuration was developed. This process was repeated several times until the finite-element model appeared to fit the stiffness and stress requirements. While the mass requirements from the lumped element models were not met, the mass had been reduced as far as was thought possible using static analysis.

The geometry of the final configuration generated from this stage is shown in table 8.5. The lengths in the tables are in units of meters and the mass is in units of grams. This configuration had two sets of curved beams, each set with a different radius ($\mathbf{r_1}$ and $\mathbf{r_2}$) and thickness ($\mathbf{h_1}$ and $\mathbf{h_2}$). Both beams had the same width ($\mathbf{w_1}$ and $\mathbf{w_2}$). The face dimensions were 0.0616 m. by 0.0616 m. with at face thickness of 0.0022 m. The total mass of the beams and the face was now 58.7 grams. Once this configuration was found and it appeared to provide the proper stiffness and stress requirements, and the mass was minimized, the next step was to put that model into a dynamic finite-element analysis where an impact with the ball was simulated.

8.8 Configuration Evaluation

The finite-element model was sent to TaylorMade golf for this step in the analysis. This was because TaylorMade already had advanced models for the ball which had been validated. This model was software-specific and it would have been inefficient to develop a new model for different software at BYU.

TABLE 8.5 Initially-curved beam refined configuration geometry.

$\mathbf{r_1}$ (m.)	$\mathbf{h_1}$ (m.)	$\mathbf{w_1}$ (m.)	$\mathbf{r_2}$ (m.)	$\mathbf{h_2}$ (m.)	$\mathbf{w_2}$ (m.)
0.028	0.0022	0.03	0.023	0.0018	0.03

The dynamic analysis was not only completed on the original configuration sent to TaylorMade, but a simple optimization plan was also implemented. This plan called for the altering of three variables, face scale, face thickness, and curved beam thickness. The original states were used as the minimums for each variable.

As with the transition from the closed-form to finite-element analyses, the transition from static to dynamic analyses produced some divergence. The initial configuration showed the stresses in the face were above what was predicted in the static analysis. In addition to this, and more importantly, the COR was below the 0.850 limit.

The entire dynamic testing program tested 33 different configurations. While the initial configuration had too low a COR, other configurations produced above 0.850 COR while keeping the stress below the limit. The minimum COR produced was 0.547. The best configuration produced a maximum COR of 0.863. The dimensions of this configuration are shown in table 8.6. In order to reduce the number of variables in the dynamic configuration refinement, all beams in the configuration were assumed to have the same thickness rather than two different thicknesses.

TABLE 8.6 Final refined initially-curved beam configuration.

Beam Radius 1	Beam Radius 2	Beam Thickness	Beam Width	Face Length and Width	Face Thickness	Beam and Face Mass
0.028	0.023	0.003	0.03	.07392	.0026	91.7

In order to understand the behaviors of these different models, the data was investigated thoroughly. The most important key trend noticed was that as the curved beam thicknesses got thinner (indicating more flexibility), the COR seemed to decrease. This signified that concepts in which the majority of the deflection came from the mechanism behind the face were not feasible unless the face mass could be very low. This was due to the fact that more of the mass of the face must then be included in the equivalent face mass $\mathbf{m_f}$. In the case of golf clubs, this would probably require the use of composite materials.

The data from the dynamic analyses represents the end of the case study. Several configurations existed which appeared to be viable. The conclusion that a curved beam mechanism succeeds in this type of application was reached. While further development is needed in order to create designs ready for production, that development would be part of a final design process, which is outside the scope of this thesis.

8.9 Summary

This case study illustrates the use of the modified concept development process to generate and evaluate a number of different compliant mechanism concepts for use in a metalwood driver golf club head. The final results of this case study are at least five configurations which are now ready for a final design process. These configurations are predicted to have a COR above 0.850 while still remaining within the acceptable stress limits. While there was some difficulty in implementing the mass specification from the lumped element model, the overall process was effective in generating concepts and refining that concept until it appears prepared for use as a starting point in a final design process.

CHAPTER 9 CONCLUSIONS

The objective of this thesis was to identify and define an approach for generating and evaluating compliant mechanism concepts for use in high-COR applications. This chapter discusses the completion of this objective and includes a discussion of additional conclusions and recommendations from this research.

9.1 Concept Development Process

The process used to generate and evaluate compliant mechanism concepts to produce a configuration that is suitable for a final design process is based upon the concept development process given by Ulrich and Eppinger (2000). The process was modified so as to be specific to compliant mechanisms used to maximize the coefficient of restitution under impact loading.

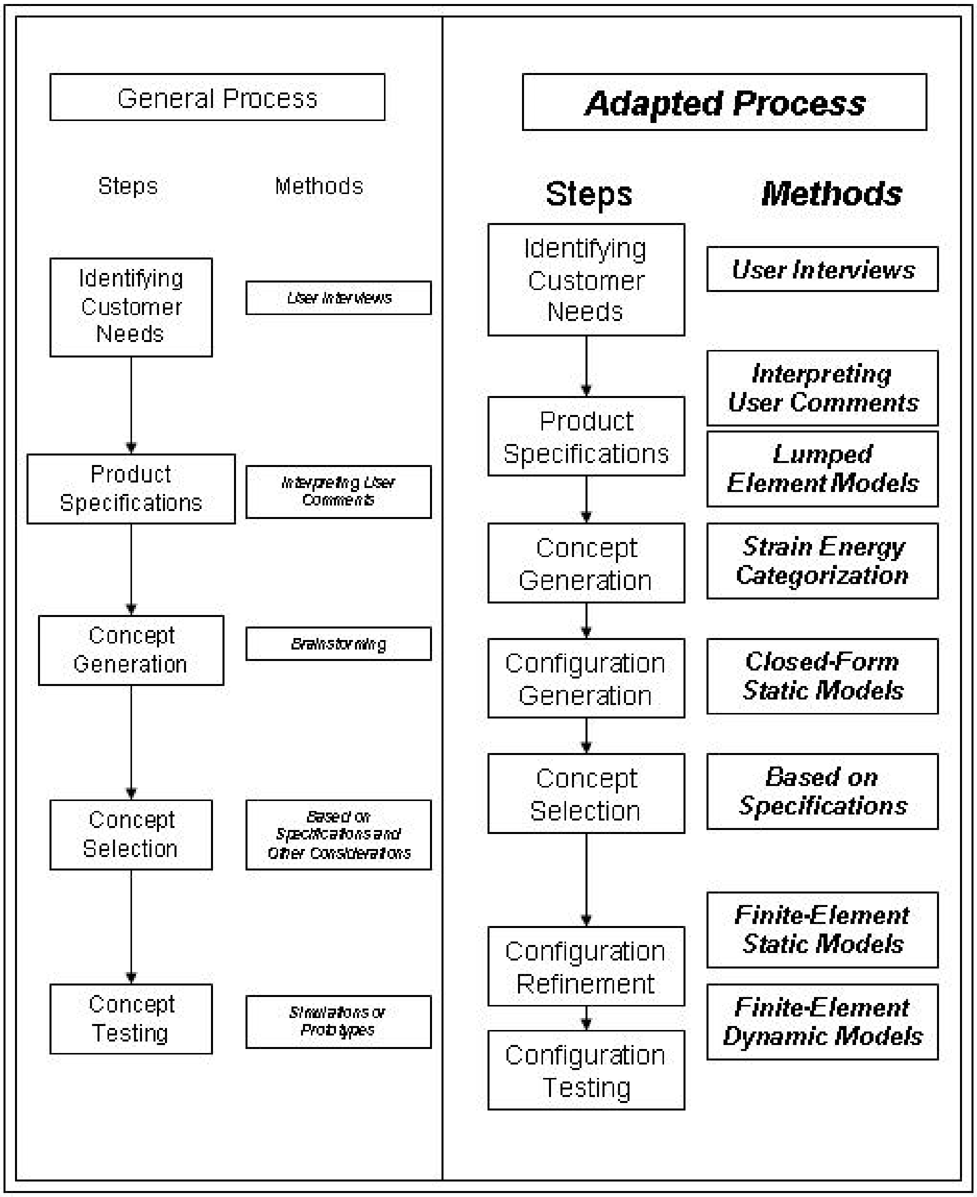


Figure 9.1 Diagram of Concept Development Process Comparison.

The new process is diagramed in figure 9.1 along with a comparison of the original concept development process. This diagram includes the steps of the process as well as the methods used to achieve those steps.

9.2 Conclusions

While the adaptation of general concept development processes narrows its focus to a specific type of application, the new process still allows for a wider variety of applications than was available before. These can range from golf clubs as shown in the case study to prosthetic legs and feet to be used by amputee athletes in competition. The process also allows for compliant mechanisms to be used in a new and different application to which they are well suited. The case study illustrated the use of the concept development and evaluation process and has produced an initially curved beam concept which is now ready to be further refined in a final design process.

The overall research has yielded the following conclusions:

- Lumped element models can be useful in determining stiffness and to some degree mass specifications for high-COR mechanisms.
- The process is most beneficial when a broad variety of strain-energy storage categories are considered.
- Closed-form static models are useful for generating configurations based on stiffness specifications, but are limited in their abilities to model the impacted surface and mass specifications.
- Static finite-element models can model the impacted surface, but are still limited in their ability to determine whether or not a configuration meets the mass specifications.
- While static models possess the limitations described above, they provide initial geometries for the next stage in the process.
- Dynamic finite-element simulations can be used to refine configurations and determine their viability for final design.

The case study also yielded two conclusions concerning golf clubs:

- Unless the impacted face is extremely light, a large portion of the required deflection must come from the impacted face itself, rather than from the mechanism behind the face.

- Initially curved beam configurations can be used to produce a high COR.

9.3 Recommendations

The recommendations for further research regarding the study of the process are:

- Additional approaches to concept development may be developed and compared with the approach outlined in this thesis.
- Additional case studies can be performed using this approach to further determine its applicability to a wide range of applications.

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