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Agent-Based Virtual Laboratories for a Novel Experimental Approach to Socio-Environmental Synthesis

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Abstract: The number of agent-based modeling (ABM) applications within the socio-environmental context has exploded over the last decade. Most of these ABMs have been designed to deepen our understanding of the decision-making processes and human-environment interactions that lead to emergent community- and/or landscape-level outcomes in specific locations and contexts. While these 'case-based' ABMs have generally been successful in this aim - to which their popularity attests - little progress has been made through this case-based approach towards the ultimate goal of building coherent theory about the structure, dynamics, and sustainability of socio-environmental systems. Moving away from case-based ABMs, this paper will introduce the agent-based virtual laboratory (ABVL) approach, which requires more generalized ABMs for cross-site experimentation, comparison, and synthesis. Broadly, the ABVL approach harnesses the process-based explanatory power of ABMs within a modeling system architecture explicitly designed for flexible, iterative experimentation and cross-site comparison. A review of practical and philosophical aspects of socio-environmental modeling purposes, epistemologies, and design and evaluation principles is presented in order to place the ABVL approach along a spectrum of existing modeling approaches. As an illustration of the novel research questions that can be asked with ABVLs, a demonstration model is used to compare a household- versus settlement-level agent representation in search of the best and most parsimonious explanation of land-use and livelihood patterns across three study sites in East and Southeast Asia. The synthesis capabilities of the ABVL approach can lead to new hypotheses and experiments to accelerate the development of theories of socio-environmental system change.

***Keywords**: cross-site comparison; agent-based modeling; socio-environmental systems; virtual laboratory; land-use change.*

1. INTRODUCTION

The use of agent-based models (ABMs) to investigate the dynamics of socio-environmental (i.e., coupled human-natural or socio-ecological) systems (SESs) has accelerated over the last decade (An, 2012; Rindfuss et al., 2008). The recognition that humans are primary agents of change in natural system structure and function (Ellis and Ramankutty, 2008), combined with the ability of ABMs to explicitly represent human decision-making processes (An, 2012; NRC, 2013), is driving the popularity of the approach. Coupled with the infeasibility of conducting field experiments with large-scale SESs, simulation modeling more broadly has become a critical tool for socio-environmental researchers. In addition, some of the most pressing questions in socio-environmental research related to sustainability and tipping points (e.g. Anderson et al., 2009) require modeling approaches that can provide process-based explanations of emergent phenomena and synthesize such findings across multiple local case studies (Parker et al., 2008; Rindfuss et al., 2008). To this point, ABMs have excelled at offering insight into the processes underlying emergent phenomena, but most (if not all) are ill-equipped for comparison and synthesis of model findings across different sites and/or SESs. This paper will articulate and demonstrate an *agent-based virtual laboratory* (ABVL) approach that facilitates cross-site comparisons and accelerates the production of generalized knowledge of SES dynamics.

The majority of ABMs that have been developed fall into two camps: simple, generalizable or complex, realistic models (Janssen and Ostrom, 2006). Many early ABMs were theoretical models intended to explain emergent phenomenon with a simple set of rules and interactions between distributed, decision-making agents. As the method gained traction, more emphasis was placed on empirically-grounding ABMs, and a shift occurred towards the development of more realistic models that could be applied to a particular case-study (Janssen and Ostrom, 2006). Increasing availability of highly detailed data sources has led to an explosion of case-based ABMs, which has prompted some to ask whether additional case-based ABMs are contributing to the ultimate goal of building coherent theory about the structure, dynamics, and sustainability of SESs? Rindfuss and colleagues (2008) have proposed that ABMs, due to their explicit representation of human decision-making, can provide a formal means to synthesize general insights into the mechanisms driving human-environmental interactions and SES sustainability. Case-based ABMs can provide insights into the influence of decision-making processes in a particular system, however synthesis across models has been difficult because of inconsistencies in how the same processes/structures are represented across models (Parker et. al., 2008).

Generalized models offer another possibility for cross-site comparison and synthesis. A generalized model, as defined here, is a minimal model based on first principles and/or phenomenological descriptions of processes and interactions that does not attempt to represent any particular system with precision (Evans et al., 2013; Roughgarden et al., 1996). More precisely for the purposes of synthesis across cases, a general model is one with an abstracted structure and process representation that reflects common patterns across systems, can be easily adapted to multiple contexts, and produces insights that are broadly applicable beyond any particular context. Various efforts towards developing such generalized models of SESs have been made in the past. One of the earliest examples was work with the SugarScape model (Epstein and Axtell, 1996). Perhaps overly simplistic to provide meaningful synthesis across sites, it was one of the first general, spatially explicit models of human natural resource use. The CORMAS modeling system (Bousquet et al., 1998) provides a more sophisticated model architecture that can be used with stakeholders for rapid prototyping and development of ABMs of SESs. While CORMAS has the flexibility to develop generalized models in different contexts, it is developed and applied as case-based ABMs for particular contexts in order to deepen stakeholder understanding of key processes and dynamics in SESs. It was never intended nor designed for simultaneous application and analysis across many different sites and contexts. The UrbanSim modeling system (Waddell, 2002) has been applied across multiple metropolitan areas to simulate development patterns and locational choices. The UrbanSim model is capable of simultaneous application across different sites, however its reliance on the 'micro-simulation' approach requires the detailed specification of large numbers of agents and is thus very data-demanding. None of these previous approaches strikes the required balance between simplicity, flexibility, and scalability for cross-site comparison and synthesis across a large number of sites.

This paper articulates the concept of an ABVL as an approach that harnesses the process-based explanatory power of ABMs within a modeling system architecture explicitly designed for cross-case comparison and synthesis. I make the distinction between the previously described ABMs as tools, and ABVLs as a unique practical and philosophical approach to model-driven research that uses ABMs with distinct design and testing requirements for the purposes of process-based synthesis. This distinction will be elaborated in the following section. I first present a conceptual framework for comparing and contrasting ABVLs with other modeling approaches based on their purpose, underlying epistemology, design principles, and evaluation requirements. I then illustrate the novel types of research questions that can be investigated with ABVLs using a demonstration model implemented across three study sites in China and Laos. Finally, I conclude with a discussion of the relative strengths and weaknesses of the ABVL approach, and how it can complement existing modeling and synthesis efforts.

2. BACKGROUND

To clearly place ABVLs within the universe of socio-ecological modeling approaches, four model attributes characterize important decision points in the model development process (Figure 1): purpose, epistemology, design, and evaluation. Decision points are part of an iterative modeling cycle in which each choice influences and is influenced by all other practical and philosophical choices. Models within the same modeling paradigm (e.g., statistical, analytical, agent-based, general equilibrium, etc.) can differ in each of these four dimensions. Conversely, models across modeling paradigms can be used for the same purpose, with the same epistemological approach, design principles, and evaluation standards. However, for the purposes of this discussion, this conceptual framework will only be applied to ABMs.

A model's purpose is perhaps the most basic element of model development, yet it has a profound influence on all other modeling choices. One cannot model everything so the model's intended purpose guides what is deemed important to include in the model and what is not. Given the inherent complexity of ABMs, the question then becomes how complex should the model be to fulfill its intended purpose? The purpose of an ABVL approach is to formulate, test, and generate new hypotheses of how and under what conditions particular processes are important for explaining observed SES outcomes. This entails both a series of case-specific model experiments, as well as a comparison of experiments across cases. Cross-site comparison and synthesis with ABMs, in particular, has the potential to provide insights into commonalities and differences in decision-making processes and agent interactions across land systems (Rindfuss et al., 2008). Thus, the primary motivation for creating ABVLs is to be able to conduct systematic comparative studies on socio-environmental system dynamics and potential trajectories across different regions and systems.

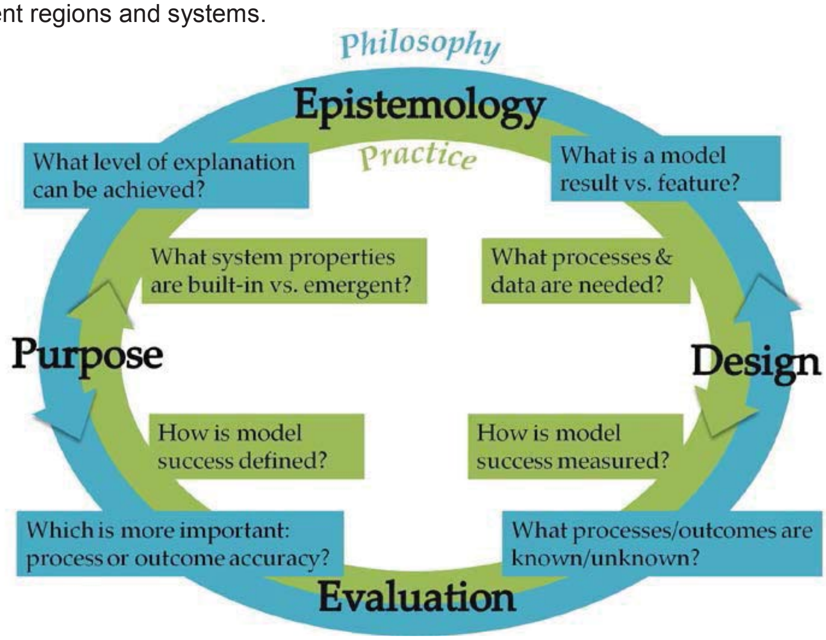

Figure 1. Conceptual framework for characterizing socio-environmental modeling approaches based on a series of practical (inner loop) and philosophical (outer loop) modeling choices.

Model epistemology can be understood as the method through which model analyses are conducted and the level of explanation a particular methodology is capable of producing. Building on the descriptions and classifications by Overmars and colleagues (2007), all socio-environmental modeling approaches can be organized along a spectrum ranging from extremes of data-driven induction to extreme deductive 'Popperian' falsification. Are model design choices primarily data- or theory driven? Is model analysis pattern- or process-based? Are model explanations correlative or causal in nature? Given their stated purpose, the ABVL approach is best described epistemologically as 'abductive modeling'. Balance between deductive and inductive approaches is necessary in many socio-environmental contexts, because fully-specified theory may not be available for the phenomenon of interest, especially when the model needs to be parameterized across many different contexts or types of SESs. Similarly, the complex nature of SESs renders inductive approaches alone insufficient, because of the possibility of non-linear, path-dependent, and/or emergent system-level outcomes arising from lower-level processes (Irwin et al., 2009). The ABVL approach uses 'factors induction' to cut through the complexity of bottom-up interactions and identify a parsimonious set of candidate processes that may improve explanatory power. This maintains the process-based, deductive approach that is the strength of ABMs, while broadening the range of theories used to inform the structure of process and agent interactions to those relevant to the processes in question rather than the specific context being modeled.

Model design and evaluation can be addressed together, because specific model inputs and outputs are closely linked to what can be measured to evaluate model performance. From a practical perspective, model *design* is a matter of which system components are represented within the model and what data is used to parameterize the model. Model design philosophy is more a question of which processes and/or outcomes are known and unknown? The design of a model must take into account which processes, system structures, and outcomes are known and can therefore be encoded directly (i.e. model features), and which are unknown and must be abstracted, represented by proxy, or are left to emerge unconstrained as model results. Model evaluation concerns both the targets for model testing and the overall goal of model development. Model evaluation is ultimately limited by the outputs of a model and how the modeler chooses to define model success. In practice, model evaluation entails the identification and comparison of patterns in model output and/or behaviors with comparable measurements from the system under study. The nature of the comparison (quantitative and/or qualitative) and thresholds for model success depend on the philosophical underpinnings of the evaluation. In other words, *should* one expect a high level of agreement between modeled and observed outcomes (Brown et al., 2005)? This question often comes down to whether outcome or process accuracy is more important to the modeling endeavor, and what processes and/or outcomes can the model be expected to represent and reproduce, respectively, given the nature of the system under study.

The purpose of the ABVL approach is to deepen understanding of which processes are important in which situations, rather than developing predictive capability. Accordingly, the design of ABMs used within with ABVL approach lies between very simple 'toy' and highly-detailed case-based ABMs. ABMs used with an ABVL approach are designed capture a parsimonious set of processes and agent attributes known to be important across cases as a starting point, rather than being as realistic as possible with a high degree of outcome accuracy. A trade-off is made between model realism and predictive power for a simpler design with a focus on process accuracy. However, the model must also be designed with sufficient detail so that meaningful comparisons can be made with empirical data. A major challenge for developing ABMs for an ABVL approach, then, is to find the proper balance between the number and types of interactions represented and the generality of their representation (Magliocca et al., 2014).

Process accuracy is essential for the ABVL approach because as much insight should be gained when the model fails to reproduce observed outcomes as when it succeeds. Conversely, a moderate degree of outcome accuracy is also critical for linking the accuracy of system-level outcomes to the relative contribution of particular processes. Successful evaluation of an ABVL application must then formally identify potentially important processes across cases, and characterize how the addition or exclusion of a particular process contributes to more realistic model outcomes. Pattern-oriented modeling (POM; Grimm et al., 2005) is a hybrid model evaluation technique that considers both process and outcome accuracy simultaneously, which makes it ideally suited to link alternative model configurations to more or less realistic model outcomes. POM is a framework for designing and testing ABMs for 'structurally realistic' processes and parameters (Kramer-Schadt et al., 2007). A pattern is defined as "any observation made at any hierarchical level or scale of the real system that displays non-random structure" (Kramer-Schadt, 2007: 1557). The main principle of POM is that a model with high process accuracy will reproduce multiple patterns observed in real systems simultaneously. If a model can accomplish this, one can conclude that the model's process representation and internal structure are reasonably consistent with those of the real system (Grimm et al., 2005; Kramer-Schadt et al., 2007).

3. AN ABVL APPLICATION

The unique configuration of model epistemology, design, and evaluation choices constituting the ABVL approach opens the door to new synthesis research questions that cannot otherwise be explored with a case-based ABM approach. A generalized ABM of smallholding farmer decision-making and LUCC is applied to three example test sites to illustrate the types of questions that can be investigated through the ABVL approach. Specifically, this demonstration explores the contexts in which detailed, household-level ABMs are or are not needed to explain observed land-use/cover patterns. Such questions can only be answered through cross-site comparisons with a generalized model structure capable of testing for the influence of local variations of land allocation processes.

Three test sites were selected that differ from one another across a set of global environmental, population density, and market influence index variables (see Magliocca et al. (2014) for detailed site-specific descriptions, data sources, and results). Sample sites include two in China (western Shandong Province, China and northern Hunan Province, China) and one in Luoang Namtha, Laos. The first site in Shandong Province, China is characterized by nearly uniformly distributed dense populations concentrated in small villages around which intensive cultivation dominates. The second site in China is located in the hilly regions of northern Hunan Province and is characterized by fairly high population density dispersed within and along the edges of two main valleys. Intensive cultivation of rice is present around settlement areas, while extensive cultivation occupies areas with moderate slopes on the edges of valleys. The site in Laos is located in a mainly swidden cultivation system in northern Laos and is characterized by very hilly terrain (median slope of 40.2 percent) with patches of extensive cultivation dispersed across the landscape.

Model simulations are conducted over a 100x100 cellular landscape with each cell representing a hectare. Landscape outcomes are generated every year over a twenty-year period (with the first ten as model spin-up) as the result of agent land-use and livelihood decisions and their interactions with their environment. Agro-ecological dynamics emerge from agent-environment interactions, which in turn provide feedbacks to agents' subsequent yield and price expectations, and result in the evolution of stable land-use and livelihood strategies by the end of the model simulations. Detailed model specifications, a full ODD protocol description, and psuedocode are provided in Magliocca et al. (2014).

Model experiments are set-up with two alternative agent representations: household- and settlement-level agents. Household-level agents represent a household with four members, and land is allocated to each agent based on a simple random seeding and area-growing algorithm. The number of household agents created is determined by local population density. Settlement-level agents represent aggregates of multiple households, the number of which varies with local population density, located in a single settlement that has 100 ha of land (10x10 cells) available for cultivation and settlement. This aggregated representation addresses scaling and implementation challenges for large systems (Rounsevell et al., 2014), and also does not require detailed knowledge of local land allocation mechanisms thus maintaining the generality of model outcomes. A more detailed explanation of this agent representation is provided in Magliocca et al. (2013).

The performance of each model structure is evaluated based on both pattern and process accuracy. First, the composition of modeled landscapes (i.e., generated by household- versus settlement-level agents) is compare to that observed in remotely-sensed land cover maps for each site. Second, process accuracy of each model version is assessed by the extent to which three behavioral patterns, or 'stylized facts', describing agent-level behaviors associated with land-use decisions (de Janvry et al., 1999) are reproduced. *Normal surplus* is a subsistence level of agricultural production commonly observed in smallholder farming systems. *Minimum aspiration level*, in this context, is defined as the minimum income needed to support farming activities and/or purchase food on the market. *Consumption smoothing* is frequently observed in smallholder consumption patterns, and is measured here as the coefficient of variation in the difference over time between agricultural production and monetary income levels relative to subsistence needs.

4. RESULTS & DISCUSSION

The composition of modeled landscapes generated with settlement-level (SM) and household-level (HH) agent representations were compared with the land-use/cover maps for each site (Figure 2). Both generalized model structures reasonably approximated observed LUCC patterns across sites. However, when considered in combination with behavioral results (Table 1), model errors show the effects of the different land allocation algorithms. Landscape composition for the Shangdong site was best reproduced with household agents, which also achieved greater than 90 percent agreement for two of the three agent-level behaviors. In contrast, settlement agents produced more realistic landscapes and agent behaviors for the Hunan and Laos sites. Both models underestimated cultivation intensity for the Hunan site, and overestimated the extent of intensive cultivation for the Laos site.

Although landscape outcomes were not predicted precisely for any site, patterns in model errors provided insights into the effects of each agent representation on outcome accuracy, and thus the relative importance of particular factors and processes operating within each site. Land constraints due to high population density in the Shangdong site were a primary influence on agent land-use and livelihood decisions. The household agent implementation set average land holdings just above one hectare (1.014 ha), which forced agents to put all of their land into intensive cultivation to meet subsistence requirements and/or pursue market-oriented agriculture or non-farm activities to generate sufficient income to buy food. Because landscape composition was similar to the real landscape and at least 90 percent of agents met both their subsistence requirements and their minimum income aspirations, the household agent model can be considered a more realistic model structure for the Shangdong site.

In contrast, the settlement agent model structure was a better predictor of landscape outcomes and agent behaviors for the Hunan and Laos sites. Land-use systems in which suitable land for agriculture is limited by terrain, as in the Hunan and Laos sites, extensive cultivation is often practiced on marginal land while intensive agriculture is focused on the best agricultural land (Magliocca et al., 2014; van Vliet et al., 2012). This is often associated with diffuse patterns of land ownership where cultivated land can be relatively far from the dwelling and cultivated and fallowed in cycles (van Vliet et al., 2012). Such land ownership patterns more closely resemble the settlement agent representation, which diffuses land-use pressure over a wider area. These findings offer a guide to further hypotheses that can be tested systematically across sites with the ABVL approach.

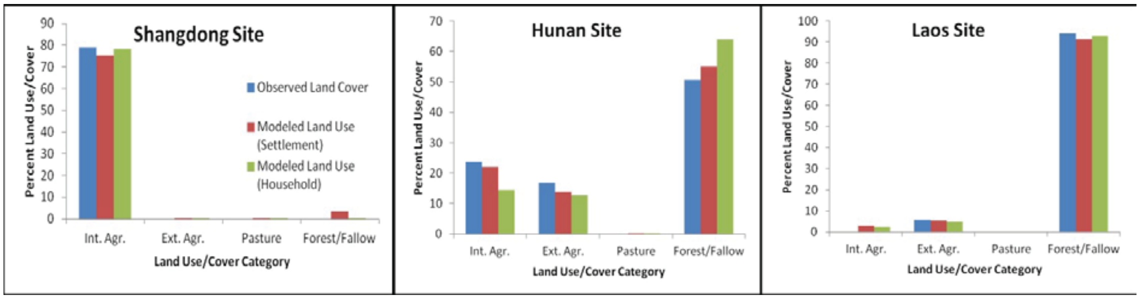

Figure 2. Comparison of modeled percent land-use/cover generated by the settlement (red) and household (green) model structures across sites.

Table 1. Percent of agents reaching target levels of covariance in consumption, surplus production, and achievement of minimum aspiration level in household versus settlement agent model structures.

Site	Model	CV Consume (%)	Surplus Level (%)	Asp. Level (%)
Shangdong	HH	14.53	99.98	90.88
	SM	24	100	82
Hunan	HH	7.77	47.32	96.69
	SM	28	98	99
Laos	HH	14.81	79.6	45.27
	SM	91	100	85

The ABVL approach brings novel challenges as well. For example, the same process (e.g., land allocation) can be represented many different ways across locations, which presents many variations to test. In addition, many local factors that influence land per capita and constrain households' livelihood choice sets, such as social networks and land tenure rules, vary widely across SESs, are heavily context dependent, and are not easily generalized (Rindfuss et al., 2004). Progress towards identifying and encoding a parsimonious set of these local processes will be gradual and require many model iterations. Fortunately, increasing computational power is now making thousands of model runs routine, and even the modest amounts of variation in land-use patterns explained by the generalized models used here suggest that the ABVL approach holds promise for cross-site hypothesis testing and synthesis.

Some social and cognitive processes may never be reliably generalized and encoded into a model. Yet, the effects of these processes can be tested indirectly with the ABVL experimental approach. Starting with simple models that capture readily generalizable processes (e.g., environmental constraints on agriculture) can set a benchmark for the explanatory power of a general model. More sophisticated and locally variable processes can then be gradually added to the model structure such that the relative importance of each process can be quantified along the way. Eventually, a point is reached at which model performance fails to improve with the addition of missing processes, and one can infer that remaining unexplained error is likely due to local, context-dependent processes and conditions. This end is consistent with the ultimate purpose, philosophy, and practice of the ABVL approach. Generalized models will often fail to predict SES dynamics and outcomes for any particular location, just as global datasets are generally poor predictors of local values. However, applying the ABVL approach systematically across sites can illustrate how the configuration of important processes for a particular location relates to global patterns in SES dynamics, and can advance our understanding of situations in which aggregate, inductive models are insufficient or when highly detailed deductive models are not necessary.

5. CONCLUSIONS

The synthesis capabilities of the ABVL approach can lead to new hypotheses and experiments to accelerate the development of generalized knowledge of SES dynamics. While the sophistication, empirical grounding, and predictive accuracy of case-based ABMs of SESs continues to grow, I argue that a complementary synthetic approach is needed to integrate local insights into more generalized and theory-oriented knowledge. The body of knowledge generated by case-based ABMs about the decision-making processes, social interactions, and adaptive behavior that drive SES dynamics is impressive and, in essence, map the possible variations of local processes that are difficult to generalize (e.g., social network structures and influence, land allocation under different land tenure regimes, and institutional arrangements). The ABVL approach can capitalize on this knowledge by generating and systematically testing new hypotheses beyond the specific locations or contexts to which the case-based ABM approach is currently limited. Ultimately, this approach enables a more integrated and dynamic global understanding of how and under what conditions driving forces of SES dynamics might differ locally from a more general model representation, which supports a more nuanced understanding of the global context and specific driving forces shaping particular regions.

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